Qbio 2019
Bilbao – 14-2-2019

Synergy as a warning sign of transitions

Sebino Stramaglia

University of Bari & INFN, Italy

Coworkers

Daniele Marinazzo (Ghent, Belgium)

Jesus M Cortes (Bilbao, Spain)

Luca Faes (Palermo, Italia)

Mario Pellicoro (Bari, Italia)

Leonardo Angelini (Bari, Italia)

Summary

- Tranfer Entropy
- Information Decomposition
- Synergy in the 2D Ising model

Transfer Entropy

X and Y two (vectorial) time series

x, the future values of X

Absence of causality:
Generalized Markov property

$$P(x \mid X) = P(x \mid X, Y)$$

$$T(Y \to X) = \int P(x, X, Y) \log\left(\frac{P(x \mid X, Y)}{P(x \mid X)}\right) dx\,dX\,dY$$

Transfer Entropy =I(x,Y|X)

The Ising model

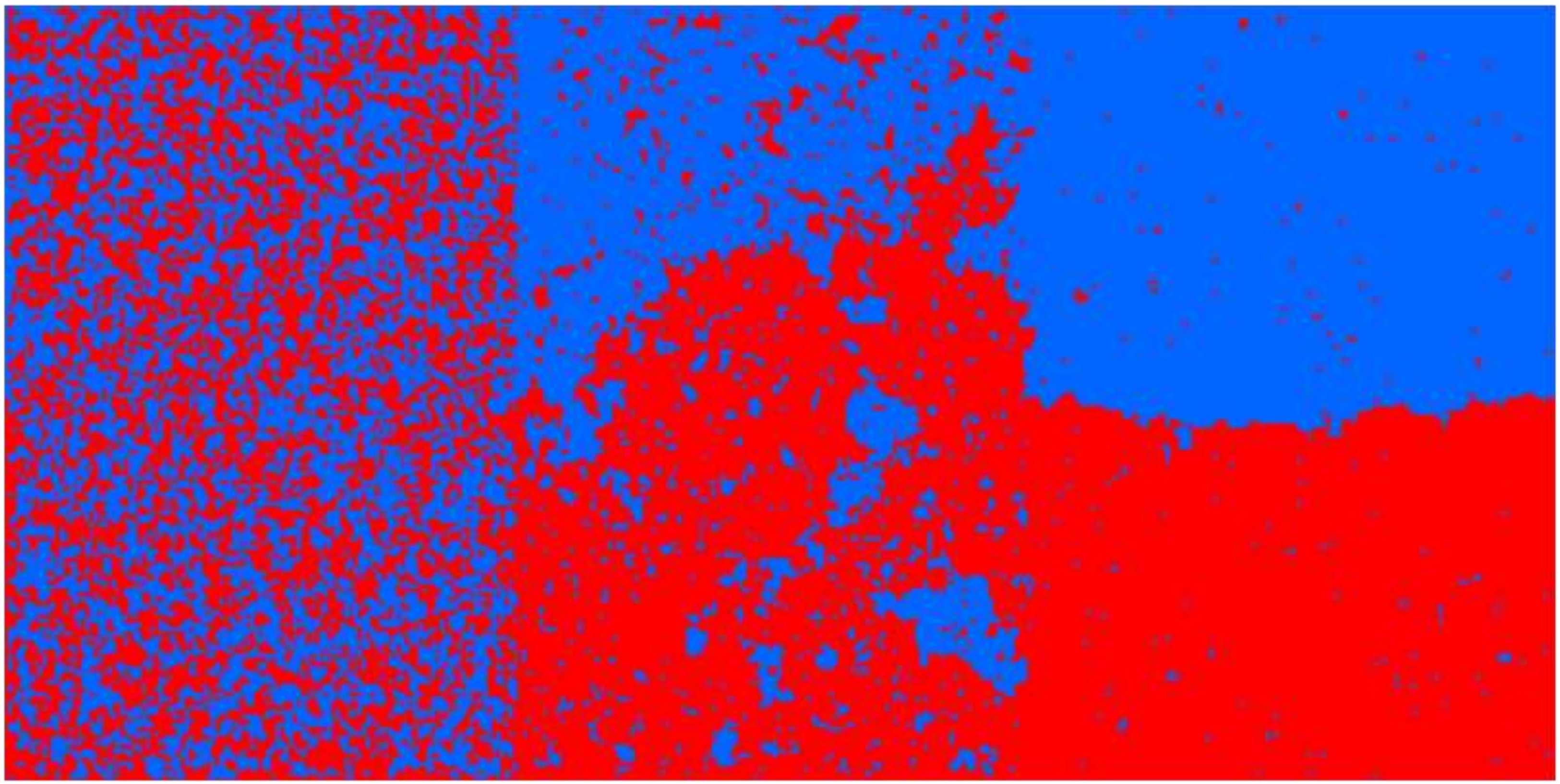
Let us consider the two dimensional Ising model, where spins on a regular lattice are characterized by the Hamiltonian

$$H = -\beta \sum_{\langle ij \rangle} s_i s_j, \tag{1}$$

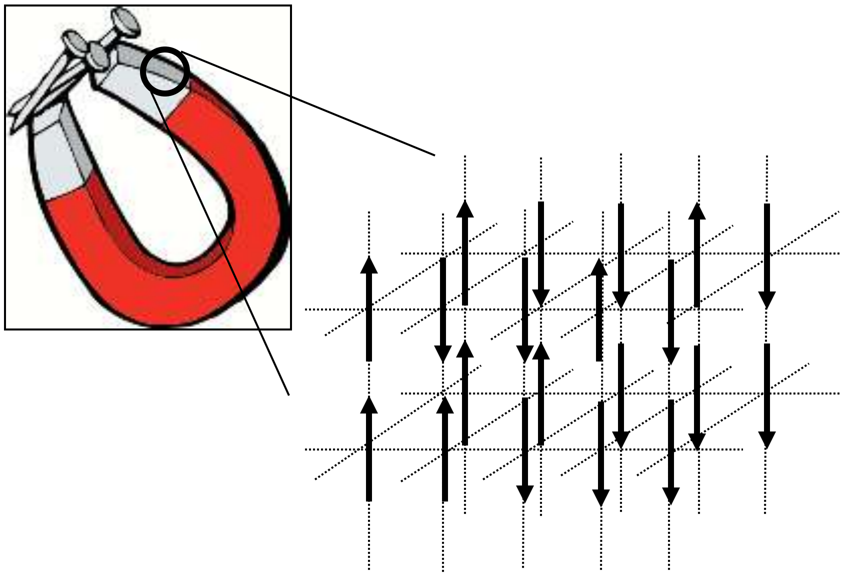
β being the coupling and the sum being performed over nearest neighbor pairs of spins. This model shows a second order phase transition at $\beta_c \approx 0.4407$, in correspondence with long range correlations in the system [7]. The mutual information of a pair of nearest neighbor

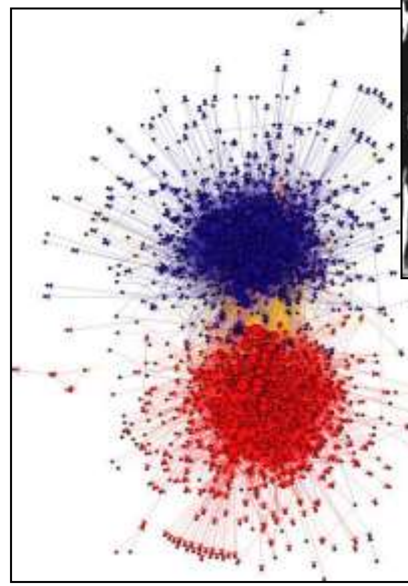
Ising 2D: Phase transition

$T \to \infty$ $T = T_{crit}$ $T \to 0$



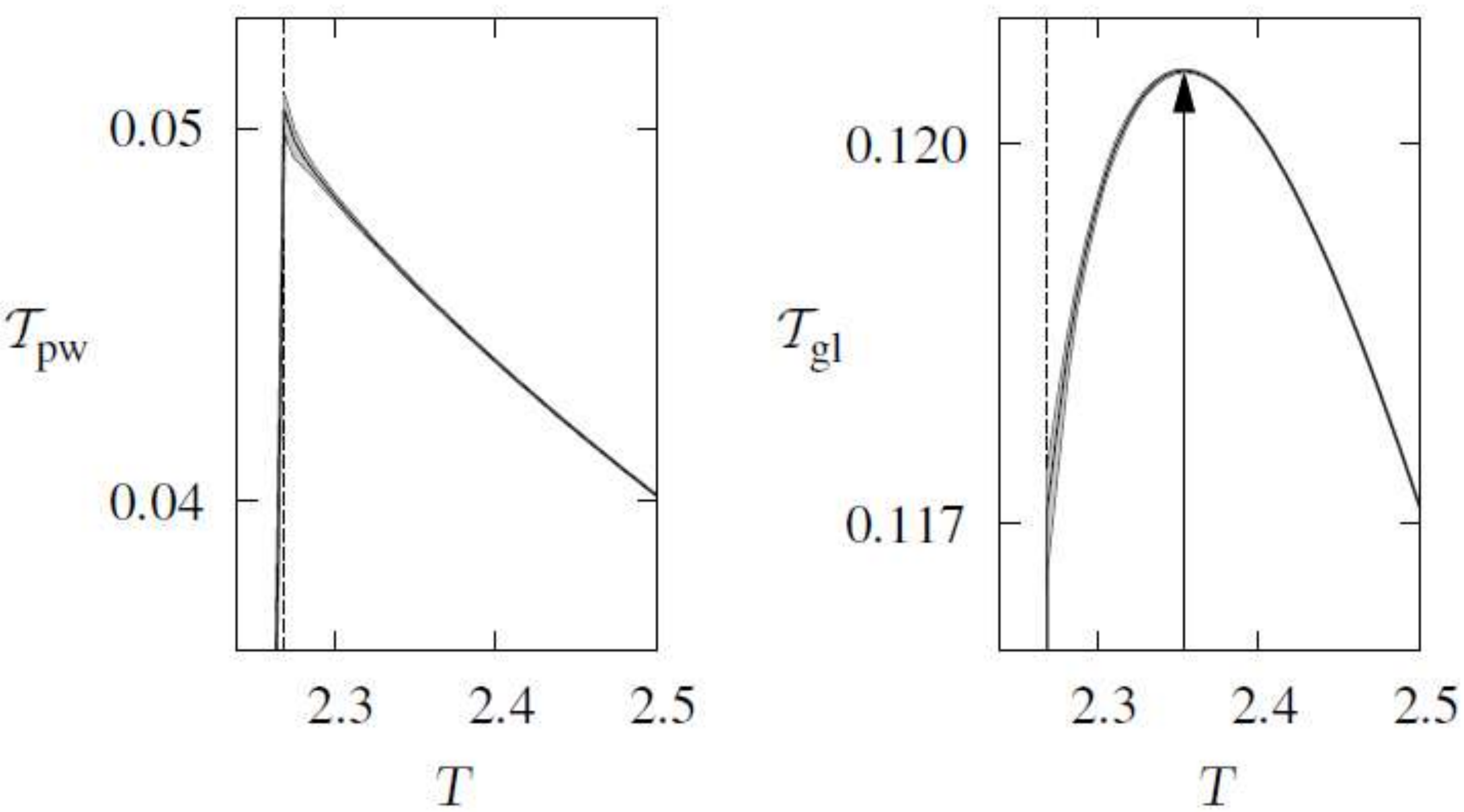
Ising Model for ferromagnetism and Opinion Dynamics and many other systems





Polarization of news, financial crashes, epileptic seizures

Transfer entropy 2D Ising model



Global Transfer Entropy as precursor of the transition !!!

- Question 1: global transfer entropy requires dynamical data. Precursors based on static data?
- Question 2: Is it mandatory to measure all the variables, or one can build precursors based on a small number (e.g. 3) of variables?

The key to answer: Information decomposition!

Example: s stimulus, r1 and r2 the response from two cells

Information Independence

$$I(\{r_1, r_2\}; s) = I(r_1; s) + I(r_2; s)$$

The two cells are sensitive to completely different features of the stimulus

E. Schneidman, W. Bialek, M.J. Berry, J. Neuroscience 23,11539 (2003).

Synergy

$$I(\{r_1, r_2\}; s) > I(r_1; s) + I(r_2; s)$$

The joint response from the two cells conveys more information than treating them separately

S is a function of both r1 and r2

Redundancy

$$I(\{r_1, r_2\}; s) < I(r_1; s) + I(r_2; s)$$

The two cells are sensitive to the same features of the stimulus

The two responses r1 and r2 share a certain amount of common information about the stimulus

The decomposition we need:

$$I\left(s_i;\{s_j s_k\}\right) = U^I_{j\to i} + U^I_{k\to i} + R^I_{jk\to i} + S^I_{jk\to i},$$

$$I\left(s_i; s_j\right) = U^I_{j\to i} + R^I_{jk\to i},$$

$$I\left(s_i; s_k\right) = U^I_{k\to i} + R^I_{jk\to i}.$$

Similar decomposition for the transfer entropy

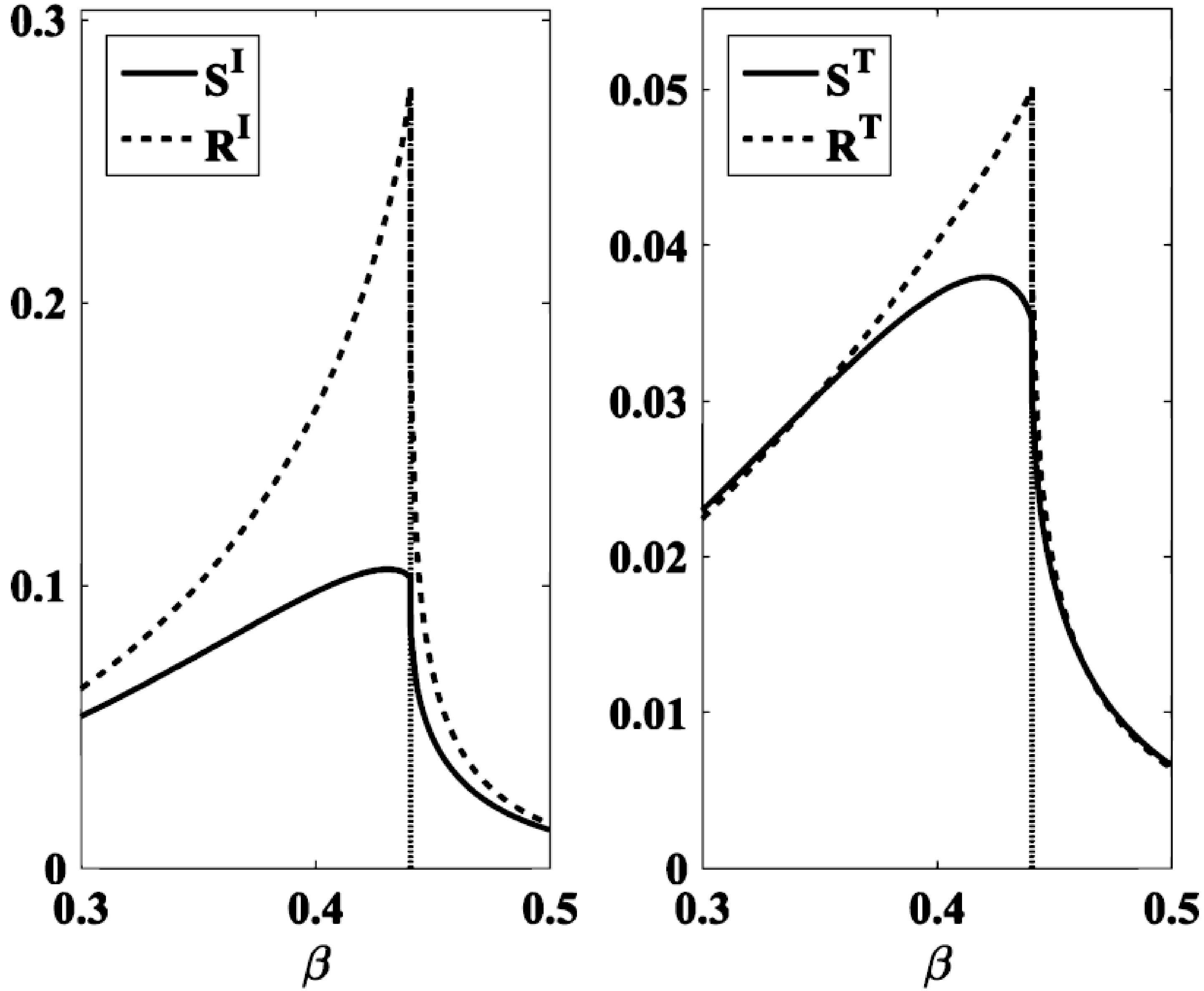
$$T_{jk\to i} = U^T_{j\to i} + U^T_{k\to i} + R^T_{jk\to i} + S^T_{jk\to i},$$

$$T_{j\to i} = U^T_{j\to i} + R^T_{jk\to i},$$

$$T_{k\to i} = U^R_{k\to i} + R^T_{jk\to i}.$$

Fourth relation (beyond Shannon theory)

Redundancy = min {T1,T2}



preprint Arxiv/1901.05405

Conclusions

The physical quantity that actually acts as a transition precursor is the synergy

This valuable marker can be found considering as few as three variables, and lagged correlations are not necessary to this scope

Preprint: arXiv.org - cond-mat

arXiv:1901.05405