Generation of three-dimensional waves by moving bottom disturbances

Hayk Nersisyan (BCAM)

BCAM,

25, January, 2013

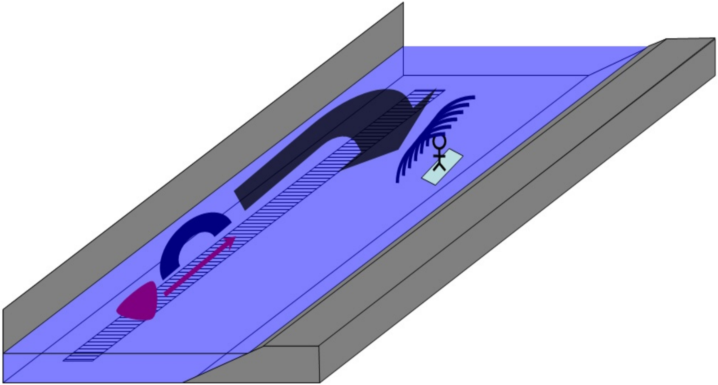

Figure: A model of wave maker

Mathematical model

[1] Generalized Boussinesq system describing the solitary waves generated by a moving bottom floor:

$$\eta_t + ((h+\eta)\,u)_x = -h_t,$$
$$u_t + uu_x + g\eta_x = -\frac{1}{\rho}p_x + \frac{1}{2}h\,(h_t + (hu)_x)_{xt} - \frac{1}{6}h^2 u_{xxt},$$

where $\eta(x,t)$ is the free-surface elevation, $u(x,t)$ is the depth-averaged velocity of the fluid, ρ and p are given density and pressure, and $h(x,t) = h_0 - \xi(x,t)$, where $\xi(x,t)$ is the topography of a moving body at depth h_0.

[1] T. Wu and Yao-Tsu. Generation of upstream advancing solutions by moving disturbances, Journal of Fluid Mechanics, 184:75–99, 1987.

[1] Generalized Boussinesq system describing the solitary waves generated by a moving bottom floor:

$$\eta_t + ((h+\eta)\,u)_x = -h_t,$$
$$u_t + uu_x + g\eta_x = -\frac{1}{\rho}p_x + \frac{1}{2}h\,(h_t + (hu)_x)_{xt} - \frac{1}{6}h^2 u_{xxt},$$

where $\eta(x,t)$ is the free-surface elevation, $u(x,t)$ is the depth-averaged velocity of the fluid, ρ and p are given density and pressure, and $h(x,t) = h_0 - \xi(x,t)$, where $\xi(x,t)$ is the topography of a moving body at depth h_0.

$$\eta_t + \left(\eta + \frac{3}{4}\eta^2 - \frac{1}{2}\xi\eta\right)_x - \frac{1}{6}\eta_{xxt} = -\frac{1}{4}\xi_{xtt}, \qquad (1)$$
$$\eta(x,0) = \eta_0(x). \qquad (2)$$

[1] T. Wu and Yao-Tsu. Generation of upstream advancing solutions by moving disturbances, Journal of Fluid Mechanics, 184:75–99, 1987.

We consider the following cost function

$$J_3(x_0, \xi_0) = \int_A^B (\eta(x, T) - \eta_f(x))^2 \mathrm{d}x,$$
$$\eta_f = (x - 2)\mathrm{sech}^2(x - 2).$$

Multi-dimensional case

[2] Let us consider

$$\eta_t + \nabla \cdot ((h+\eta)\, u) = -h_t,$$

$$u_t + g\nabla\eta + (u \cdot \nabla)u = \frac{h}{2}\nabla(\nabla \cdot h u_t) - \frac{h^2}{6}\nabla(\nabla \cdot u_t) + \frac{h}{2}\nabla h_{tt}.$$

[2] Peregrine 1967

FVCF(Finite volume characterized Flux)

Consider the following conservation of lows

$$\partial_t v + \partial_x f(v) = g.$$

Integrating over Ω_i we get

FVCF(Finite volume characterized Flux)
Consider the following conservation of lows

$$\partial_t v + \partial_x f(v) = g.$$

Integrating over Ω_i we get

$$\int_{\Omega_i} \partial_t v \mathrm{d}x + \int_{\Omega_i} \partial_x f(v) \mathrm{d}x = \int_{\Omega_i} g \mathrm{d}x.$$

For $v_i(t) = \frac{1}{|\Omega_i|} \int_{\Omega_i} v \mathrm{d}x$, we obtain

FVCF(Finite volume characterized Flux)
Consider the following conservation of lows

$$\partial_t v + \partial_x f(v) = g.$$

Integrating over Ω_i we get

$$\int_{\Omega_i} \partial_t v \mathrm{d}x + \int_{\Omega_i} \partial_x f(v) \mathrm{d}x = \int_{\Omega_i} g \mathrm{d}x.$$

For $v_i(t) = \frac{1}{|\Omega_i|} \int_{\Omega_i} v \mathrm{d}x$, we obtain

$$\partial_t v_i + \frac{1}{|\Omega_i|} \left(f(v)|_{i+1/2} - f(v)|_{i-1/2} \right) = g_i.$$

Let us introduce the numerical flux:

$$f(v)|_{i+1/2} = \Phi(v_i, v_{i+1}),$$

where

$$\Phi(v, w) = \frac{1}{2}(f(v) - f(w)) - U(\nu)\frac{f(w) - f(v)}{2},$$

Let us introduce the numerical flux:

$$f(v)|_{i+1/2} = \Phi(v_i, v_{i+1}),$$

where

$$\Phi(v,w) = \frac{1}{2}(f(v) - f(w)) - U(\nu)\frac{f(w) - f(v)}{2},$$

where $U = sign(A(\nu))$, $A = \frac{\partial f}{\partial v}$ and

$$sign(A) = Lsign(\Lambda)R$$

for $A = L\Lambda R$, $L = R^{-1}$

FVCF(Time discretization) [3] Strong Stabilization Method Preserving Runge-Kutta schemes (SSP-RK3).
Third order 4-stage scheme

$$
\begin{aligned}
V_1 &= V + \frac{1}{2} dtRHS(V); \\
V_2 &= V_1 + \frac{1}{2} dtRHS(V_1); \\
V_3 &= \frac{2}{3} V + \frac{1}{3} V_2 + \frac{1}{6} RHS(V_2) \\
U &= V_3 + \frac{1}{2} dtRHS(V_3).
\end{aligned}
$$

[3] from : Spiteri and Ruuth (2002), SIAM J. Numer. Anal.

Optimization using IPOPT (Interior Point OPTimizer)

https://projects.coin-or.org/Ipopt

Thank you for your attention!