Studies on the short term fairness of schedulers exploiting multiuser diversity

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Introduction
Background

- Since the utilization of multiuser diversity (MD) in wireless networks can increase the information theoretic capacity (ITC), much attention has been paid to schedulers exploiting MD.

- MD is a diversity existing between the wireless channel states of different users.
- MD comes from the fact that the wireless channel state processes of different users are usually independent for the same shared medium.
Tradeoff in schedulers exploiting MD

• Tradeoff between ITC (on downlink) and capacity consumed by feedback (on uplink)
  o Schedulers exploiting MD consume the bandwidth for the feedback load, which is defined as the amount of channel information that needs to be fed back from MSs (mobile stations) to BS (base station).

• Tradeoff between ITC and fairness achieved by schedulers exploiting MD
Schedulers exploiting MD

• For example, MD can be exploited in such a way that the scheduler at the **BS (Base Station)** selects the **MS (Mobile Station)** whose received SNR is the best, and transmits packets to the selected MS.

• This scheduler maximizes ITC of the overall system, but it is highly unfair.
To solve this unfair problem, proportional fair (PF) scheduler was proposed. PF considers the normalized SNRs of MSs (defined by the received SNR/the average received SNR), and selects the MS whose normalized SNR is the largest.
Quantized PF (QPF) scheduler

• In practice, probably the normalized SNR values are quantized, the quantized normalized SNR values are reported to the BS from MSs, and the PF scheduler is performed based on the quantized normalized SNR values.

• This PF scheduler is called QPF (Quantized PF) scheduler.
One-bit feedback fair (1FF) scheduler

• From a view of reducing the amount of feedback information, a small number of quantization levels is desirable.

• One-Bit Feedback fair (1FF) scheduler is QPF scheduler with two quantization levels (i.e., with only one threshold for quantization).

It has been reported that the 1FF scheduler can achieve a relatively good ITC, if the quantization threshold is appropriately determined.
Feedback load (FL) and Fairness of 1FF

- 1FF can reduce FL (by introducing 1-bit feedback with feedback threshold) and achieve the ideal **long term fairness** (by considering normalized SNRs), while having considerable performance gain.

- However, 1FF may still have some difficulty for FL.
- FL of 1FF linearly increases with the number of MSs, although the performance gain for the capacity also grows as the number of MSs becomes large.
Opportunistic feedback fair (OFF) scheduler

One way to overcome the difficulty for FL to introduce a random access-based feedback scheme.

- As a scheduler with random access-based feedback scheme, the opportunistic feedback scheduler has been proposed.
- Feedback resources are random access minislots.
- MSs transmit feedback information with some probability in each minislot only when their SNR values are greater than or equal to threshold.
• Contrary to 1FF, FL of the opportunistic feedback scheduler is independent of # of MSs.
There exists a tradeoff between ITC and fairness achieved by schedulers exploiting MD. The fairness is classified into short term fairness and long term fairness.

- Short term (ST) fairness: the ability of the scheduler on how equally it can distribute network resources (e.g., service times) over multiple MSs in a finite observation period.
- Long term (LT) fairness: the ability of the scheduler on how equally it can distribute network resources over multiple MSs in an infinite observation period.
Significance of short term fairness

- ST fairness greatly affects packet level performances such as delay and loss probability of individual MSs.
- LT fairness governs the (long run) average throughput of individual MSs.

1FF scheduler and OFF scheduler provide an ideal LT fairness property. However, their ST fairness properties have not been sufficiently explored yet.
Statistical-time access fairness index (STAFI)

- As an index of short term fairness, Liu et al. propose a statistical time-access fairness index (STAFI) defined as

\[
P \left( \left| \frac{\alpha^{(i)}(t_1, t_2)}{\phi(i)} - \frac{\alpha^{(j)}(t_1, t_2)}{\phi(j)} \right| \geq x \right) \leq f^{(i,j)}(x),
\]

\(\alpha^{(i)}(t_1, t_2)\): the amount of the service in time (not in bits) for flow i in \([t_1, t_2]\)

\(\phi(i)\): assigned weight for flow i

\(f^{(i,j)}(x)\): some function
Purpose of this study

- Study the short term fairness properties of 1FF schedulers and those of OFF schedulers.
- Consider STAFI as a measure of short term fairness
  - In this paper, call the probability on the left hand side of the inequality the STAFI
  - In particular, consider STAFI where the assigned weights are all equal to one
What we do in this study

- **Develop two numerical methods** to examine the transient properties of the STAFI of the two schedulers.

- The first method calculates the exact value of the STAFI by using the **inverse discrete FFT method**.
  - It enables us to precisely observe how the STAFI changes as the progress of time.

- The second method estimates the asymptotic decay rate of the STAFI by using the **theory of large deviations**.
  - It enables us to predict how fast the STAFI approaches to ideal fairness as the progress of time.
• Provide some numerical results to investigate the short term fairness properties of 1FF and OFF schedulers.
System Model
Consider a wireless network consisting of a BS and K MSs as shown in Fig. 1.
BS employs 1FF scheduler or OFF scheduler for downlink transmission.
Focus on downlink transmission.
Analyze the STAFI of 1FF scheduler and that of OFF scheduler.

Figure 1: System model
Channel model

• Assume that the downlink channel of MS is described by a flat Rayleigh fading channel model.

• The received SNR process \( \{ z^{(i)}(t) \} \) (\( t = 0, 1, \ldots \)) of MS \( i \) (\( i=3,\ldots,K \)) is a stationary process. But we don’t assume the stationarity for \( i=1,2 \).

• \( z^{(i)}(t) \) for any \( t \) is according to the following exponential distribution (\( i=3,\ldots,K \))

\[
P \{ z^{(i)}(t) \leq x \} = 1 - \exp(-x/\bar{z}^{(i)})
\]

where \( \bar{z}^{(i)} \) denotes the average received SNR of MS \( i \).

• Assume that the received SNR processes of the \( K \) MSs are independent with each other.
1FF scheduler

- Under 1FF scheduling, the normalized SNR processes \( \{ \frac{z^{(i)}(t)}{\overline{z}^{(i)}} \} \) of MSs are considered.
- Each MS partitions the entire normalized SNR range into 2 grades with threshold \( \gamma_1 \).
- MSs with the normalized SNR values greater than or equal to \( \gamma_1 \) transmit one-bit feedback information to the BS.
• At every time $t$, MS $i$ estimates its received normalized SNR $\tilde{z}^{(i)}(t)/\tilde{z}^{(i)}$ and examines if $\frac{\tilde{z}^{(i)}(t)}{\tilde{z}^{(i)}} \geq \gamma_1$.

• If $\frac{\tilde{z}^{(i)}(t)}{\tilde{z}^{(i)}} \geq \gamma_1$ (i.e., if the wireless channel state of MS $i$ is in state 1), MS $i$ transmits one-bit feedback information to BS.
• For downlink transmission, 1FF scheduler at the BS randomly selects one of MSs which feed back.
• If there are no MSs which feed back, 1FF scheduler randomly selects one of $K$ MSs.
• The scheduling is performed frame-by-frame.
**OFF scheduler**

- Under OFF scheduling, the **normalized SNR processes** $\{ z^{(i)}(t)/\bar{z}^{(i)} \}$ of MSs are considered.
- Each MS partitions the entire normalized SNR range into 2 grades with **threshold** $\gamma_1$.
- If $z^{(i)}(t)/\bar{z}^{(i)} < \gamma_1$ ( $z^{(i)}(t)/\bar{z}^{(i)} \geq \gamma_1$), we say that the wireless channel state of MS $i$ is in state 0 (1) at time $t$. 
• Suppose that OFF scheduler is employed in a frequency-division-duplex (FDD) system.
• In FDD, at the beginning of the downlink frame, BS broadcasts a message containing the information for opportunistic feedback to all the MSs.
• $N$ minislots in an uplink frame for random access feedback follow the downlink message.
• At every time $t$, MS $i$ estimates its received normalized SNR $\tilde{z}^{(i)}(t)/\bar{z}^{(i)}$ and examines if $\tilde{z}^{(i)}(t)/\bar{z}^{(i)} \geq \gamma_1$.

• If $\tilde{z}^{(i)}(t)/\bar{z}^{(i)} \geq \gamma_1$ (i.e., if the wireless channel state of MS $i$ is in state 1), MS $i$ attempts to transmit feedback information to BS with a probability $\pi$ in every minislot.

• We hereafter call the probability $\pi$ the feedback probability.
• If $\frac{z^{(i)}(t)}{\bar{z}^{(i)}} < \gamma_1$ (i.e., if the wireless channel state of MS $i$ is in state 0), MS $i$ does not feedback any information to BS in the random access minislots.

• The feedback information can be successfully fed back to BS if and only if one MS attempts to transmit feedback information in the minislot.

• Otherwise, either a collision happens or there is no MS to feed back in the minislot.
• If multiple MSs successfully feed back during the random access period consisting of $N$ minislots, BS randomly selects one of the successful MSs.
  o If an MS successfully fed back multiple times in the minislots, it is multiply counted in the random selection.
• If there is no successful feedback in the $N$ minislots, BS randomly selects one MS among all the MSs.
• The scheduling is performed frame-by-frame.
• Assume that the random access attempts are independent among MSs and also independent among random access minislots.
Wireless channel model

- \(\{s^{(i)}(t)\} \ (t = 0, 1, \ldots; i = 1, \ldots, K\) : the wireless channel state process of MS \(i\), where \(s^{(i)}(t) = 1\) if \(\frac{z^{(i)}(t)}{\bar{z}^{(i)}} \geq \gamma_1\) and \(s^{(i)}(t) = 0\) otherwise.

- Assume that the channel state process \(\{s^{(i)}(t)\}\) is well described by a discrete-time 2-state Markov chain (MC).

- Further assume that for \(i = 3, \ldots, K\), the MC \(\{s^{(i)}(t)\}\) is stationary from the assumption of the stationarity of the received SNR process \(\{z^{(i)}(t)\}\) for \(i = 3, \ldots, K\).

- On the other hand, for \(i = 1, 2\), we do not assume the stationarity of the MC \(\{s^{(i)}(t)\}\).
• To determine the state transition probabilities of the MC, we first consider the level crossing rate $\chi(\gamma)$ of the received normalized SNR at $\gamma$ given by

$$\chi(\gamma) = \sqrt{2\pi \gamma f_d} \exp(-\gamma), \quad (3)$$

where $f_d$ denotes the mobility-induced Doppler spread of MSs. We assume that for all the MSs, the mobility-induced Doppler spreads are identical.

• From this assumption, the transition matrices of the MCs become identical.
• The stationary probability vector of each MC is given by

\[ s_0 = 1 - e^{-\gamma_1}, \quad s_1 = e^{-\gamma_1}. \]  

(4)

• Let \( P = (p_{i,j}) \ (i, j = 0, 1) \) denote the transition probability matrix of the 2-state MCs. The transition probabilities are determined by

\[ p_{0,1} = \frac{\chi(\gamma_1)T_f}{s_0}, \quad p_{1,0} = \frac{\chi(\gamma_1)T_f}{s_1}, \]  

(5)

\[ p_{0,0} = 1 - p_{0,1}, \quad p_{1,1} = 1 - p_{1,0}, \]  

(6)

where \( s_i \ (i = 0, 1) \) and \( \chi(\gamma_1) \) are given by (4) and (3), respectively.

Analysis
STAIFI of schedulers

• Without loss of generality, analyze the STAIFI between MS 1 and MS 2.

• Let $G_n(x) \ (n = 1, 2, \ldots)$ denote the STAIFI between MS 1 and MS 2 during $n$ slots.

• The STAIFI is then given by

$$G_n(x) = P(|\alpha^{(1)}(0, n) - \alpha^{(2)}(0, n)| \geq x)$$

where $\alpha^{(i)}(t_0, t_1)$ denotes the amount of service for MS i in $[t_0, t_1]$.

• Further define the probability mass function by

$$g_n(x) = P(|\alpha^{(1)}(0, n) - \alpha^{(2)}(0, n)| = x).$$
Analysis of STAFI

- We first define a \((K - 1) \times (K - 1)\) matrix \(R\) by

\[
[R]_{i,j} = \sum_{k=\max(0,i+j-K+2)}^{\min(i,j)} \binom{i}{k} p_{1,1}^k p_{1,0}^{i-k} \cdot \binom{K-2-i}{j-k} p_{0,1}^{j-k} p_{0,0}^{K-2-i-j+k},
\]

where \([R]_{i,j} (i, j = 0, \ldots, K - 2)\) denotes the \((i, j)\)th element of \(R\).

- \([R]_{i,j}\) denotes the conditional probability that \(j\) MSs among the \((K - 2)\) MSs excluding MS 1 and MS 2 are in state 1 at time \(t\) given that \(i\) MSs among the \((K - 2)\) MSs was in state 1 at time \(t - 1\).
• Let $r$ denote the stationary probability vector of $R$.

• The stationary probability vector $r$ is given by

$$[r]_j = \binom{K - 2}{j} s_0^{K - 2 - j} s_1^j,$$  \hspace{1cm} (8)

where $s_0$ and $s_1$ are given by (4).

• We next define a $4(K - 1) \times 4(K - 1)$ matrix $Q$ by

$$Q = P \otimes P \otimes R,$$  \hspace{1cm} (9)

where $\otimes$ denotes the Kronecker product, $P$ is determined by (5) and (6), and $R$ is defined by (7).
Next we consider a $4(K - 1) \times 4(K - 1)$ diagonal matrix $D(z)$ expressing the service rule of scheduler. So $D(z)$ depends on schedulers.

- We first consider the matrix $D(z)$ of 1FF scheduler.

- The matrix $D(z)$ of 1FF scheduler is given by

$$D(z) = \text{diag}(d_{0,0}(z), d_{0,1}(z), d_{1,0}(z), d_{1,1}(z)),$$

where $d_{i,j}(z)$ ($i, j = 0, 1$) is a $1 \times (K - 1)$ vector given by

$$[d_{0,0}(z)]_k = \begin{cases} 
\frac{z + z^{-1} + K - 2}{K} & (k = 0), \\
1 & (\text{otherwise}),
\end{cases}$$

$$[d_{0,1}(z)]_k = \frac{z^{-1} + k}{k + 1},$$

$$[d_{1,0}(z)]_k = \frac{z + k}{k + 1}, \quad [d_{1,1}(z)]_k = \frac{z + z^{-1} + k}{k + 2},$$

for $k = 0, \ldots, K - 2$. 
• We next consider the matrix $D(z)$ of OFF scheduler.

• Let $\psi(k, n, x)$ denote the probability that given that $k$ MSs is in state 1, the number of minislots is equal to $n$ and the feedback probability is equal to $x$, the $k$ MSs fail to feed back. For $k = 0, \ldots, K - 2, n = 1, 2, \ldots$ and $0 \leq x \leq 1$, $\psi(k, n, x)$ is given by

$$\psi(k, n, x) = [1 - k(1 - x)^{k-1}x]^n.$$
We then define a $4(K-1) \times 4(K-1)$ diagonal matrix $D(z)$ by

$$D(z) = \text{diag}(d_{0,0}(z), d_{0,1}(z), d_{1,0}(z), d_{1,1}(z)),$$  \hspace{1cm} (10)

where $d_{i,j}(z)$ ($i,j = 0, 1$) is a $1 \times (K-1)$ vector given by

$$[d_{0,0}(z)]_k = \psi(k, N, u) \frac{z + z^{-1} + K - 2}{K} + 1 - \psi(k, N, u),$$

$$[d_{0,1}(z)]_k = \psi(k + 1, N, u) \frac{z + z^{-1} + K - 2}{K} + (1 - \psi(k + 1, N, u)) \frac{z^{-1} + k}{k + 1},$$

$$[d_{1,0}(z)]_k = \psi(k + 1, N, u) \frac{z + z^{-1} + K - 2}{K} + (1 - \psi(k + 1, N, u)) \frac{z + k}{k + 1},$$

$$[d_{1,1}(z)]_k = \psi(k+2, N, u) \frac{z + z^{-1} + K - 2}{K} + (1 - \psi(k+2, N, u)) \frac{z + z^{-1} + k}{k + 2},$$

for $k = 0, \ldots, K - 2$. 
• We further define $4(K - 1) \times 4(K - 1)$ matrix $C(z)$ by

$$C(z) = D(z)Q,$$

(11)

where $D(z)$ and $Q$ are defined by (10) and (9), respectively.

• Finally, we define $\eta_n(z) \ (n = 1, 2, \ldots)$ by

$$\eta_n(z) = (r^{(1)} \otimes r^{(2)} \otimes r)C(z)^n e,$$

where $r^{(i)}$ denotes the initial state probability vector of the MC $\{s^{(i)}(t)\}$ for $i = 1, 2$, $r$ denotes the probability vector given by (8), $e$ denotes a $4(K - 1) \times 1$ vector whose elements are all equal to one, and $C(z)$ is defined by (11).
• We are now ready to present the analysis of the STAIFI $G_n(x)$.

• Note that $\eta_n(z)$ can also be expressed in the power series of $z$ as $\eta_n(z) = \sum_{l=-n}^{n} c_l z^l$, where $c_l$ ($l = -n, \ldots, n$) is a (unknown) real constant satisfying $0 \leq c_l \leq 1$ and $\sum_{l=-n}^{n} c_l = 1$.

• Then the probability mass function $g_n(x)$ is expressed as $g_n(x) = c_x + c_{-x}$.

• Thus, if we determine the unknown real constants $\{c_l\}_{l=-n}^{n}$, we obtain the probability mass function $g_n(x)$.

• The STAIFI $G_n(x)$ is then given by

$$G_n(x) = \sum_{l=x}^{n} g_n(l) = 1 - \sum_{l=0}^{x-1} g_n(l) = 1 - c_0 - \sum_{l=1}^{x-1} (c_l + c_{-l}).$$
Inverse discrete FFT method

• The STAFI $G_n(x)$ can be expressed in terms of $2n+1$ unknown constants $\{c_l\}_{l=-n}^{n}$ (depend on $n$) as

$$G_n(x) = 1 - c_0 - \sum_{l=1}^{x-1} c_l + c_{-l}$$

• We can determine the unknown constants by using the inverse discrete FFT method [17].

• By using this numerical method, we can calculate the exact value of the STAFI.
Large deviations

- Although the numerical method based on the inverse discrete FFT method provides the exact value of the STAFI $G_n(x)$, it is very time-consuming when $n$ is large.

- By using the theory of large deviations, we can estimate how fast the STAFI $G_n(nx)$ decreases as $n \to \infty$. 
The following Proposition shows that the STAFI $G_n(nx)$ exponentially decreases as $n \to \infty$.

**Proposition 1:**
For $0 \leq x < 1$,

$$\lim_{n \to \infty} \frac{1}{n} \log G_n(nx) = -\Lambda^*(x),$$

where $\Lambda^*(a)$ is defined by $\Lambda^*(a) = \sup_{\theta} [\theta a - \log \delta_C(\theta)]$, $\delta_C(\theta)$ denotes the Perron-Frobenius eigenvalue of the matrix $C(e^{\theta})$. Here the matrix $C(z)$ is defined in (12).

We call the term $\Lambda^*(x)$ the asymptotic decay rate (ADR) of the STAFI $G_n(nx)$.
Numerical Results
• Provide numerical results to investigate the properties of the STAFI of 1FF scheduler and those of OFF scheduler

• Fix the parameters
  - mobility-induced Doppler spread of MSs = 10 Hz
  - the length of one slot = 1 msec
Numerical results for 1FF scheduler
Initial State Setting

- Initial states \("ij\) \((i, j = 0, 1)\): MS 1 and 2 start with state \(i\) and \(j\) w.p.1, respectively. All the other MSs start according to the stationary probability.

- Initial states \("ss\): All the MSs start according to the stationary probability.
Effect of threshold value on STAFI

• Observe how the initial state setting affects the effect of the threshold $\gamma_1$ on STAFI $G_{256}(x)$.

• Figs. 1-3 show the STAFI of 1FF as a function of $x$.

• "1FF(ydB)": 1FF scheduler whose threshold $\gamma_1$ is equal to $y$ dB.

• For comparison, also shows the STAFI of the random scheduler (RS) which randomly selects a MS among $K$ MSs irrespective of their received SNRs.

• "RS": the random scheduler
Fig. 1: STAFI $G_{256}(x)$ as a function of $x$ for initial state setting “00”.
Fig. 2: STAFI $G_{256}(x)$ as a function of $x$ for initial state setting “01”.
Fig. 3: STAFI $G_{256}(x)$ as a function of $x$ for initial state setting “ss”.
**Observation in Figs. 1, 2 and 3**

- For initial state settings “00” and “ss”, for small $x$ of $G_{256}(x)$, 1FF with larger threshold provides better fairness than that with smaller threshold.
- But the situation is converse for large $x$ of $G_{256}(x)$.
- Thus, 1FF scheduler with larger threshold can keep the probability of moderate unfairness lower, but it can cause serious unfairness with high probability.
On the other hand, for initial state settings “01”, for all $x$ of $G_{256}(x)$, 1FF with smaller threshold simply provides better fairness than that with larger threshold.
Reason of serious unfairness with large threshold

• First, suppose that threshold is large.
• In many sample paths, all MSs including MS 1 and MS 2 start in state 0 and likely to stay in state 0 during a certain period.
• The realization probability of such sample paths is large.
• If such sample paths are realized, serious unfairness is not caused, because one MS among all the MSs is randomly selected for service slot-by-slot.
• On the other hand, there exist sample paths where MS 1 and all the other MSs including MS 2 start in state 1 and in state 0, respectively.

• In these sample paths, MS 1 is surely selected for service in the first slot and MS 1 is likely to be continuously selected during a certain period due to the positive correlation of the normalized SNR processes in time.

• Although the realization probability of such sample paths is small, if such sample paths are realized, serious unfairness is caused.
Conversely, suppose that threshold is small.

In many sample paths, all MSs including MS 1 and MS 2 start in state 1 and likely to stay in state 1 during a certain period.

The realization probability of such sample paths is large.

If such sample paths are realized, serious unfairness is not caused, because one MS among all the MSs is randomly selected for service slot-by-slot.
• On the other hand, there exist sample paths where MS 1 and all the other MSs including MS 2 start in state 0 and in state 1, respectively.

• In these sample paths, one MS of all the other MSs except for MS 1 is randomly selected for service slot-by-slot during a certain period.

• Thus, even in such sample paths, the probability of causing serious unfairness is quite low, compared to the case where threshold is large, because MS 2 is selected for service with probability $1/(K-1)$. 
How STAFI changes as the progress of time

- Investigate how STAFI changes as the progress of time and approaches to the ideal fairness.
- For this purpose, consider the STAFI $G_n(hn/K)$ as a function of $n$, where $h$ is a parameter and $K$ denotes the number of MSs.

- Note here that the term $n/K$ denotes the expected access-time which each user receives when the ideal fairness is achieved.
- Thus, the STAFI $G_n(hn/K)$ is considered as a measure indicating a deviation from the ideal fairness, where $h$ is a parameter.
• Figs. 4 and 5 display the STAFI $G_n(hn/K)$ as a function of the observation period $n$ for $h = 2.0, 4.0$, respectively.

• In addition, to confirm that the estimated ADR is identical to the actual ADR, show the exponential decay lines $\exp(-n\Lambda^*(h/K))$ for $h = 2.0, 4.0$, respectively, where $\Lambda^*(h/K)$ is the estimated ADR of the STAFI from Proposition 1.

• Set the number of MSs to 16 and the threshold to 2.74dB.
Fig. 4: STAFI $G_n(hn/K)$ as a function of $n$ ($h = 2.0$)
Fig. 5: STAFI $G_n(\frac{hn}{K})$ as a function of $n$ ($h = 4.0$)
Observation in Figs. 4 and 5

• In asymptotic sense, the STAFI $G_n(hn/K)$ decreases exponentially.

• For all the initial state settings, the decay rates of the STAFI become nearly equal to the asymptotic decay rate after around 512msec.
Change of STAFl as progress of time

- Examine how the STAFl of 1FF scheduler changes as the progress of time.
- Figs. 3 and 4 exhibit the STAFl $G_n(hn)$ as a function of $h$ for $n = 64, 128, 256, 512$.
- Number of MSs=30, threshold $\gamma_1 = 3.78$ dB in Fig 3 and $\gamma_1 = 2.00$ dB in Fig 4.
Numerical results for OFF scheduler
• Here set the initial state probability vectors of MS 1 and MS 2 to the stationary probability vector.
Effect of threshold value on STAFI

- Observe the effect of the threshold value \( \gamma_1 \) on STAFI \( G_{256}(x) \).
- Fig. 6 shows the STAFI of OFF as a function of \( x \).
- “OFF(\( y dB \))”: OFF scheduler whose threshold \( \gamma_1 \) is equal to \( y dB \).
- For comparison, also show the STAFI of the random scheduler (RS) which randomly selects a MS among \( K \) MSs irrespective of their received SNRs.
- “RS”: the random scheduler
• Number of MSs: $K = 30$
• Number of minislots: $N = 8$
• Feedback probability: $u = 0.8$
Fig. 6: STAFI \( G_{256}(x) \) as a function of \( x \)
Observation in Fig. 6

• For whole range, the STAFIs of OFF schedulers are greater than the STAFI of RS.
  o In other words, the short term fairness provided by OFF schedulers is worse than that provided by RS.
  o This is due to the positive correlation of the normalized SNR process in time.
• For whole range, OFF (2.00 dB) yields better short term fairness than OFF (4.00 dB) and OFF (6.00 dB).
• Comparing OFF (4.00 dB) and OFF (6.00 dB), we observe that for small $x$ of $G_{256}(x)$, OFF (6.00 dB) provides better fairness than OFF (4.00 dB). However, the situation is converse for large $x$ of $G_{256}(x)$.
• Thus, OFF with larger threshold can keep the probability of moderate unfairness lower, but it can cause serious unfairness with higher probability, compared to OFF with smaller threshold.
How STAFI changes as the progress of time

• Investigate how STAFI changes as the progress of time and approaches to the ideal fairness.

• For this purpose, consider the STAFI \( G_n(hn) \) as a function of \( h \). It shows the probability that the service difference is greater than or equal to \( h \times n \) (observation period).

• Figs. 7 and 8 show the STAFI \( G_n(hn) \) as a function of \( h \).
• Number of MSs: $K = 30$

• Number of minislots: $N = 5$

• Feedback probability: $u = 0.8$
Fig. 7: STAFI $G_n(hn)$ as a function of $h$ ($\gamma_1 = 4.00$ dB)
Fig. 8: STAFI $G_n(hn)$ as a function of $h$ ($\gamma_1 = 2.00$ dB)
Observation in Figs. 7 and 8

- STAFI $G_n(hn)$ rapidly decreases with increase of the observation period.
- In other words, the STAFI rapidly approaches the ideal long term fairness as the progress of time.
Effect of number of minislots

- Observe the effect of the number of minislots \( N \) on the STAFI \( G_n(x) \).
- Fig. 9 shows the STAFI \( G_{256}(x) \) as a function of \( x \).
- Number of MSs: \( K = 30 \)
- Threshold: \( \gamma_1 = 4.00 \) dB
- Feedback probability: \( u = 0.80 \)
Fig. 9: Effect of number of minislots $N$ on $\text{STAFI} \ G_{256}(x)$
Observation in Fig. 9

- With the increase in the number of minislots $N$, the short term fairness of OFF schedulers becomes worse.
Effect of feedback probability

- Observe the effect of the feedback probability $\mu$ on the STAFI $G_n(x)$.
- Fig. 10 displays the STAFI $G_{256}(x)$ as a function of $x$.
- Number of MSs: $K = 30$
- Threshold: $\gamma_1 = 4.00$ dB
- Number of minislots: $N = 10$
Fig. 10: Effect of feedback probability $u$ on STAFI $G_{256}(x)$
Observation in Fig. 10

- Among the four OFF schedulers \((u = 0.2, 0.4, 0.6, 0.8)\), the schedulers with the feedback probability \(u = 0.4\) yields the worst short term fairness.
Conclusion
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• We focus on 1FF scheduler and OFF scheduler and numerically study the STAFIs of 1FF scheduler and OFF scheduler to understand their short term fairness properties.
• For this purpose, we develop the two numerical methods.
• Numerical results show that the threshold greatly affects the short term fairness properties of 1FF scheduler and those of OFF scheduler.