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THE PLEMELJ JUMP FORMULA FOR SINGULAR INTEGRAL OPERATORS ON SHRINKING DOMAINS

Let $\Omega \subset \mathbb{R}^3$ be a bounded C^2 domain, $\Sigma := \partial\Omega$, σ the surface measure on Σ and ν outward (with respect to Ω) unit normal vector field on Σ. For $\epsilon > 0$, set $\Omega_\epsilon := \{x \in \mathbb{R}^3 : \text{dist}(x, \Sigma) < \epsilon\}$. Due to the regularity of Σ, there exists $\eta > 0$ so that for every $0 < \epsilon \leq \eta$, Ω_ϵ can be represented (uniquely) as follows:

$$\Omega_\epsilon := \{x_\Sigma + t\nu(x_\Sigma) : x_\Sigma \in \Sigma, t \in (-\epsilon, \epsilon)\}$$

For $t \in [-\eta, \eta]$, we set

$$\Sigma_t := \{x_\Sigma + t\nu(x_\Sigma) : \ x_\Sigma \in \Sigma\},$$

and let σ_t denote the surface measure on Σ_t and ν_t the normal vector field to Σ_t.

Let

$$k(x) := \frac{x}{4\pi|x|^3}, \quad \text{for } x \neq 0,$$

and for $(x_\Sigma, t), (y_\Sigma, s) \in \Sigma \times (-1, 1)$ set $x_{\epsilon t} = x_\Sigma + \epsilon t\nu(x_\Sigma)$, $y_{\epsilon s} = y_\Sigma + \epsilon s\nu(y_\Sigma)$.

For $\epsilon > 0$ small enough, let T_ϵ, $T : L^2(\Sigma \times (-1, 1)) \to L^2(\Sigma \times (-1, 1))$ be the bounded operators defined as follows:

$$T_\epsilon f(x_\Sigma, t) = \int_{-1}^{1} \int_{\Sigma_{\epsilon s}} k(x_{\epsilon t} - y_{\epsilon s}) \cdot \nu_{\epsilon s}(y_{\epsilon s}) f(y_\Sigma, s)\, d\sigma_{\epsilon s}(y_{\epsilon s})\, ds,$$

$$Tf(x_\Sigma, t) = \lim_{\delta \to 0} \int_{-1}^{1} \int_{|x_\Sigma - y_\Sigma| > \delta} k(x_\Sigma - y_\Sigma) \cdot \nu(y_\Sigma) f(y_\Sigma, s)\, d\sigma(y_\Sigma)\, ds + \frac{1}{2}\int_{-1}^{1} \text{sgn}(t-s) f(x_\Sigma, s)\, ds.$$

The aim of the talk is to prove that, for any $f \in L^2(\Sigma \times (-1, 1))$,

$$\lim_{\epsilon \to 0} ||T_\epsilon f - Tf||_{L^2} = 0.$$

A key ingredient of the proof is to relate T_ϵ to the Hardy-Littlewood maximal operator and some maximal singular integral operators from Calderón-Zygmund theory.