Aeroacoustics of Aircraft Engine

Seminar Presented at
Basque Center for Applied Mathematics (BCAM)

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June 13, 2013
Background

Assistant Professor, Colorado Mesa University (since Oct. 2012)

Mechanical Engineer, General Electric (GE) Research Center (USA)
- Aerodynamics and aeroacoustics (wind turbines, GE9X and Open Rotor engines)

Post-doctoral Fellow (McGill University, Canada)
- Rotorcraft noise simulation

Ph.D. Mechanical Engineering (McGill University, Canada)
- Parallel simulations of turbofan inlet noise propagation

M.Sc. Aerospace Engineering (Wichita State University, USA)
- Propeller/rotor noise simulation & performance analysis

M.A.Sc. Mechanical Engineering (Concordia University, Canada)
- Least-squares finite element method for Euler equations with mesh adaptation

B.Sc. Mechanical Engineering (University of Tabriz, Iran)
- Thermodynamic design of a 4-stage gas turbine
3D Parallel Computations of Turbofan Inlet Noise Propagation
Why fan noise simulation?

- Environment Impact (noise around the airports)
- Passenger comfort
- Stricter standards:
  - ICAO Chapter 4 (2006)
  - 10dB Lower margin
- NASA Challenge
  - cum margin below Stage 4:
    - N+2: -42 dB (2020)
    - N+3: -71 dB (2025)
- Business competitiveness
Motivation

- Accurate and reliable state-of-the-art analysis and design tool
- Moving from 2D to 3D simulations
- Practical solution times: hours and not weeks
- Handling complex geometries with industrial relevance
- Taking advantage of parallel computers
Mathematical Formulation

- Governing equations
  - Continuity: \[ \frac{\partial \rho^*}{\partial t^*} + \nabla \cdot (\rho^* \vec{V}^*) = 0 \]
  - Momentum: \[ \frac{\partial \vec{V}^*}{\partial t^*} + \vec{V}^* \cdot \nabla \vec{V}^* = -\frac{\nabla p^*}{\rho^*} \]

- Assumptions:
  - Mean flow in the x-direction; Air as an ideal gas
  - Irrotational flow: \( \vec{V}^* = \nabla \Phi^* \)
  - Linearization: \( \rho^* = \rho_0 + \rho, \quad \text{and} \quad \Phi^* = \Phi_0 + \Phi \)
Linearization

Mean flow

- **Continuity:** \( \nabla \cdot (\rho_0 \nabla \Phi_0) = 0 \)

- **Momentum:** \( \rho_0 = \left[ 1 - \left( \frac{\gamma - 1}{2} \right) (\nabla \Phi_0 \cdot \nabla \Phi_0 - M_\infty^2) \right]^{1/(\gamma-1)} \)

Acoustics

- **Continuity:** \( \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho_0 \nabla \Phi + \rho \nabla \Phi_0) = 0 \)

- **Momentum:** \( \rho = -\frac{\rho_0}{c_0^2} \left[ \frac{\partial \Phi}{\partial t} + (\nabla \Phi_0 \cdot \nabla \Phi) \right] \)

Decoupled equations
Spatial Discretization

- Spectral Element Method
  - Geometric flexibility of FEM
  - Superior to $p$-version FEM
  - Reduced points per wavelength (PPW)
  - Spectral accuracy
Spatial Discretization

- Function approximation:
  \[ \phi = \sum_{ijk} h_i(\xi) h_j(\eta) h_k(\zeta) \phi^e_{ijk} \]

- Interpolating polynomials:
  \[ h_m(\sigma) = \frac{2}{Nc_m} \sum_{n=0}^{N} \frac{1}{c_n} T_n(\sigma_m) T_n(\sigma) \]

- Chebyshev function:
  \[ T_n(x) = \cos(n \cos^{-1} x) \]

- Gauss-Chebyshev-Lobatto points:
  \[ \sigma_i = -\cos \left( \frac{\pi i}{N} \right), \quad i = 0, \ldots, N \]
Mean Flow Problem

- Method of weighted residuals (MWR)
  - Weak form (Newton method):
    \[
    \int_{\Omega} \left[ \rho_0 (\nabla \Psi_0 \cdot \nabla \delta \Phi_0) - \rho_0^{2-\gamma} (\nabla \Psi_0 \cdot \nabla \Phi_0) (\nabla \Phi_0 \cdot \nabla \delta \Phi_0) \right] d\Omega = \\
    - \int_{\Omega} \nabla \Psi_0 \cdot (\rho_0 \nabla \Phi_0) d\Omega + \int_{\Gamma} \Psi_0 (\rho_0 \nabla \Phi_0 \cdot \vec{n}) d\Gamma
    \]
  - Update: \( \Phi_0^{k+1} = \Phi_0^k + \delta \Phi_0 \)

- Mean flow velocity: \( \nabla \Phi_0 = (u_0, v_0, w_0) \)
Mean Flow Problem

- **Boundary conditions**

  - **Solid surfaces:** \( \nabla \Phi_0 \cdot \vec{n} = 0 \)

  - **Inlet (fan face):**
    \[
    \int_{\Gamma} \Psi_0 (\rho_0 \nabla \Phi_0 \cdot \vec{n}) \, d\Gamma = \frac{\dot{m}}{A_f} \int_{\Gamma_f} \Psi_0 \, d\Gamma
    \]

  - **Far-field:**
    \[ \Phi_0 = M_\infty x \]
Acoustic Problem

- Method of weighted residuals (MWR)

**Assumptions:**

- **Acoustic potential**
  \[ \Phi = \phi(x, y, z) e^{-i\omega t} \]

- **Test function**
  \[ \Psi = \psi(x, y, z) e^{i\omega t} \]

- **Reduced frequency**
  \[ \tilde{\omega} = \omega R/c_\infty \]

- **Weak form:**

\[
\int_{\Omega} \frac{\rho_0}{c_0^2} \left[ \tilde{\omega}^2 \phi \psi + \left( u_0^2 - c_0^2 \right) \phi_x \psi_x + \left( v_0^2 - c_0^2 \right) \phi_y \psi_y + \left( w_0^2 - c_0^2 \right) \phi_z \psi_z + u_0 v_0 \left( \phi_x \psi_y + \phi_y \psi_x \right) + u_0 w_0 \left( \phi_x \psi_z + \phi_z \psi_x \right) + v_0 w_0 \left( \phi_y \psi_z + \phi_z \psi_y \right) + i\tilde{\omega} u_0 \left( \phi \psi_x - \phi_x \psi \right) + i\tilde{\omega} v_0 \left( \phi \psi_y - \phi_y \psi \right) + i\tilde{\omega} w_0 \left( \phi \psi_z - \phi_z \psi \right) \right] \, d\Omega = \]

\[
- \int_{\Gamma} \Psi \left( \rho_0 \nabla \Phi + \rho \nabla \Phi_0 \right) \cdot \bar{n} \, d\Gamma,
\]
Acoustic Problem

- **Boundary conditions**
  - **Solid surfaces:** \( \nabla \Phi \cdot \mathbf{n} = 0 \)
  - **Inlet (acoustic source):**
    - Circular/annular
      \[ \Phi(x, r, \theta) = \varphi(r) e^{i(k_x x + m \theta - \omega t)} \]
    - Without centerbody
      \[ \varphi_{ms}(r) = \sum_s A_s J_m(k_{ms} r) \]
    - With centerbody
      \[ \varphi_{ms}(r) = \sum_s A_s J_m(k_{ms} r) + Y_m(k_{ms} r) \]
      - \( m \): spinning mode
      - \( s \): radial mode

Farzad Taghaddosi (6/13/2013)
Acoustic Problem

- **Far-field:**
  - Damping layer
  - Modified continuity equation
  - Damping function
  - Modified weak form

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho_0 \nabla \Phi + \rho \nabla \Phi_0) = -\nu(x) \rho
\]

\[
\nu(x) = \nu_0 \left| \frac{x - x_I}{D} \right|^n
\]

\[
-\int_{\Omega} (\nu \rho) \Psi \ d\Omega = \int_{\Omega} \frac{\nu \rho_0}{c_0^2} \left( i\omega \phi \psi + u_0 \phi_x \psi + v_0 \phi_y \psi + w_0 \phi_z \psi \right) \ d\Omega
\]
Solution Method

- Parallel approach
- Domain decomposition method:
  - Overlapping
  - Non-overlapping
Solution of the Mean Flow Problem

- Linear system: \( Au = f \)
  - Symmetric and positive-definite (SPD) matrix

- Parallel iterative solver: Conjugate Gradient (CG)

- Preconditioning
  - Additive Schwarz method (ASM)
  - Example:
Solution of the Acoustic Problem

- Linear system: \( Au = f \)
  - Complex-valued, non-symmetric & indefinite, ill-conditioned

- Parallel direct solver: SPOOLES, SuperLU, etc.
  - Prohibitively costly for large systems

- Parallel iterative solver
  - **Schur complement method**
    - Denser matrix
    - Better conditioned
    - Smaller in size
    - Computer architecture compatible
    - Complex parallel algorithm
Domain Decomposition

- Subdomain matrix:

\[ A^{(i)} = \begin{bmatrix}
A^{(i)}_{II} & A^{(i)}_{IB} \\
A^{(i)}_{BI} & A^{(i)}_{BB}
\end{bmatrix} \]

- Reduced system:

\[
\begin{bmatrix}
A^{(1)}_{II} & A^{(1)}_{IB} \\
A^{(2)}_{II} & A^{(2)}_{IB} \\
\vdots & \vdots \\
A^{(i)}_{II} & A^{(i)}_{IB}
\end{bmatrix}
\begin{bmatrix}
\begin{bmatrix}
u^{(1)}_I \\
u^{(1)}_I \\
\vdots \\
u^{(i)}_I \\
u_B
\end{bmatrix}
\end{bmatrix} =
\begin{bmatrix}
f^{(1)}_I \\
f^{(2)}_I \\
\vdots \\
f^{(i)}_I \\
g
\end{bmatrix}
\]
Building Schur Matrix

- Schur system:

\[ Su_B = g, \quad \text{where} \quad S = \sum_{i=1}^{p} \tilde{R}_i^T S_i \tilde{R}_i, \quad \text{and} \quad g = \sum_{i=1}^{p} \tilde{R}_i^T g^{(i)} \]

\[ S_i = A_{BB}^{(i)} - A_{BI}^{(i)} A_{II}^{(i)}^{-1} A_{IB}^{(i)} \quad g^{(i)} = f_B^{(i)} - A_{BI}^{(i)} A_{II}^{(i)}^{-1} f_I^{(i)} \]

- Restriction matrix (scatter operator): \[ u_B^{(i)} = \tilde{R}_i u_B \]

- Interior solve: \[ u_I^{(i)} = A_{II}^{(i)}^{-1} (f_I^{(i)} - A_{IB}^{(i)} u_B^{(i)}), \quad i = 1, 2, \ldots, p \]

- Solution method: matrix-free GMRES
Matrix-free Algorithm

Sample operations:

\[ w = Sv = \sum_{i=1}^{p} \tilde{R}_i^T \left( A_{BB}^{(i)} - A_{BI}^{(i)} A_{II}^{(i)}^{-1} A_{IB}^{(i)} \right) \tilde{R}_i v \]

- scatter \( v \): \( v_B^{(i)} = \tilde{R}_i v \),
- calculate: \( v_I^{(i)} = A_{IB}^{(i)} v_B^{(i)} \),
- solve for \( u_I^{(i)} \): \( A_{II}^{(i)} u_I^{(i)} = v_I^{(i)} \),
- calculate: \( w_B^{(i)} = A_{BB}^{(i)} v_B^{(i)} \),
- calculate: \( v_B^{(i)} = A_{BI}^{(i)} u_I^{(i)} \),
- update: \( w_B^{(i)} = w_B^{(i)} - v_B^{(i)} \),
- assemble: \( w = \sum_{i=1}^{p} \tilde{R}_i^T w_B^{(i)} \).
Preconditioner

- Proposed a preconditioner based on sub-domain Schur matrices:

$$M_P^{-1} = \sum_{i=1}^{p} \tilde{R}_i^T \tilde{D}_i A_{BB}^{(i)} \tilde{D}_i \tilde{R}_i$$

- Shorter calculations
- Local operator
- Elements of $A_{BB}^{(i)}$ are clustered around diagonal

- Other preconditioners examined:

$$M_B^{-1} = A_{BB}^{-1} = \left( \sum_{i=1}^{p} \tilde{R}_i^T A_{BB}^{(i)} \tilde{R}_i \right)^{-1}$$

$$M_J^{-1} = \left( \text{Diag}(A_{BB}) \right)^{-1}$$
Preconditioner Performance

<table>
<thead>
<tr>
<th>Preconditioner</th>
<th>None</th>
<th>$M_J^{-1}$</th>
<th>$M_P^{-1}$</th>
<th>$M_B^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution Time (min:sec)</td>
<td>10:55</td>
<td>9:39</td>
<td>9:29</td>
<td>14:58</td>
</tr>
<tr>
<td>No. of Iterations</td>
<td>261</td>
<td>223</td>
<td>173</td>
<td>129</td>
</tr>
</tbody>
</table>
Numerical Results
Uniform Cylinder

- $\bar{\omega} = 5.91$ (320 Hz)
- $L/R = 2.5$
- $E = 3258$, $N = 4$ (~220 000 Eqs)
Uniform Cylinder (Effect of Mean Flow)

- Effect of mean flow
  - $\bar{\omega} = 5.91$
  - $e/R = 1/8$

- $M_\infty = 0.2$
- (Mass flow rate)$_f = 50$ kg/s
Generic Scarfed Nacelle

- $\bar{\omega} = 17$ (1058 Hz)
- Propagating mode: $(13,0)$
- $E = 15328$, $N = 5$ ($\sim 2e6$ Eqs)

<table>
<thead>
<tr>
<th>No. of CPU’s</th>
<th>8</th>
<th>16</th>
<th>24</th>
<th>32</th>
<th>40</th>
<th>48</th>
</tr>
</thead>
<tbody>
<tr>
<td>Building Matrix</td>
<td>4:25</td>
<td>2:11</td>
<td>1:27</td>
<td>1:04</td>
<td>0:52</td>
<td>0:44</td>
</tr>
<tr>
<td>Solving Equations</td>
<td>7:41</td>
<td>3:40</td>
<td>2:07</td>
<td>2:19</td>
<td>2:06</td>
<td>1:05</td>
</tr>
<tr>
<td>Total (hr:min)</td>
<td>12:17</td>
<td>5:51</td>
<td>3:34</td>
<td>3:23</td>
<td>2:58</td>
<td>1:49</td>
</tr>
<tr>
<td>No. of Iterations</td>
<td>216</td>
<td>288</td>
<td>406</td>
<td>484</td>
<td>586</td>
<td>614</td>
</tr>
</tbody>
</table>
Generic Scarfed Nacelle

- Parallel efficiency

![Graph showing CPU Time vs. No. of Processors]
Acoustic Pressure Contours
Directivity

SPL directivity on horizontal plane
Directivity

SPL directivity on vertical plane (not shown)
Conclusions

What has been accomplished:
- Successful migration from 2D to 3D technology
- Richer physical modeling
- Practical solution times: hours instead of days, through:
  - Efficient new preconditioned iterative solvers
  - Efficient massive parallelism on commodity clusters
- CAD-based solution for complex industrial problems
- Thorough verification and validation against analytical, numerical and experimental results

Contributions can be classified as
- Engineering
- Mathematical
- Computational
Conclusions

- Development of a 3D aeroacoustics code
  - 14,000 line of instructions
  - Fan noise and ducted acoustic simulation
  - Accurate results
  - General 3D geometries
  - Accounting for mean flow effects
  - Symmetry formulation
  - Parallel computations

- Iterative solution
  - Schur complement
  - Novel preconditioner

- Excellent parallel efficiency
Open Rotor Project

Historical Perspective

- Born out of the energy crisis of the 1970’s

- Promise: speed and performance of a turbofan with the fuel economy of a turboprop

- 10-20% lower fuel consumption
- Thrust $\sim mv$
- Kinetic energy $\sim v^2$

- Major concern: noise

https://www.youtube.com/watch?v=zxVAalsfPIY
Noise Sources

- Blade self-noise
- Blade-Blade interaction
  - Tip vortex
  - Wake
- Pylon-Blade interaction
- Compressor
- Core

Courtesy: www.kaist.ac.kr
Noise Analysis

- **Tone**
  - Linearized Euler Equations (LEE)
  - Blade response to incoming gusts (1BPF, 2BPF, etc.)

- **Broadband**
  - Integral formulations based on acoustic analogy
    - (Amiet, etc.)
Noise Reduction

- Typically highest during take-off

- Reduce tip vortex & wake strength
  - Better blade aerodynamic design
  - Optimize blade spacing
  - Trim aft blade to avoid/reduce vortex impingement
  - Increase front rotor RPM at take-off & reduce incidence

- Wind tunnel tests done at NASA Glenn

- In July 2012, GE announced:

  “Open rotor noise NOT a barrier for entry into service”