FRACTAL PROPERTIES OF GENERALIZED BESSEL FUNCTIONS

The fractal oscillatority of solutions $x = x(t)$ of ordinary differential equations at $t = \infty$ is measured by oscillatory and phase dimensions defined through the box dimension. Oscillatory and phase dimensions are defined as box dimensions of the graph of $X(\tau) = x(1/\tau)$ near $\tau = 0$ and trajectory $(x, \dot{x})$ in $\mathbb{R}^2$, respectively, assuming that $(x, \dot{x})$ is a spiral converging to the origin. The box dimension of a plane curve measures the accumulation of a curve near a point, which is in particular interesting for non-rectiable curves. The oscillatory dimension of solutions of Bessel equation has been determined by Pasic and Tanaka (2011). Here, we compute the phase dimension of solutions of a class of ordinary differential equations, including Bessel equation. These solutions we call generalized Bessel functions. The phase dimension of Bessel functions has been computed to be equal to $4/3$. More generally, we determined the box dimension of a specific type of spirals that we called wavy spirals, which are nonmonotonously converging to the origin.

Computation of the phase dimension of generalized Bessel functions use asymptotic expansions of Bessel functions. Some expressions in this computation have very large number of terms. It would be practically impossible to make this computation by hand. So, here we explain the usage of the software for symbolic computation, Wolfram Mathematica, for this job.