

Geometry and analysis on the Heisenberg group

We give a short introduction to the theory of Lie groups, in particular the definition of invariant vector fields.

We give examples of invariant sub-Riemannian structures on Lie groups, in particular examples related to non-holonomic systems. We define the sub-Riemannian distance.

We then define hypoelliptic heat operators on sub-Riemannian manifolds, and give examples of results connecting the knowledge about sub-Riemannian distance and the behavior of solutions of hypoelliptic heat equations.

We finally study the example of the Heisenberg group in detail.