Thursday, May 3, 12:00 – 13:00

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THE KATO SQUARE ROOT PROBLEM FOR DIVERGENCE FORM OPERATORS WITH POTENTIAL

The Kato square root problem for divergence form elliptic operators is the equivalence statement $\left\Vert \sqrt{- \mathrm{div} \left(A \nabla \right)}u \right\Vert \simeq \left\Vert \nabla u \right\Vert$, where A is a complex matrix-valued function. In 2006, a few years after the first proof of this statement, A. Axelsson, S. Keith and A. McIntosh developed a general framework for proving square function estimates associated with Dirac-type operators and they showed that the Kato problem followed as an immediate application.

In this talk I will give an overview of the Kato square root problem and run through a sketch of the proof of Axelsson, Keith and McIntosh. I will then discuss a generalisation of the Kato problem to include positive potentials V, namely $\left\Vert \sqrt{- \mathrm{div} \left(A \nabla \right) + V}u \right\Vert \simeq \left\Vert \nabla u \right\Vert + \left\Vert V^{\frac{1}{2}} u \right\Vert$. I will discuss how the Axelsson-Keith-McIntosh framework can be altered to allow for dependence of the Dirac-type operator on the potential. The Kato estimate for certain potentials will then follow as a result.