Non–conforming meshes with curved elements and local-stepping for the simulation of wave propagation in elasto-acoustic media

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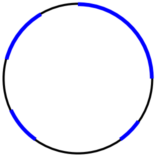
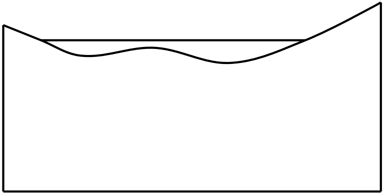
Introduction

We want to take into account the influence of the oceans on seismic wave propagation:

- For global simulations:



- For local simulations:



Parameters of the experiment

- Physical Parameters
  - Wave velocities : c_f, V_P and V_S, $c_f < V_S < V_P$
  - Frequency of the source f
  - Wavelengths : $\lambda_f = c_f/f$, $\lambda_P = V_P/f$, $\lambda_S = V_S/f$
- Numerical parameters
  - Space step $h_f \approx \lambda_f/20$, $h_s \approx \lambda_S/20$
  - Time step : $\Delta t_f \leq \alpha h_f/c_f$, $\Delta t_s \leq \alpha h_s/V_P$

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In our configurations, the height of the oceans can be smaller than one wavelength. We need to use non-conforming meshes and local time stepping.

Outline

- Equations and Variational Formulation
- Non-conforming meshes using curved elements
- Local Time-Stepping
- HPC issues

Equations

We consider here

- The second order formulation of the acoustic wave equation (in pressure) :

$$\frac{1}{c_f^2 \rho_f} \frac{\partial^2 p}{\partial t^2} + \text{ div } \frac{1}{\rho_f} \nabla \mathbf{p} = f$$

- The second order formulation of the elastic wave equation (in velocity):

$$\rho_s \frac{\partial^2 \mathbf{v}_s}{\partial t^2} - \mathbf{div}\, \underline{\underline{C}}\, \underline{\underline{\nabla}} \mathbf{v}_s = 0$$

- The classical transmission conditions:

$$\begin{cases} \mathbf{v}_s \cdot \mathbf{n} = \mathbf{v}_f \cdot \mathbf{n} \\ p\, \mathbf{n} = -\underline{\underline{\sigma}} \mathbf{n} \end{cases}$$

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Non-conforming meshes

Time Stepping

After time discretization, we end up with the following ODE:

$$\begin{cases} M_f \dfrac{\mathrm{d}^2 P}{\mathrm{d}t^2} - K_f P + C \dfrac{\mathrm{d}U}{\mathrm{d}t} = 0 \\ M_s \dfrac{\mathrm{d}^2 U}{\mathrm{d}t^2} - K_s U - C^t \dfrac{\mathrm{d}P}{\mathrm{d}t} = 0 \end{cases}$$

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$$\begin{cases} M_f \dfrac{P^{n+1} - 2P^n + P^{n-1}}{\Delta t^2} - K_f P^n + C \dfrac{U^{n+1} - U^{n-1}}{2\Delta t} = 0 \\ M_s \dfrac{U^{n+1} - 2U^n + U^{n-1}}{\Delta t^2} - K_s U^n - C^t \dfrac{P^{n+1} - P^n}{\Delta t} = 0 \end{cases}$$

Local Time Stepping for fluid/structure coupling

$$\begin{cases} M_f \dfrac{P^{n+\frac{1}{2}} - 2P^n + P^{n-\frac{1}{2}}}{\Delta t^2} - K_f P^n + C \left[\dfrac{\mathrm{d}U}{\mathrm{d}t}\right]_1 = 0 \\ M_f \dfrac{P^{n+1} - 2P^{n+\frac{1}{2}} + P^n}{\Delta t^2} - K_f P^{n+\frac{1}{2}} + C \left[\dfrac{\mathrm{d}U}{\mathrm{d}t}\right]_2 = 0 \\ M_s \dfrac{U^{n+1} - 2U^n + U^{n-1}}{\Delta t^2} - K_s U^n - C^t \left[\dfrac{\mathrm{d}P}{\mathrm{d}t}\right] = 0 \end{cases}$$

Local Time Stepping for fluid/structure coupling

$$
\left\{
\begin{array}{l}
M_f \dfrac{P^{n+\frac{1}{2}} - 2P^n + P^{n-\frac{1}{2}}}{\Delta t^2} - K_f P^n + C \dfrac{U^{n+1} - U^n}{\Delta t} = 0 \\
M_f \dfrac{P^{n+1} - 2P^{n+\frac{1}{2}} + P^n}{\Delta t^2} - K_f P^{n+\frac{1}{2}} + C \dfrac{U^{n+1} - U^n}{\Delta t} = 0 \\
M_s \dfrac{U^{n+1} - 2U^n + U^{n-1}}{\Delta t^2} - K_s U^n - C^t \dfrac{P^{n+1} - P^n}{\Delta t} = 0
\end{array}
\right.
$$

HPC issues

- Solving the linear system on the interface
- Load balancing between computations in the fluid (1 unknown) and in the solid (3 unknown).
- Effect of non-conforming meshes
- Effect of local time stepping