

Thursday, February 21st, 17:00

Bruno Volzone (UniUniversità degli Studi di Napoli Parthenope)

RECENT RESULTS ON NONLINEAR AGGREGATION-DIFFUSION EQUATIONS: RADIAL SYMMETRY AND LONG TIME ASYMPTOTICS

One of the archetypical aggregation-diffusion models is the so-called classical parabolic-elliptic Patlak-Keller-Segel (PKS for short) model. This model was classically introduced as the simplest description for chemotactic bacteria movement in which linear diffusion tendency to spread fights the attraction due to the logarithmic kernel interaction in two dimensions. For this model there is a well-defined critical mass. In fact, here a clear dichotomy arises: if the total mass of the system is less than the critical mass, then the long time asymptotics are described by a self-similar solution, while for a mass larger than the critical one, there is finite time blow-up. In this talk we will show some recent results concerning the symmetry of the stationary states for a nonlinear variant of the PKS model, of the form

$$(1) \quad \partial_t \rho = \Delta \rho^m + \nabla \cdot (\rho \nabla (W * \rho)),$$

being $W \in C^1(\mathbb{R}^d \setminus \{0\})$, $d \geq 2$, a suitable aggregation kernel, in the assumptions of dominated diffusion, i.e. when $m > 2 - 2/d$. In particular, if W represents the classical logarithmic kernel in the two-dimensional case, we will show that there exists a unique stationary state for the model (1) and it coincides, according to one of the main results in the work [1], with the global minimizer of the free energy functional associated to (1). In the case $d = 2$ we will also show how such steady state coincides with the asymptotic profile of (1). Finally, we will also discuss some recent results concerning the model (1) with a Riesz potential aggregation, namely when $W(x) = c_{d,s}|x|^{2s-d}$ for $s \in (0, d/2)$, again in the diffusion dominated regime, i.e. for $m > 2 - (2s)/d$. In particular, all stationary states of the model are shown to be radially symmetric decreasing and that global minimizers of the associated free energy are compactly supported, uniformly bounded, Hölder regular, and smooth inside their support. These results are objects of the joint works [2], [3].

REFERENCES

- [1] J. A. CARRILLO, D. CASTORINA, B. VOLZONE, *Ground States for Diffusion Dominated Free Energies with Logarithmic Interaction*, SIAM J. Math. Anal. 47 (2015), no. 1, 1–25.
- [2] J. A. CARRILLO, S. HITTMEIR, B. VOLZONE, Y. YAO, *Nonlinear Aggregation-Diffusion Equations: Radial Symmetry and Long Time Asymptotics*, arXiv:1603.07767.
- [3] J. A. CARRILLO, F. HOFFMANN, E. MAININI, B. VOLZONE, *Ground States in the Diffusion-Dominated Regime*, arXiv:1705.03519.