

Thursday, January 31st, 12:00

Felix del Teso (BCAM)

THE LIOUVILLE THEOREM FOR NONLOCAL DIFFUSION OPERATORS (and its relation to irrational numbers and subgroups of \mathbb{R}^N)

The classical Liouville Theorem states that

$$u \in L^\infty(\mathbb{R}^N) \quad \& \quad \Delta u = 0 \quad \text{in } \mathbb{R}^N \quad \iff \quad u \equiv C \in \mathbb{R}.$$

In this talk we will revisit this result for the following class of nonlocal operators

$$\mathcal{L}^\mu[\psi](x) = \text{P.V.} \int_{|z|>0} (\psi(x+z) - \psi(x)) \, d\mu(z),$$

where μ is any positive symmetric Radon measure such that

$$\int_{\mathbb{R}^N} \min\{|z|^2, 1\} \, d\mu(z) < +\infty.$$

This class of operators naturally includes the fractional Laplacian $(-\Delta)^s$ for $s \in (0, 1)$, Relativistic Schrodinger operators $(-\Delta + m^2)^s - m^{2s}$, convolution operators $(J * \psi)(x) - \psi(x)$ as well as discretizations of both local and nonlocal symmetric diffusion operators.

First, we will treat the one dimensional case. Here we give a precise classification, in terms of the measure μ , for which the Liouville Theorem holds. The condition will be related to *irrational numbers* ([1]).

In \mathbb{R}^N such a characterization is also proved. This time it will be given in terms of the *group* generated by the support of the measure μ ([1]).

This nonlocal result will allow us ([2]) to give a full characterization of the Liouville property for any operator (local + nonlocal, and not necessarily symmetric) with constant coefficients satisfying the maximum principle (see [3]), i.e

$$\underbrace{\text{Tr}(\sigma\sigma^T D^2\psi(x)) + b \cdot \nabla\psi}_{\text{Local}} + \underbrace{\int_{|z|>0} (\psi(x+z) - \psi(x) - z \cdot \nabla\psi(x) \mathbf{1}_{|z|\leq 1}) \, d\nu(z)}_{\text{Nonlocal}}.$$

References

- [1] N. Alibaud, F. del Teso, J. Endal, and E. R. Jakobsen. *Characterization of nonlocal diffusion operators satisfying the Liouville theorem. Irrational numbers and subgroups of \mathbb{R}^d* . Preprint: arXiv:1807.01843.
- [2] N. Alibaud, F. del Teso, J. Endal, and E. R. Jakobsen. *The Liouville theorem and linear operators satisfying the maximum principle. A complete characterization in the constant coefficient case*. Work in progress.
- [3] P. Courrège. *Générateur infinitésimal d'un semi-groupe de convolution sur \mathbb{R}^n , et formule de Lévy-Khinchine*. Bull. Sci. Math. (2), 88:3–30, 1964.

Joint work with N. Alibaud (University of Besançon), J. Endal and E. R. Jakobsen (Norwegian University of Science and Technology).