Footballs, phase change and nanotubes: practical applications of mathematics

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May 20, 2012
Freezing in gas flow channels of fuel cells
Microfluidic valves - Biomedical (PCR), chemical devices

Often miniature versions of their macro counterparts
Conventional valves: take up space, continuous energy consumption but fast - order ms

But, some new valves exploit small scale
Phase change valve: do not take up space (in flow channel), virtually leakproof but slow - order s
Goals

- Understand flow
- Determine factors that control freezing (and hence speed it up)
- Determine when closing will never occur

Figure: Problem configuration
Governing equations

\[ \frac{\rho l}{d} \frac{du}{dt} = -\nabla p + \mu \nabla^2 u \]

\[ \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \alpha_l \nabla^2 T - h(x, t) < z < h(x, t) \]

\[ \frac{\partial \theta}{\partial t} = \alpha_s \nabla^2 \theta \quad h(x, t) < |z| < R \]

\[ \rho_s L_f \frac{dh}{dt} = k_s \left. \frac{\partial \theta}{\partial z} \right|_{z=h} - k_l \left. \frac{\partial T}{\partial z} \right|_{z=h} \]
Making life easier

Non-dimensionalisation

\[
\begin{align*}
\bar{x} &= \frac{x}{L} \quad \bar{z} = \frac{z}{R} \\
\bar{u} &= \frac{u}{U} \quad \bar{w} = \frac{w}{W} \quad \bar{p} = \frac{p}{P} \\
\bar{t} &= \frac{t}{\tau} \quad \bar{T} = \frac{T - T_f}{T_o - T_f} \quad \bar{\theta} = \frac{\theta - T_f}{T_f - T_w}
\end{align*}
\]

\[\frac{\partial^2 u}{\partial z^2} = \frac{\partial p}{\partial x} + O(\epsilon^2, \epsilon^2 Re) \quad \frac{\partial p}{\partial z} = O(\epsilon^2, \epsilon^4 Re) \quad \nabla \cdot \mathbf{u} = 0\]

Subject to

\[u(x, \pm h(x, t), t) = 0, \quad w(x, 0, t) = 0, \quad w(x, h(x, t), t) = \left(1 - \frac{\rho_s}{\rho_l}\right) \frac{L}{U \tau} \frac{\partial h}{\partial t}\]

\[\tau = \rho_s L_f R^2 / (k_s (T_f - T_w)) \quad \text{... freezing is driven through the contact with the wall}\]
Making life easier?

\[ u = \frac{p_x}{2} (z^2 - h^2) \]

\[ w = -\frac{\partial}{\partial x} \int_0^z u(x, \xi, t) \, d\xi = -\frac{\partial}{\partial x} \left[ \frac{p_x}{6} (z^3 - 3h^2z) \right] \]

Boundary fixing transformation

Let \( \hat{z} = z/h(x, t) \) so \( \hat{z} \in [-1, 1] \)

\[ u = \frac{p_x h^2}{2} (\hat{z}^2 - 1) = -\frac{3Q}{4h} (\hat{z}^2 - 1) \]

\[ w(x, \hat{z}, t) = -\frac{\partial}{\partial x} \left[ \frac{p_x h^3}{6} (\hat{z}^3 - 3\hat{z}) \right] = -\frac{3Q}{4h} \hat{z} h_x (\hat{z}^2 - 1) = \hat{z} h_x u \]
Making life easier??

Heat equations reduce to

\[ \frac{\partial^2 \theta}{\partial \hat{z}^2} \approx 0 \quad \theta = -\frac{h(\hat{z} - 1)}{1 - h}. \]

\[ -\frac{3QPe}{4} (\hat{z}^2 - 1) \frac{\partial T}{\partial x} = \frac{1}{h} \frac{\partial^2 T}{\partial \hat{z}^2} \]

\[ h = h(x, t) \text{ therefore equation is separable} \]

The Stefan condition

\[ h \frac{\partial h}{\partial t} = \left( \frac{\partial \theta}{\partial \hat{z}} - k \frac{\partial T}{\partial \hat{z}} \right) \bigg|_{\hat{z}=1} \]

\[ k = k_l (T_0 - T_f)/(k_s (T_f - T_w)) \]
All boils down to solving an integro-differential equation

\[ h \frac{\partial h}{\partial t} = -\frac{h}{1 - h} - k \sum_{n=1}^{\infty} A_n e^{-\lambda_n^2 \beta(t) H(x,t)} \frac{\partial Z_n}{\partial \hat{z}} \bigg|_{\hat{z}=1} \]

\[ h(x,0) = 1 \]

\[ H(x, t) = \int_0^x 1/h(\xi, t) d\xi \]

\[ Z_n \text{ are eigenfunctions} \]

Radially symmetric pipe slightly simpler

Solve numerically - problem at \( t = 0 \) ...
Limiting behaviour as $t \to 0$ ($h \to 1$) or small $k$ etc

Cartesian problem

$$h \frac{\partial h}{\partial t} = - \frac{h}{1 - h} \quad \Rightarrow \quad h = 1 - \sqrt{2t}$$

We use this as the initial condition to avoid singularity

Cylindrical problem

$$h \frac{\partial h}{\partial t} \approx \frac{1}{\ln h} \quad \Rightarrow \quad h^2 (1 - 2 \ln h) = 1 - 4t.$$
Variation of $h$ at different times for
a) $Pe = k = 1$, b) $Pe = 10, k = 1$
Variation of closure time with $Pe$ for Cartesian problem (Gui 2004 expts have $Pe \sim 1$) $k = \frac{k_l(T_0 - T_f)}{k_s(T_f - T_w)}$, $Pe = \frac{UR^2}{\alpha_1 L}$
Contact melting and comparison with experimental results of Moallemi et al IJHMT 108, 1986
Current work

Size dependent melting temperature
Classical Gibbs-Thompson

\[ T(s(t), t) = T_m = T_m^* \left( 1 - \frac{\sigma \kappa}{\rho_s L_m} \right) \]

Or ... Full Gibbs-Thompson

\[
\left( \frac{1}{\rho_l} - \frac{1}{\rho_s} \right) (p_l - p_{atm}) = L_m \left( \frac{T_m}{T_m^*} - 1 \right) + (c_l - c_s) \left[ T_m \ln \left( \frac{T_m}{T_m^*} \right) + T_m^* - T_m \right] + \frac{\sigma \kappa}{\rho_s}
\]
(Almost) complete Stefan problem in spherical co-ordinates

\[
\frac{c_l \rho_l}{\partial t} \left( \frac{\partial T}{\partial t} + \frac{\Delta \rho}{\rho_s} \frac{dR}{dt} \frac{\partial T}{\partial r} \right) = k_l \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right), \quad R(t) < r < R_b(t)
\]

\[
\frac{c_s \rho_s}{\partial t} \frac{\partial \theta}{\partial t} = k_s \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \theta}{\partial r} \right), \quad 0 < r < R(t)
\]

\[
R_b(t)^3 = \frac{\rho_s}{\rho_l} R_0^3 + (1 - \frac{\rho_s}{\rho_l}) R(t)^3
\]

Boundary conditions

\[
T(R_b, t) = T_H \quad T(R, t) = \theta(R, t) = T_m
\]

\[
\theta_r(0, t) = 0 \quad \theta(r, 0) = T_m^*
\]

\[
\rho_l \left[ L_m + \Delta c(T_m - T_m^*) \right] \frac{dR}{dt} + \frac{\rho_l}{2} \left( \frac{\Delta \rho}{\rho_s} \right)^2 \left( \frac{dR}{dt} \right)^3 = k_s \frac{\partial \theta}{\partial r} - k_l \frac{\partial T}{\partial r} \bigg|_{r=R}
\]
Melting results

Comparison of results for $T_m$ constant, $T_m(t), \Delta c = 0$, $T_m(t), \Delta c \neq 0$

Figure: Melting a) gold b) ice and $\beta = 10$.

$\beta = L_m/(c_l \Delta T)$
Carbon nanotubes

Carbon nanotubes have unique electrical properties, extraordinary strength, and efficiency in heat conduction. They have potential in fields such as nanotechnology, electronics, optics, materials science, and architecture, for example:

- **Textiles** – CNT can make waterproof and/or tear-resistant fabrics: MIT is working on combat jackets that use CNT fibers to stop bullets and to monitor the condition of the wearer.
- **Space elevator** – CNT are under investigation as possible components of the tether up which a space elevator can climb. This requires tensile strengths of more than about 70 GPa.
- **Flywheels** – The high strength/weight ratio enables very high rotational speeds.
- **Air pollution filter** – CNT membranes can filter carbon dioxide from power plant emissions.
- **Nanotube membrane** – Liquid flows up to five orders of magnitude faster than predicted by classical fluid dynamics (can you trust Wikipedia?)
Enhanced flow has been reported (by orders of magnitude), see Majumder et al Nature 438, 2005, Holt et al Science 312 2006, Whitby et al Nature Nanotechnology 2 2007 (Table above from Whitby).

These estimates have come down. Whitby et al Nanoletters 8(9) 2008 quote a maximum factor of around 45
Theoretical predictions

Unidirectional flow, no-slip \( u(r = R) = 0 \)

\[
\begin{align*}
u_{HP} &= - \frac{p_z R^4}{4\mu} \left(1 - \frac{r^2}{R^2}\right) \\
Q_{HP} &= 2\pi \int_0^R ru_{HP} \, dr = -\frac{\pi R^4 p_z}{8\mu}
\end{align*}
\]

Navier slip condition

\[
u(R) = -L_s \left. \frac{\partial u}{\partial r} \right|_{r=R}
\]

leads to

\[
\begin{align*}
u_{slip} &= -\frac{R^2 p_z}{4\mu_1} \left[1 - \frac{r^2}{R^2} + \frac{2L_s}{R}\right] \\
Q_{slip} &= Q_{HP} \left(1 + \frac{4L_s}{R}\right)
\end{align*}
\]

Flow enhancement \( \epsilon = Q_{slip}/Q_{HP} \)

Initial enhancement predictions imply \( L_s = \mathcal{O}(\mu m) \), recent estimates have \( L_s = \mathcal{O}(30nm) \)
When can we use a continuum theory?

Does it make sense to use a continuum theory?

Comparison of MD and Navier-Stokes predictions for CNTs of 5.1 and 10.2 molecular diameters (water molecule = 0.29nm), taken from Travis et al. Phys. Rev. E 55(4) 1997

Thomas et al. IJTS 2010 state MD matches continuum down to 1.66nm
What is happening near the wall?

Experiments have shown the existence of a depletion layer (due to dissolved gas or formation of vapour?) this has led to MD studies with a gas gap, frictionless interface and slip length

Experiments and MD simulations imply depletion layer of around 0.7nm from wall
Biviscosity model

Leads to

$$u_1 = \frac{p_z}{4\mu_1}(r^2 - \alpha^2) - \frac{p_z}{4\mu_2}(R^2 - \alpha^2)$$

$$u_2 = \frac{p_z}{4\mu_2}(r^2 - R^2)$$

$$\alpha = R - 0.7\text{nm}$$

$$Q_\mu = Q_{HP} \frac{\alpha^4}{R^4} \left[ 1 + \frac{\mu_1}{\mu_2} \left( \frac{R^4}{\alpha^4} - 1 \right) \right]$$

The flow rate enhancement is

$$\epsilon_\mu = \frac{Q_\mu}{Q_{HP}} = \frac{\alpha^4}{R^4} + \frac{\mu_1}{\mu_2} \left( 1 - \frac{\alpha^4}{R^4} \right)$$
Whitby *et al* 2008 work with water in a CNT with $R = 20\text{nm}$. Their results require $L_s \approx 35\text{nm}$, hence $\epsilon_{slip} = 8$

$$\mu_2 = \mu_1 \left[ \frac{R^4 - \alpha^4}{\epsilon_\mu R^4 - \alpha^4} \right]$$

Taking $\epsilon_\mu = 8$ gives $\mu_2 = 0.018\mu_1$

Oxygen viscosity $\approx 0.018$ that of water
Phase Change

Enhanced flow in CNTs

Choosing a football

Analogy with slip length

\[ \alpha = R - \delta \]

\[ \epsilon_\mu = 1 + \frac{4\delta}{R} \left( \frac{\mu_1}{\mu_2} - 1 \right) \left[ 1 - \frac{3}{2} \frac{\delta}{R} + \left( \frac{\delta}{R} \right)^2 - \frac{1}{4} \left( \frac{\delta}{R} \right)^3 \right] \]

maximum enhancement predicted by setting \( R = \delta, \mu_2/\mu_1 = 0.018, \delta = 0.7\text{nm} \) to give \( \epsilon_\mu \approx 50 \)

\[ \epsilon_{\text{slip}} = 1 + \frac{4L_s}{R} \]

Whitby et al predict a maximum enhancement of up to 45 times theoretical predictions
Noting that $\mu_1/\mu_2 \gg 1$, we can identify three distinct regimes:

1. **For sufficiently wide tubes:** $(\delta/R)(\mu_1/\mu_2) \ll 1 \Rightarrow \epsilon_\mu \approx 1$. The no-slip boundary condition will be sufficient (Requires $R > 3\mu m$). On smooth surfaces slip is not observed in wide tubes.

2. **For moderate tubes:** $(\delta/R)(\mu_1/\mu_2) = \mathcal{O}(1)$, $\delta/R \ll 1$ then

$$
\epsilon_\mu \approx 1 + \frac{4\delta}{R} \frac{\mu_1}{\mu_2}
$$

corresponds to a constant slip length, $L_s = \delta \mu_1/\mu_2$ ($R \in [21nm, 3\mu m]$)

*Numerous papers report constant slip-lengths around 20-40nm for $R \in \text{'some nanometers up to several hundreds of nanometers'}$*

3. **For very small tubes:** $\delta/R = \mathcal{O}(1)$ then the full expression for $\epsilon_\mu$ is required and the slip-length varies with tube radius. *Thomas et al suggest $L_s$ varies with $R$ for $R \in [1.6, 5]nm$*

*Thomas et al also show $\epsilon \approx 32$ when $R = 3.5nm$: we predict $\epsilon \approx 33.2$.***
Conclusions

- Flow enhancement can be plausibly related to a reduced viscosity model
- The flow enhancement is only an order of magnitude (not orders as reported in some papers)
- The reduced viscosity model provides one possible explanation for the Navier slip boundary condition when the solid is smooth down to the nanoscale

So why are slip-lengths so large in carbon nanotubes? The term 'slip length' is rather misleading. Not a real length but a length-scale proportional to the product of the viscosity ratio and the width of the depletion region.
In football matches the home team provides the ball. There are obvious restrictions on this choice.

- **FIFA Law 2** specifies that the ball is an air-filled sphere with a circumference of 68 - 70 cm, a (dry) weight 410 - 450 g, inflated to a pressure of 0.6 to 1.1 atmospheres (59 - 108 kPa) "at sea level", and covered in leather or "other suitable material”

- **Sponsorship** Most teams have a sponsor and are contracted to use their balls.

At the 8th Mathematics in Industry Study Group, held in January 2011, at The University of the Witwatersrand, a South African premiership team, Bidvest Wits, posed a question on the best choice of football to disadvantage the opposition.

Bidvest Wits play at 2000m above sea-level.
What to study?

Focus on the motion of the ball in the air and in particular from a free kick or corner.

Relatively controlled situations and there is much data on a ball’s motion through the air.

Free kicks are an important factor in scoring: in the 1998 world cup 42 of the 171 goals scored came from set-plays, with 50% of these from free kicks [1].

Due to the high altitude of Johannesburg, and so decreased drag effect, the motion through the air is where most difference in the ball’s behaviour is likely to occur.

Does the ball make a difference?

Trajectory of Adidas Terrapass, Nike T90, Adidas Replica, Puma V1.08

Yes!
Factors affecting motion

Q. What affects a balls motion through the air?
A. Gravity and drag

Best to leave gravity alone

Q. What affects drag?
A. Atmospheric conditions, ball design (size, pattern, seams, roughness)

Atmospheric conditions fixed hence ...

What causes swerve?

Does a ball with top-spin go up or down?

With top-spin
The air close to top of ball has higher velocity
The air close to bottom of ball has lower velocity

Bernouilli’s equation

\[ P_1 + \frac{1}{2} \rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho gh_2 \]

At fixed height \( h_1 = h_2 \) this implies \( v \uparrow, p \downarrow \)

*Top-spin therefore has ball curving up*

**Magnus effect**

\( v \uparrow \Rightarrow \) higher drag, earlier detachment of boundary layer
\( v \downarrow \Rightarrow \) lower drag, later detachment of boundary layer

Drag = Force = pressure \( \times \) area

*Top-spin therefore has ball curving down*

Taken from www.physicscentral.com

Note, Bernouilli above is not correct form
Magnus effect

1666 Isaac Newton noticed that motion of tennis ball was affected by spin ‘For a circular as well as a progressive motion ... its parts on that side, where the motions conspire, must press and beat the contiguous air more violently than on the other’

1742 Benjamin Robins showed a transverse aerodynamic force on a rotating sphere.

1877 Lord Rayleigh credited Magnus (1852) with the correct explanation of the irregular motion of a tennis ball (now accepted as Robins - but too late)

1890 Tait used this on golf balls to show they travelled further with spin.

This was all with frictionless fluids. Correct explanation required Prandtl’s boundary layer theory (1904) which involved very small friction.

Taken from Van Dyke 1982 Album of fluid motion

Taken from pencilcricket.blogspot.com
Not Bernouilli!!!!!!

In the good old days leather balls absorbed water and so were less responsive to aerodynamic forces - players would usually kick ball at wall to break it or hope it would deflect past the keeper.

Early swerve ... 2nd FIFA World Cup final 1934, Czech. vs Italy 1-0 after 82min. Italy’s Orsi ran through Czech defence and shot. The ball swerved wildly and curled past the keeper into the net. Italy scored again in extra time to win.

The next day, Orsi tried 20 times to repeat his shot and failed (see Fifa.com).

Now footballers exploit Magnus effect (see Beckham as physicist? Physics in Sport 2001) ...

Top and side spin ...

Beckham (Eng. vs Gr. 2001) ...

Top-spin is easily confused with gravity

Side-spin ...

Carlos (Br. vs Fr. 1997) ...

This is not Bernouilli!
Equations of motion

*Trajectory of a soccer ball*

\[ \mathbf{F} = m \mathbf{g} + \mathbf{F}_d + \mathbf{F}_l \]

where

\[ \mathbf{F}_d = -\frac{1}{2} \rho A |\mathbf{v}|^2 C_d \vec{v} \]
\[ \mathbf{F}_l = \frac{1}{2} \rho A |\mathbf{v}|^2 C_l \vec{\sigma} \times \vec{v} \]

\( m \) is the mass of the ball, \( A \) is its cross-sectional area, \( \rho \) is the density of air, \( C \) are drag coefficients, \( \vec{v} = \mathbf{v} / |\mathbf{v}| \)

\[ |\mathbf{v}| = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} \]
Two-dimensional equations of motion

Focus on 2D - which gives sufficient information

*Solve numerically?*

\[
\begin{align*}
\ddot{x} &= -|v| \left\{ k_d \dot{x} - k_l \sin \gamma \dot{y} \right\} \\
\ddot{y} &= -|v| \left\{ k_d \dot{y} + k_l \sin \gamma \dot{x} \right\}
\end{align*}
\]

\[k_d = \frac{\rho A C_d}{2m}, \quad k_l = \frac{\rho A C_l}{2m}, \quad \gamma \text{ is angle of spin axis}\]

Numerical solution does not indicate important factors

*Or analytically?*

Ball kicked mainly in \( y \)-direction hence \( \dot{x} \ll \dot{y} \quad |v| \approx \dot{y} \)

\[
\begin{align*}
\ddot{x} &\approx k_l \sin \gamma \dot{y}^2 \\
\ddot{y} &\approx -k_d \dot{y}^2
\end{align*}
\]

Initial conditions

\[
x(0) = y(0) = 0 \quad \dot{x}(0) = 0 \quad \dot{y}(0) = v
\]

Then

\[
\begin{align*}
y &= \frac{1}{k_d} \ln(1 + k_d vt) \\
x &= -\frac{k_l}{k_d} (y - vt) \sin \gamma
\end{align*}
\]

Important quantities for swerve - \( x \propto \sin(\text{spin axis}), \text{ lift and drag coefficients, } (y - vt) \)
Perturbation solution

Formalise the simplification ...

\[ \ddot{x} = -|\mathbf{v}| \left\{ k_d \dot{x} - k_l \sin \gamma \dot{y} \right\} \]

Scale with typical value \( \hat{x} = x/L_1 \), \( \hat{y} = y/L_2 \), \( \hat{t} = t/\tau \)

\( L_2 \sim \) distance of the free kick - 20m \quad (also possible \( L_2 = 1/k_d \approx 100m \))

\( \tau \sim L_2/\nu \) time taken for the ball to travel the distance \( L_2 \sim 1s \)

\( L_1 \) unknown

\[ |\hat{\mathbf{v}}| = \sqrt{\frac{L_1^2}{\tau^2} \dot{x}^2 + \frac{L_2^2}{\tau^2} \dot{y}^2} = \frac{L_2}{\tau} \sqrt{1 + \frac{L_1^2}{L_2^2} \dot{x}^2 \dot{y}^2} \]

\[ \ddot{x} = -\dot{y} \sqrt{1 + \frac{L_1^2}{L_2^2} \dot{x}^2 \dot{y}^2} \left( k_d L_2 \dot{x} - k_l \sin \gamma \frac{L_2^2}{L_1} \dot{y} \right) \]

Indicates \( L_1 = k_l \sin \gamma L_2^2 \) \quad taking \( k_l = 0.013, \gamma = \pi/2, L_2 = 20 \Rightarrow L_1 = 5.2m \)

Denote \( \epsilon = k_d L_2 (\approx 0.3 \text{ for the given parameter values}) \)

\[ L_1/L_2 = 5.2/20 = 0.26 \Rightarrow L_1/L_2 = a\epsilon \]
Perturbation 2

Initial conditions are now

\[ x(0) = y(0) = 0, \quad \dot{x}(0) = 0, \quad \dot{y}(0) = 1 \]

To highlight spin take \( \gamma = \pi/2 \)

Now let (series solution)

\[ x = x_0 + \epsilon x_1 + \epsilon^2 x_2 + \epsilon^2 x_3 + \cdots, \quad y = y_0 + \epsilon y_1 + \epsilon^2 y_2 + \epsilon^3 y_3 + \cdots \]

Leads to

\[ y = t - \epsilon \frac{t^2}{2} + \epsilon^2 (2 - b) \frac{t^3}{6} - \epsilon^3 (6 + a^2 - 7b) \frac{t^4}{24} + \mathcal{O}(\epsilon^4) \]

\[ x = \frac{t^2}{2} - \epsilon \frac{t^3}{2} + \epsilon^2 (11 + a^2 - 2b) \frac{t^4}{24} - \epsilon^3 (50 + 15a^2 - 25b) \frac{t^5}{120} + \mathcal{O}(\epsilon^4) \]
Perturbation 3

Returning to dimensional form

\[ x \to x/L_1 = x/(k_l L_2^2 \sin \gamma), \quad y \to y/L_2, \quad t \to tv/L_2 \]

\[ x = k_l \sin \gamma \frac{v^2 t^2}{2} \left( 1 - k_d vt + (11 + a^2 - 2b) \frac{(k_d vt)^2}{12} - (50 + 15a^2 - 25b) \frac{(k_d vt)^3}{60} + \cdots \right) \]

\[ y = vt \left( 1 - k_d \frac{vt}{2} + (2 - b) \frac{(k_d vt)^2}{6} - (6 + a^2 - 7b) \frac{(k_d vt)^3}{24} + \cdots \right) \]

First term is most important

Comparison of numerical solution (solid line) and series solution (to \( O(\epsilon^2) \)) (dot-dashed) for a) \( x(t) \), b) \( y(t) \)
Comparison of results

Analytical solution makes the important factors clear

\[ x = k_l \sin \gamma \frac{v^2 t^2}{2} \left( 1 - k_d vt + (11 + a^2 - 2b) \frac{(k_d vt)^2}{12} - (50 + 15a^2 - 25b) \frac{(k_d vt)^3}{60} \right) \]

So, how to get swerve

- large spin (⇒ make \( \gamma \) high)
- kick ball hard (⇒ make \( v \) high)
- do it from far away (⇒ allow \( t \) to get high - but beware \( \dot{y} \) then decreases)

For example ...

**Swerve goals**

- Increase \( k_l = \rho AC_l/2m \)
Drag and lift coefficients

To choose a ball we need more information on drag coefficients.

Spin puts one side of ball at different $Re$ (and hence $C_D$) to other - generates swerve.
Smooth ball - lower drag, lower lift or swerve, transition at higher $Re$.
Rough ball - higher drag, higher lift or swerve, transition at low $Re$.
Decrease in density equivalent to decrease in velocity - density at coast is 20% higher than at 2000m.

Is it going in?

So, rough or smooth balls?

Coastal teams expect swerve, high altitude teams do not
High altitude team choose ball that swerves the least (i.e. smooth) to confuse opposition

Before 2nd Feb Wits had gone eight games without a win
Recent results in South African premiership (since study group):
2 March, Bidvest Wits 2 - 0 Ajax Cape Town; 26 Feb, Sundowns 2 - 0 Wits;
22 Feb, Free State 2 - 2 Wits; 16Feb, Wits 3 - 1 Santos;
19 Feb, Wits 3 - 0 Mpumalanga; 6 Feb Wits 6 - 0 Vasco da Gama

Did not lose a single league home game after meeting
And the Jabulani?

Construction: 8 bonded 3d moulded Polyurethane panels make for a near-waterproof 'roundest ever' ball

<table>
<thead>
<tr>
<th>Feature</th>
<th>FIFA Reg.</th>
<th>Jabulani</th>
<th>Standing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>420 - 445</td>
<td>440</td>
<td>Heavy</td>
</tr>
<tr>
<td>Circumference</td>
<td>68.5-69.5cm</td>
<td>69.0 +/- 0.2</td>
<td>Average</td>
</tr>
<tr>
<td>Roundness</td>
<td>1.5%</td>
<td>Variance1.0%</td>
<td>Good</td>
</tr>
<tr>
<td>Water Absorption</td>
<td>Max 10%</td>
<td>0%</td>
<td>Amazing</td>
</tr>
<tr>
<td>Bounce Variance</td>
<td>10cm</td>
<td>6cm</td>
<td>Good</td>
</tr>
</tbody>
</table>

"This table shows that the moulding technique means the ball retains its shape, and the lack of seaming means there is essentially zero water retention, which will reduce sluggishness of the ball if South Africa defies the odds and rains throughout the World Cup The weight is interesting - being towards the higher end of the allowed scale which means the flight will be truer and more predictable making keepers happy, but also rewarding accuracy for strikers." see http://www.jabulaniball.com/

Lionel Messi (Argentina) "The ball is very complicated for the goalkeepers and for us"
Julio Cesar (Brasil) "Its terrible, horrible. Its like one of those balls you buy in the supermarket"
Iker Casillas (Spain) "Its a little sad that in a competition as big as the World Cup to have such a poor ball"
Luis Fabiano (Brasil) "supernatural - as it unpredictably changed direction when traveling through the air"

Fabio Capello (England coach) "the ball behaved completely different at altitude"
Reverse Magnus

Jabulani is smooth ball with no seams

Transition from turbulent to laminar at relatively high speed

Leads to reverse Magnus effect – ball swerves in other direction – hence S shaped trajectory

To Fabiano ("supernatural - as it unpredictably changed direction when traveling through the air") – do your maths first!

To Capello ("the ball behaved completely different at altitude") – true, but don’t blame the ball for Englands performance
Acknowledgements

CRM - Michelle de Decker, Francesc Font
U. Limerick - Sarah Mitchell

Feel free to read

- T.G. Myers *Why are slip lengths so large in carbon nanotubes?* Microfluidics and Nanofluidics 2011 DOI 10.1007/s10404-010-0752-7
- M. de Decker & T.G. Myers *Contact melting of a three-dimensional phase change material on a flat substrate* Submitted to Int. J. Heat Mass Trans. 2011