Numerical performance of a stabilized DG method with plane waves and Lagrange multipliers for 2D Helmholtz problems

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Motivation and context
Outline

- Motivation and context
- A new DG solution methodology for Helmholtz problems

Magdalena Grigoroscultu-Strugaru

Euskadi-Kyushu Workshop, March 2011
Motivation and context

A new DG solution methodology for Helmholtz problems

Solution strategy
Motivation and context

A new DG solution methodology for Helmholtz problems

Solution strategy

Computation complexity
Outline

- Motivation and context
- A new DG solution methodology for Helmholtz problems
  - Solution strategy
  - Computation complexity
  - Performance assessment
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- Motivation and context
- A new DG solution methodology for Helmholtz problems
  - Solution strategy
  - Computation complexity
  - Performance assessment
- Summary and perspectives
Applications
Applications

- Radar
Applications

- Radar
- Sonar
Motivation and Context

Applications

- Radar
- Sonar
- Medical imaging
Applications

- Radar
- Sonar
- Medical imaging
- Nondestructive testing
Applications

- Radar
- Sonar
- Medical imaging
- Nondestructive testing
- Geophysical exploration
Motivation and Context

Numerical Difficulties

$k\alpha = 1$
Numerical Difficulties

\[ k \alpha = 1, \quad \frac{h}{a} = \frac{1}{10} \]
Numerical Difficulties

\[ k\alpha = 1, \quad \frac{h}{a} = \frac{1}{10} \]
Numerical Difficulties

\[ k a = 1, \quad \frac{h}{a} = \frac{1}{10} \]
$k\alpha = 3$
Numerical Difficulties

$k\alpha = 3, \quad \frac{h}{a} = \frac{1}{10}$
Numerical Difficulties

\[ k\alpha = 3, \quad \frac{h}{a} = \frac{1}{20} \]
Numerical Difficulties

\[ k\alpha = 3, \quad \frac{h}{a} = \frac{1}{30} \]
Numerical Difficulties

\[ ka = 3, \quad \frac{h}{a} = \frac{1}{30} \quad \Rightarrow \quad kh = \frac{1}{10} \]
Numerical Difficulties

Resolution necessary to achieve 10% on the relative error
Numerical Difficulties

Resolution necessary to achieve 10% on the relative error

⚠️ $kh \neq \text{constant}$
Numerical Difficulties

\[ \frac{|u-u^h|_1}{|u|_1} \leq C_1 k h + C_2 k^3 h^2; \quad k h < 1 \]

(Babuška et al (95, 00))
Motivation and Context

Numerical Difficulties

“Realistic” simulation (Tezaur et al (02))

\[ ka = 10 \]

System of about 9.6 million complex unknowns
Motivation and Context

Prominent Plane Waves Based Approaches

- Weak Element Method
  Rose (75)

- Partition of Unity Method
  Babuška-Melenk (97), Laghouache-Bettes (00)

- Ultra-Weak Variational Method
  Cessenat-Desprès (98)

- Least-Squares Method (LSM)
  Monk-Wang (99)

- Trefftz-Type Wave-Based Method
  Desmet et al (98, 02, 10), Stojek (98)

- Discontinuous Galerkin Method (DGM)
  Farhat et al (01, 03, 04, 05)
Motivation and Context

DGM Formulation (Farhat et al)
DGM Formulation (Farhat et al)
Mathematical model

\[ \Delta u + k \sum u = 0 \quad \text{in} \quad \Omega \]
\[ \partial_n u = iku + g \quad \text{on} \quad \Sigma \]
Motivation and Context

DGM Formulation (Farhat et al)

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Motivation and Context

DGM Formulation (Farhat et al)

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\[ \partial_n u = iku + g \quad \text{on} \quad \Sigma \]
Motivation and Context

DGM Formulation (Farhat et al)

Approximation

\[
\begin{align*}
    u & \equiv \sum_{l=1}^{4} u_{Kl} \phi_{Kl} \\
    \phi_{Kl} & = e^{i k x \cdot d_l} \\
    \lambda & \in \mathbb{C} \cap \partial K \cap \partial K'
\end{align*}
\]
DGM Formulation (Farhat et al)

Approximation

\[ u \approx \sum_{l=1}^{L} u_{K_l} \phi_{K_l} \in K \]

\[ \phi_{K_l} = e^{ikx \cdot d_l} \lambda \in C \cap \partial K \cap \partial K' \]
Motivation and Context

DGM Formulation (Farhat et al)

Approximation

\[ u \equiv \sum_{l=1}^{4} u^K_l \phi^K_l \text{ in } K \]

\[ \phi^K_l = e^{ikx \cdot d_l} \]
Motivation and Context

DGM Formulation (Farhat et al)

Approximation

\[ u \equiv \sum_{l=1}^{4} u^K_l \phi^K_l \text{ in } K \]

\[ \phi^K_l = e^{ikx \cdot d_l} \]

\[ \lambda = \lambda^K = -\lambda^{K'} \in \mathbb{C} \text{ on } \partial K \cap \partial K' \]
Motivation and Context

DGM Formulation (Farhat et al)

Variational Formulation

\[
\begin{align*}
\left\{ \begin{array}{c}
a(u, v) + b(v, \lambda) &= F(v) \\ b(u, \mu) &= 0
\end{array} \right.
\end{align*}
\]

\[
a(u, v) = \sum_{K \in \tau_h} \left( \int_K (\nabla u \cdot \nabla v - k^2 u v) \, dx - ik \int_{\partial K \cap \Sigma} uv \, ds \right)
\]
DGM Formulation (Farhat et al)

Variational Formulation

\[ a(u, v) + b(v, \lambda) = F(v) \]
\[ b(u, \mu) = 0 \]

\[ b(v, \mu) = \sum_{K \in \tau_h} \int_{\partial K \cap \partial K'} \mu v ds \]
DGM Formulation (Farhat et al)

Variational Formulation

\[
\begin{align*}
\begin{cases}
  a(u, v) + b(v, \lambda) &= F(v) \\
  b(u, \mu) &= 0
\end{cases}
\end{align*}
\]

\[
F(v) = \sum_{K \in \tau_h} \int_{\partial K \cap \Sigma} gvds
\]
Motivation and Context

DGM Formulation (Farhat et al)

Algebraic Formulation

\[ Au + B\lambda = f \]
\[ B^T u = 0 \]
DGM Formulation (Farhat et al)

Algebraic Formulation

\[
\begin{align*}
Au + B\lambda &= f \\
B^T u &= 0
\end{align*}
\]

\[\implies B^T A^{-1} B \lambda = B^T A^{-1} f\]
DGM Formulation (Farhat et al)

Main Features
DGM Formulation (Farhat et al)

Main Features

- Plane waves for local approximation of the field
Motivation and Context

DGM Formulation (Farhat et al)

Main Features

- Plane waves for local approximation of the field
- Lagrange multipliers to enforce continuity
Motivation and Context

DGM Formulation (Farhat et al)

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- Plane waves for local approximation of the field
- Lagrange multipliers to enforce continuity
- Analytical evaluation of the matrices
Motivation and Context

DGM Formulation (Farhat et al)

Main Features

- Plane waves for local approximation of the field
- Lagrange multipliers to enforce continuity
- Analytical evaluation of the matrices
- Global system: symmetric and sparse
Motivation and Context

DGM Formulation (Farhat et al)

Main Features

- **Plane waves** for local approximation of the field
- **Lagrange multipliers** to enforce continuity
- **Analytical** evaluation of the matrices
- **Global system**: symmetric and sparse
- **Size** of the global system \( \equiv \# \text{ dofs for the Lagrange multiplier} \)
Motivation and Context

DGM Formulation (Farhat et al)

2D Numerical Performance
Motivation and Context

DGM Formulation (Farhat et al)

2D Numerical Performance

DGM outperforms high-order FE methods:
DGM Formulation (Farhat et al)

2D Numerical Performance

DGM outperforms high-order FE methods:

- **R-4-1, R-8-2** require 5 to 7 times fewer dof than **Q2**
DGM Formulation (Farhat et al)

2D Numerical Performance

DGM outperforms high-order FE methods:

- **R-4-1, R-8-2** require 5 to 7 times fewer dof than **Q2**

- **Q-16-4** requires 6 times fewer dof than **Q4**
DGM Formulation (Farhat et al)

2D Numerical Performance

DGM outperforms high-order FE methods:

- **R-4-1, R-8-2** require 5 to 7 times fewer dof than **Q2**
- **Q-16-4** requires 6 times fewer dof than **Q4**
- **Q-32-8** requires 25 times fewer dof than **Q4**
DGM Formulation (Farhat et al)

Issues
Motivation and Context

DGM Formulation (Farhat et al)

Issues

Inf-Sup condition: Discrete spaces compatibility
DGM Formulation (Farhat et al)

Issues

Inf-Sup condition: Discrete spaces compatibility

# plane waves vs. # Lagrange multipliers
Motivation and Context

DGM Formulation (Farhat et al)

Issues

Inf-Sup condition: Discrete spaces compatibility

Relative error, $kh=1/2$

R-8-2 element
Motivation and Context

DGM Formulation (Farhat et al)

Issues

Inf-Sup condition: Discrete spaces compatibility

Relative error, $kh=1/2$
Motivation and Context

DGM Formulation (Farhat et al)

Issues

Inf-Sup condition: Numerical instabilities

Total relative error, $ka=1$
Objective: build on top of DGM
Objective: build on top of DGM

- Preserve the good features of DGM
Objective: build on top of DGM

- Preserve the good features of DGM
- Overcome the numerical instabilities
DGM Formulation: Another point of view

\[\begin{align*}
\Delta u + k^2 u &= 0 \quad \text{in } \Omega \\
\partial_n u &= ik u + g \quad \text{on } \Sigma
\end{align*}\]
DGM Formulation: Another point of view

\[
\begin{align*}
\Delta u + k^2 u &= 0 \quad \text{in } \Omega \\
\partial_n u &= iku + g \quad \text{on } \Sigma
\end{align*}
\]

- Split the solution \( u \):

\[ u = \phi + \Phi(\lambda) \]
DGM Formulation: Another point of view

\[
\begin{align*}
\Delta \varphi^K + k^2 \varphi^K &= 0 \quad \text{in } K \\
\Delta \Phi^K(\lambda) + k^2 \Phi^K(\lambda) &= 0 \quad \text{in } K
\end{align*}
\]
DGM Formulation: Another point of view

\[ \begin{align*}
\Delta \phi^K + k^2 \phi^K &= 0 \quad \text{in } K \\
\partial_n \phi^K &= ik\phi^K + g \quad \text{on } \partial K \cap \Sigma
\end{align*} \]

\[ \begin{align*}
\Delta \Phi^K(\lambda) + k^2 \Phi^K(\lambda) &= 0 \quad \text{in } K \\
\partial_n \Phi^K(\lambda) &= ik\Phi^K(\lambda) \quad \text{on } \partial K \cap \Sigma
\end{align*} \]
DGM Formulation: Another point of view

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\begin{align*}
\Delta \varphi^K + k^2 \varphi^K &= 0 \quad \text{in } K \\
\partial_n \varphi^K &= ik \varphi^K + g \quad \text{on } \partial K \cap \Sigma \\
\partial_n \varphi^K &= 0 \quad \text{on } \partial K \cap \Omega
\end{align*}
\]

\[
\begin{align*}
\Delta \Phi^K(\lambda) + k^2 \Phi^K(\lambda) &= 0 \quad \text{in } K \\
\partial_n \Phi^K(\lambda) &= ik \Phi^K(\lambda) \quad \text{on } \partial K \cap \Sigma \\
\partial_n \Phi^K(\lambda) &= \lambda \quad \text{on } \partial K \cap \Omega
\end{align*}
\]
DGM Formulation: Another point of view

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\Delta \varphi^K + k^2 \varphi^K &= 0 \quad \text{in } K \\
\partial_n \varphi^K &= ik \varphi^K + g \quad \text{on } \partial K \cap \Sigma \\
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\partial_n \Phi^K(\lambda) &= \lambda \quad \text{on } \partial K \cap \hat{\Omega}
\end{align*} \]

\[ \lambda = \lambda^K = -\lambda^{K'} \quad \text{on } \partial K \cap \partial K' \]
DGM Formulation: Another point of view

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\begin{align*}
\Delta \varphi^K + k^2 \varphi^K &= 0 & \text{in } K \\
\partial_n \varphi^K &= i k \varphi^K + g & \text{on } \partial K \cap \Sigma \\
\partial_n \varphi^K &= 0 & \text{on } \partial K \cap \hat{\Omega}
\end{align*}
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\[
\begin{align*}
\Delta \Phi^K(\lambda) + k^2 \Phi^K(\lambda) &= 0 & \text{in } K \\
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\partial_n \Phi^K(\lambda) &= \lambda & \text{on } \partial K \cap \hat{\Omega}
\end{align*}
\]

\[
\lambda = \lambda^K = -\lambda^K' \text{ on } \partial K \cap \partial K'
\]

\[
[u] = [\varphi + \Phi(\lambda)] = 0 \text{ on each interior edge}
\]
DGM Formulation: Another point of view

★ Solve local variational problems in each $K$: 

$$\int_{\partial K} (\partial_n v - i kv \chi_\Sigma) w = L(w)$$
DGM Formulation: Another point of view

★ Solve local variational problems in each $K$:

$$\int_{\partial K} (\partial_n \nu - 0)w = L(w)$$

if $K$ is an interior element
DGM Formulation: Another point of view

★ Solve local variational problems in each $K$:

$$\int_{\partial K} (\partial_n v - i k v \chi_\Sigma) w = L(w)$$

★ Solve one global variational problem:

$$\sum_{e \subset \hat{\Omega}} \frac{1}{|e|} \int_e [\Phi(\lambda)] \mu = - \sum_{e \subset \hat{\Omega}} \frac{1}{|e|} \int_e [\varphi] \mu$$
Solution strategy: two key features
Solution strategy: Feature 1
Solution strategy: Feature 1

Modify the local problems:

\[ \begin{align*}
\Delta \varphi^K + k^2 \varphi^K &= 0 \quad \text{in } K \\
\partial_n \varphi^K - i k \varphi^K &= g \quad \text{on } \partial K \cap \Sigma \\
\partial_n \varphi^K &= 0 \quad \text{on } \partial K \cap \hat{\Omega}
\end{align*} \]

\[ \begin{align*}
\Delta \Phi^K(\lambda) + k^2 \Phi^K(\lambda) &= 0 \quad \text{in } K \\
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\partial_n \Phi^K(\lambda) &= \lambda \quad \text{on } \partial K \cap \hat{\Omega}
\end{align*} \]
A new DG solution methodology

Solution strategy: Feature 1

Modify the local problems:

\[
\begin{align*}
\Delta \varphi^K + k^2 \varphi^K &= 0 \quad \text{in} \ K \\
\partial_n \varphi^K - ik \varphi^K &= g \quad \text{on} \ \partial K \cap \Sigma \\
\partial_n \varphi^K - i \alpha \varphi^K &= 0 \quad \text{on} \ \partial K \cap \hat{\Omega}
\end{align*}
\]

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\]

Gain: Well-posed local problems for \( \alpha \in \mathbb{R}^*_+ \)
Solution strategy: Feature 1

Modify the local problems:

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\partial_n \Phi^K(\lambda) - i \alpha \Phi^K(\lambda) &= \lambda \quad \text{on} \ \partial K \cap \hat{\Omega}
\end{aligned} \]

Gain: Well-posed local problems for \( \alpha \in \mathbb{R}^*_+ \)

Price: \( \lambda = \lambda^K \neq -\lambda^{K'} \) on \( \partial K \cap \partial K' \)

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Euskadi-Kyushu Workshop, March 2011
Solution strategy: Feature 1
Solution strategy: Feature 1

Local variational problems:

\[ \int_{\partial K} (\partial_n v - i k u \chi \Sigma) w \, ds = l(w) \]
Solution strategy: Feature 1

Local variational problems:

\[ \int_{\partial K} (\partial_n v - i k v)(\partial_n \bar{w} + i k \bar{w}) \, ds = l(w) \]
Solution strategy: Feature 1

Local variational problems:

\[ \int_{\partial K} (\partial_n v - i \ k v)(\partial_n \overline{w} + i \ k \overline{w}) \ ds = l(w) \]

Gain: Hermitian and positive definite local matrix
A new DG solution methodology

Solution strategy: Feature 2
Solution strategy: Feature 2

💡 Restore the continuity of the field and its normal derivative in a different manner:

\[
A(\lambda, \mu) = \sum_{e \subset \Omega} \frac{1}{|e|} \int_e [\Phi(\lambda)] \mu
\]
A new DG solution methodology

Solution strategy: Feature 2

💡 Restore the continuity of the field and its normal derivative in the least-squares sense:

\[
\lambda = \arg\min_{\mu} \sum_{e \subset \Omega} \left( \beta_e \| [\varphi + \Phi(\mu)] \|_{0,e}^2 + \gamma_e \| [\partial_n \varphi + \partial_n \Phi(\mu)] \|_{0,e}^2 \right)
\]
A new DG solution methodology

Solution strategy: Feature 2

 Restore the continuity of the field and its normal derivative in the least-squares sense:

\[
A(\lambda, \mu) = \sum_{e \subset \Omega} \left( \beta_e \int_e [\Phi(\lambda)] [\Phi(\mu)] + \gamma_e \int_e [[\partial_n \Phi(\lambda)]] [[\partial_n \Phi(\mu)]] \right)
\]
Solution strategy: Feature 2

 Restore the continuity of the field and its normal derivative in the least-squares sense:

\[ A(\lambda, \mu) = \sum_{e \subset \Omega} \left( \beta_e \int_e [\Phi(\lambda)][\Phi(\mu)] + \gamma_e \int_e [[\partial_n \Phi(\lambda)]] [[\partial_n \Phi(\mu)]] \right) \]

Gain: Hermitian and positive semi-definite global matrix

Price: Stencil width larger than in DGM
Solution strategy: Summary
Solution strategy: Summary

- **Step 1**: Compute $\varphi^K$ and $\Phi^K(\mu)$ for each $\mu$ by solving local variational problems in each $K$:
  \[ a_K(v, w) = l_K(w) \]

  with
  \[ a_K(v, w) = \int_{\partial K} (\partial_n v - i k v)(\partial_n \bar{w} + i k \bar{w}) \]
Solution strategy: Summary

- **Step 1:** Compute $\varphi^K$ and $\Phi^K(\mu)$ for each $\mu$ by solving local variational problems in each $K$:
  
  $$a_K(v, w) = l_K(w)$$

  with
  
  $$a_K(v, w) = \int_{\partial K} (\partial_n v - i kv)(\partial_n \bar{w} + i k\bar{w})$$

- **Algebraic level:** solve one linear system
  
  ★ $n^K \times n^K$ Hermitian positive definite matrix
  ★ $n^K = \#$ plane waves
  ★ multiple right-hand side
Solution strategy: Summary

- **Step 2:** Compute the Lagrange multiplier $\lambda$ by solving one global variational problem:

$$A(\lambda, \mu) = L(\mu)$$

with

$$A(\lambda, \mu) = \sum_{e \subset \hat{\Omega}} \left( \beta_e \int_e [\Phi(\lambda)][\overline{\Phi(\mu)}] 
+ \gamma_e \int_e [[\partial_n \Phi(\lambda)]][[\partial_n \overline{\Phi(\mu)}]] \right)$$
Solution strategy: Summary

- **Step 2**: Compute the Lagrange multiplier $\lambda$ by solving one global variational problem:

$$A(\lambda, \mu) = L(\mu)$$

with

$$A(\lambda, \mu) = \sum_{e \subset \tilde{\Omega}} \left( \beta_e \int_e [\Phi(\lambda)][\Phi(\mu)] ight) + \gamma_e \int_e [[\partial_n \Phi(\lambda)][[\partial_n \Phi(\mu)]]$$

- Algebraic level: solve a sparse linear system
  - $n^\lambda \times n^\lambda$ Hermitian positive semi-definite matrix
  - $n^\lambda = \#$ Lagrange multipliers
A new DG solution methodology

Comparison with LSM (Monk et al)
Comparison with LSM (Monk et al)

- The proposed method:

\[ A(\lambda, \mu) = \sum_{e \subset \hat{\Omega}} \left( \beta_e \int_e [\Phi(\lambda)]\overline{[\Phi(\mu)]} + \gamma_e \int_e [[\partial_n \Phi(\lambda)]]\overline{[[\partial_n \Phi(\mu)]]} \right) \]

- Least-Squares Method (LSM)

\[ A(u, v) = \sum_{e \subset \hat{\Omega}} \left( k^2 \int_e [u][v] + \int_e [[\partial_n u]]\overline{[[\partial_n v]]} \right) \]

No local problems required!
A new DG solution methodology

Computation complexity
A new DG solution methodology

Computation complexity

- Uniform $n \times n$ mesh
A new DG solution methodology

Computation complexity

- Uniform $n \times n$ mesh
- $n^K$ shape functions at the element level
A new DG solution methodology

Computation complexity

- Uniform $n \times n$ mesh
- $n^K$ shape functions at the element level
- $n^{\lambda^K}$ dofs for the Lagrange multiplier on each interior edge of the element $K$
A new DG solution methodology

**Computation complexity**

- **Uniform** $n \times n$ mesh
- $n^K$ shape functions at the element level
- $n^{\lambda K}$ dofs for the Lagrange multiplier on each interior edge of the element $K$

<table>
<thead>
<tr>
<th>Method</th>
<th>Asymptotic size of the solution vector</th>
<th>Stencil width</th>
</tr>
</thead>
<tbody>
<tr>
<td>New method</td>
<td>$4n^{\lambda K} n^2$</td>
<td>$20n^{\lambda K}$</td>
</tr>
<tr>
<td>DGM</td>
<td>$2n^{\lambda K} n^2$</td>
<td>$7n^{\lambda K}$</td>
</tr>
<tr>
<td>LSM</td>
<td>$n^K n^2$</td>
<td>$5n^K$</td>
</tr>
</tbody>
</table>
A new DG solution methodology

Performance assessment
A new DG solution methodology

Performance assessment
Comparison with DGM

Plane wave solution: $u = e^{ikx \cdot d^*}$, $d^* = (\cos \theta^*, \sin \theta^*)$
A new DG solution methodology

Performance assessment
Comparison with DGM

Plane wave solution: \( u = e^{ikx \cdot d^*} \), \( d^* = (\cos \theta^*, \sin \theta^*) \)

- Modified \( H^1 \) norm:

\[
\| u - u_h \|_{\hat{H}^1} = \sqrt{\sum_{K \in \mathcal{T}_h} \| u - u_h \|^2_{H^1(K)} + \sum_{e-interior} \| [u_h] \|^2_{L^2(e)}}
\]
A new DG solution methodology

Compatibility condition
A new DG solution methodology

Compatibility condition

DGM

Relative error, $kh=1/2$
A new DG solution methodology

Compatibility condition

The proposed method

Relative error, $\text{kh}=1/2$

R-8-2 element

R-8-3 element
A new DG solution methodology

Compatibility condition

Relative error, $kh=1/2$
A new DG solution methodology

Sensitivity to the mesh refinement

Total relative error, $ka=1$

R-8-2 element

R-8-3 element
A new DG solution methodology

Source of the oscillations

The smallest eigenvalue, $ka=1$

8 plane waves
A new DG solution methodology

Source of the oscillations

The smallest eigenvalue, $ka=1$

Loss of the linear independence
Source of the oscillations

The smallest eigenvalue, $ka=1$
Sensitivity to the mesh refinement

Total relative error, $ka=1$

The proposed method, R-8-3
The proposed method, R-7-2
Sensitivity to the mesh refinement

Total relative error, $ka=1$
A new DG solution methodology

Sensitivity to the mesh refinement

Total relative error, $ka=20$
A new DG solution methodology

Application in geophysical exploration
Application in geophysical exploration

- **Objective**: produce images of the subsurface from tomography measurements
A new DG solution methodology

Application in geophysical exploration

- **Objective**: produce images of the subsurface from tomography measurements
A new DG solution methodology

Application in geophysical exploration

- **Objective:** produce images of the subsurface from tomography measurements
- **Wave propagation in time domain:** Discrete Fourier Transform
A new DG solution methodology

Application in geophysical exploration

- **Objective:** produce images of the subsurface from tomography measurements

- Wave propagation in **time** domain: Discrete Fourier Transform

- Solve Helmholtz equation (reduced wave equation)
A new DG solution methodology

Application in geophysical exploration

- **Objective:** produce images of the subsurface from tomography measurements
- Wave propagation in **time** domain: Discrete Fourier Transform
- Solve Helmholtz equation (reduced wave equation)
- Build the solution in **time** domain: Inverse Discrete Fourier Transform
A new DG solution methodology

Application in geophysical exploration

Stratified medium

\[ C_1 = 1000 \text{ m/s} \]

\[ C_2 = 1500 \text{ m/s} \]
A new DG solution methodology

Application in geophysical exploration

R-4-1 element

\[ \frac{1}{50} \leq kh \leq 2 \]

Stratified medium

\[ C_1 = 1000 \text{ m/s} \]

\[ C_2 = 1500 \text{ m/s} \]
A new DG solution methodology

Application in geophysical exploration

R-4-1 element

\[ \frac{1}{50} \leq kh \leq 2 \]

Stratified medium

\begin{align*}
C_1 &= 1000 \text{ m/s} \\
C_2 &= 1500 \text{ m/s}
\end{align*}
A new DG solution methodology

Application in geophysical exploration

R-4-1 element

\[ \frac{1}{50} \leq kh \leq 2 \]

Stratified medium

\[ C_1 = 1000 \text{ m/s} \]

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A new DG solution methodology

Application in geophysical exploration

$\frac{1}{50} \leq kh \leq 2$

R-4-1 element

Stratified medium

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A new DG solution methodology

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Application in geophysical exploration

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Stratified medium

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A new DG solution methodology

Application in geophysical exploration

R-4-1 element
\[ \frac{1}{50} \leq k h \leq 2 \]

Stratified medium

\[ C_1 = 1000 \text{ m/s} \]
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A new DG solution methodology

Application in geophysical exploration

R-4-1 element

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A new DG solution methodology

Application in geophysical exploration

R-4-1 element

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A new DG solution methodology

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Application in geophysical exploration

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Stratified medium

C_1 = 1000 m/s

C_2 = 1500 m/s

R-4-1 element
A new DG solution methodology

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Application in geophysical exploration

R-4-1 element

\[ \frac{1}{50} \leq \frac{k h}{1} \leq 2 \]

Stratified medium

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A new DG solution methodology

Application in geophysical exploration

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Application in geophysical exploration

$\frac{1}{50} \leq kh \leq 2$

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Application in geophysical exploration

*R-4-1 element*

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**R-4-1 element**

\[ \frac{1}{50} \leq kh \leq 2 \]

Stratified medium

- \( C_1 = 1000 \text{ m/s} \)
- \( C_2 = 1500 \text{ m/s} \)
A new DG solution methodology

Application in geophysical exploration

- Multi-frequency solver; same mesh!
A new DG solution methodology

Application in geophysical exploration

- Multi-frequency solver; same mesh!
- Resolution: 3 to 300 elements per wavelength
A new DG solution methodology

Application in geophysical exploration

- Multi-frequency solver; same mesh!
- Resolution: 3 to 300 elements per wavelength
- R-4-1 element: accurate and stable element
Sensitivity to the frequency
### Sensitivity to the frequency

<table>
<thead>
<tr>
<th>ka</th>
<th>R-7-2</th>
<th>R-11-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td></td>
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<tr>
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<td></td>
<td></td>
</tr>
<tr>
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Total relative error, $kh=2$

- **R-7-2 element**
- **R-11-3 element**

Computational cost increased by 50%

Gain of more than two orders of magnitude
Sensitivity to the frequency

<table>
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Total relative error, $kh=2$

R-7-2 element

R-11-3 element
A new DG solution methodology

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Total relative error, $kh=2$

- R-7-2 element
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Total relative error, $kh=2$

R-7-2 element

R-11-3 element
A new DG solution methodology

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Total relative error, $kh=2$

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Total relative error, $kh=2$

$\Rightarrow$ Use higher-order elements for higher frequencies
A new DG solution methodology

Sensitivity to the mesh refinement
High-frequency regime

Total relative error, $ka=200$
Sensitivity to the mesh refinement
High-frequency regime

Total relative error, $ka = 400$
Resolution vs. fixed accuracy

<table>
<thead>
<tr>
<th>Level of accuracy</th>
<th>R-11-3</th>
<th>R-13-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>1.88</td>
<td>1.32</td>
</tr>
<tr>
<td>5%</td>
<td>2.10</td>
<td>1.44</td>
</tr>
<tr>
<td>1%</td>
<td>2.51</td>
<td>1.73</td>
</tr>
</tbody>
</table>

\[ k_a = 200 \]
A new DG solution methodology

Resolution vs. fixed accuracy

<table>
<thead>
<tr>
<th>Level of accuracy</th>
<th># degrees of freedom</th>
<th>R-11-3</th>
<th>R-13-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>42,480</td>
<td>27,552</td>
<td></td>
</tr>
<tr>
<td>5%</td>
<td>53,064</td>
<td>33,120</td>
<td></td>
</tr>
<tr>
<td>1%</td>
<td>75,840</td>
<td>47,520</td>
<td></td>
</tr>
</tbody>
</table>

\( \text{ka} = 200 \)

Computational cost reduced by about 40%
A new DG solution methodology

Resolution vs. fixed accuracy

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\( ka = 200 \)

Computational cost reduced by about 40%
## Resolution vs. fixed accuracy

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<tr>
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<td>R-11-3: 1.88, R-13-4: 1.49</td>
</tr>
<tr>
<td>5%</td>
<td>R-11-3: 2.43, R-13-4: 1.60</td>
</tr>
<tr>
<td>1%</td>
<td>R-11-3: 2.82, R-13-4: 1.99</td>
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\[ ka = 400 \]

**R-11-3 element**

**R-13-4 element**
A new DG solution methodology

Resolution vs. fixed accuracy

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<td></td>
<td>R-11-3</td>
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<tr>
<td>10%</td>
<td>171,360</td>
<td>142,880</td>
</tr>
<tr>
<td>5%</td>
<td>286,440</td>
<td>164,832</td>
</tr>
<tr>
<td>1%</td>
<td>386,640</td>
<td>256,032</td>
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$\text{R-11-3 element}$

$\text{R-13-4 element}$

Computational cost reduced by about $40\%$
Resolution vs. fixed accuracy

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\[ ka = 400 \]

⇒ Higher-order elements rather than refinement
A new DG solution methodology

Sensitivity to the mesh distortion

DGM, $Q$-8-2

The proposed method, $Q$-7-2

Relative error, $ka=30$, $\theta^* = 67.5^\circ$
A new DG solution methodology

Sensitivity to the mesh distortion

DGM, $Q$-8-2

The proposed method, $Q$-7-2

Relative error, $ka=30, \theta^* = 67.5^\circ$
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DGM, $Q$-8-2

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Relative error, $ka = 30$, $\theta^* = 67.5^\circ$
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DGM, $Q$-8-2

The proposed method, $Q$-7-2

Relative error, $k\alpha=30$, $\theta^* = 67.5^\circ$
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Sensitivity to the mesh distortion

DGM, \(Q\)-8-2

The proposed method, \(Q\)-7-2

Relative error, \(ka=30, \theta^* = 67.5^\circ\)
Summary and Perspectives

A new DG solution methodology

Small systems: Hermitian and positive definite

Global system: Hermitian, sparse and positive semi-definite

Performance: Promising numerical results (accuracy and stability)

Adaptive strategy (mesh and order of the element): easy to adopt and implement
A new DG solution methodology
A new DG solution methodology

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Magdalena Grigorescuta-Strugaru
Euskadi-Kyushu Workshop, March 2011
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Summary and Perspectives

Ongoing work

- Test other shape functions
Ongoing work

- Test other shape functions
- Perform the mathematical analysis
Ongoing work

- Test other shape functions
- Perform the mathematical analysis
- Analyze the performance for 2D scattering problems
Ongoing work: scattering problems

\[ \Delta u_{\text{scat}} + k^2 u_{\text{scat}} = 0 \text{ in } \Omega \]
\[ \partial_n u_{\text{scat}} = i k u_{\text{scat}} \text{ on } \Sigma \]
\[ \partial_n u_{\text{inc}} = -\partial_n u_{\text{scat}} \text{ on } \Gamma \]
Ongoing work: scattering problems

\[ \begin{align*}
\Delta u^{\text{scat}} + k^2 u^{\text{scat}} &= 0 \quad \text{in } \Omega^c \\
\partial_n u^{\text{scat}} &= iku^{\text{scat}} \quad \text{on } \Sigma \\
\partial_n u^{\text{scat}} &= -\partial_n u^{\text{inc}} \quad \text{on } \Gamma
\end{align*} \]
Ongoing work: scattering problems

Jump error, $ka=1$: $1.38 \cdot 10^{-1}$

R-8-2 element
Ongoing work: scattering problems

Jump error, $ka=1$: $3.40 \cdot 10^{-2}$
Ongoing work: scattering problems

Jump error, $ka=1$: $7.99 \cdot 10^{-3}$
Summary and Perspectives

Mid-term goals

- Three-dimensional acoustic scattering problems
Mid-term goals

- Three-dimensional acoustic scattering problems
- Elasto-acoustic scattering problems
Thank you for your attention!
A new DG solution methodology

Comparison with LSM
Comparison with LSM

Exact solution: \( u = e^{ikxy} \)
Comparison with LSM

Exact solution: \( u = e^{ikxy} \)

Relative error, \( ka = 1 \)

Sensitivity to the mesh refinement
Comparison with LSM

Exact solution:  \[ u = e^{ikxy} \]

Relative error, \( ka = 20 \)

Sensitivity to the mesh refinement