HEURISTIC OPTIMIZATION TO DESIGN SOLAR POWER TOWER SYSTEMS

Carmen-Ana Domínguez Bravo

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1. Introduction

2. Optimization problem

3. Analytical and computational model

4. Design of Heliostats Field

5. Design with multiple receivers
THE DESIGN OF A SOLAR POWER TOWER (SPT)

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Thesis tutor: Manuel Quero (Abengoa Solar)
Research contract: CapTorSol 1200-0494 (Abengoa Solar and FIUS)
Solar Power Plants

Solar Thermal
- Non-Concentrating
  - Solar Chimney
- Concentrating
  - Linear
    - Linear Fresnel
  - Point
    - Parabolic Through
    - Parabolic Dish
    - Heliostat

Photovoltaic
- PV cell
SOLAR POWER TOWER PLANTS

- Heliostat: reflective mirror.
- Receiver: sunlight collector.

1. Sunlight is reflected by the heliostats field onto a receiver at the top of the tower.
2. Thermal energy is transferred through a thermodynamic cycle to produce electricity.

Source: Abengoa
Heliostat

Helio: Greek word for sun.
Stat: stationary (reflected image is fixed).

- Pedestal mounted mirror.
- Two-axis tracking system to follow the sun.

Source Abengoa and CIEMAT-PSA
**HELIOSTAT**

**Helio**: Greek word for sun.
**Stat**: stationary (reflected image is fixed).

- Pedestal mounted mirror.
- Two-axis tracking system to follow the sun.

Source: Abengoa and CIEMAT-PSA
Tower-Receiver(s)

$$\Theta := (h, \xi, \alpha, r)$$

Heliostats Field

$$S := \{(x_i, y_i) \in \mathbb{R}^2 : i \in [1, N]\}$$
MATHEMATICAL MODELLING
OPTIMIZATION PROBLEM

\[
(P) \begin{cases}
\min_{\Theta, S} F(\Theta, S) = C(\Theta, S) / E(\Theta, S) \\
\text{subject to} \quad \Theta \in \Theta \\
\quad \quad S \in \mathcal{S} \\
\quad \quad \Pi_0 \leq \Pi_{T_d}(\Theta, S) \leq \Pi^+
\end{cases}
\]

- Optimization criteria
  - 2 objectives: total construction cost and collected annual energy.
  - Deal with the aggregation function,
MATHEMATICAL MODELLING

Tower (height), receiver aperture (position and dimensions), and heliostats field (positions and number).

▶ Main difficulties
  ▶ unknown number of variables,
  ▶ high number of heliostats in commercial plants (up to 200,000),
  ▶ expensive objective function evaluation,
  ▶ nonclosed-form of the objective function,
  ▶ non-convex constraints.

▶ Assumptions
  ▶ circular aperture,
  ▶ the aim point is the aperture center and it is static,
  ▶ the receiver heat flux is homogeneous,
  ▶ shadowing & blocking effects consider parallel heliostats,
  ▶ costs associated with hel. are independent on the position.
**ANALYTICAL AND COMPUTATIONAL MODEL**

**COST FUNCTION**

\[
C(h, r, |S|) = \beta_1 (h + \lambda_1)^{\lambda_2} + \beta_2 \pi r^2 + c_f + c |S|
\]

- \(h\): tower height.
- \(r\): aperture radius.
- \(|S|\): number of heliostats.
- Tower-receiver and heliostats field costs.
- Easy to compute and low number of variables.
ANALYTICAL AND COMPUTATIONAL MODEL
ANNUAL THERMAL ENERGY FUNCTION

▶ Analytical models
▶ Analytical formula and numeric algorithms to evaluate the plant performance.
▶ Accurate enough for the optimization process, e.g. Fortran code NSPOC\(^1\).

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Source Article\(^3\).


**Analytical and Computational Model**

*Annual Thermal Energy Function*

- **Analytical models**
  - Analytical formula and numeric algorithms to evaluate the plant performance.
  - Accurate enough for the optimization process, e.g. Fortran code *NSPOC*\(^1\).

- **Ray-tracing models**
  - Tracing light trajectories and simulate the effects of its interactions with digitized objects.
  - Traditionally used to evaluate the plant performance, e.g. *SolTRACE*\(^2\).

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ANNUAL THERMAL ENERGY FUNCTION

\[ E(\Theta, S) = \int_0^T \Pi_t(\Theta, S) \, dt - \gamma_1 \]

\[ l(t) \sum_{(x,y) \in S} \varphi(t, x, y, \Theta, S)A(x, y) - \gamma_2 \pi r^2 \]
ANNNUAL THERMAL ENERGY FUNCTION

\[ E(\Theta, S) = \int_0^T \Pi_t(\Theta, S) \, dt - \gamma_1 \]

\[ l(t) \sum_{(x, y) \in S} \varphi(t, x, y, \Theta, S) A(x, y) - \gamma_2 \pi r^2 \]

- \( I \): solar radiation,
- \( A \): heliostat area,
- \( \varphi \): solar efficiency functions: \( \prod_{i=1}^{5} \nu_i \) with \( \nu_i \in [0, 1] \).
ANNUAL THERMAL ENERGY FUNCTION
SOLAR EFFICIENCY FUNCTIONS

1. Reflectivity

2. Cosine

\[
\sqrt{\frac{1}{2} + \frac{\vec{w}(x,y) \cdot \vec{v}_{\text{sun}}(t)}{2 ||\vec{w}(x,y)||}}
\]

3. Interception

\[
f_1(t, x, y) \int \int_S \exp \left( -\frac{f_2(u,v,x,y)}{2 f_3^2(t,x,y,\Theta)} \right) \, du \, dv
\]

4. Atmospheric

\[
\alpha_1 - \alpha_2 ||\vec{w}(x,y)|| + \alpha_3 ||\vec{w}(x,y)||^2
\]

5. Shading & blocking

Sassi’s algorithm\(^1\)

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\(^1\) G. Sassi. “Some notes on shadow and blockage effects”. In: Solar Energy 31.3 (1983), pp. 331–333

**Solar Efficiency Functions Variations**

- **Cosine**
- **Interception**
- **Atmospheric**
- **Shading and blocking**
OPTIMIZATION PROBLEM
TOWER-RECEIVER AND HELIOSTATS FIELD

\[
(\mathcal{P}) \begin{cases}
\min_{\Theta, S} & F(\Theta, S) = \frac{C(\Theta, S)}{E(\Theta, S)} \\
\text{subject to} & \Theta \in \Theta \\
& S \in \mathcal{S} \\
& \Pi_0 \leq \Pi_{T_0}(\Theta, S)
\end{cases}
\]

- Separate \((\mathcal{P})\) into 2 sub-problems:
  1. \((\mathcal{P}_S)\): fixed heliostats field, optimize tower and receiver.
  2. \((\mathcal{P}_\Theta)\): fixed tower-receiver, optimize heliostat field.
Optimal Design ($\mathcal{P}$)
* Tower-Receiver
* Heliostat Field

Alternating algorithm

Start
Initial Tower-Receiver Design

Calculate Initial Hel. Field

Optimize Tower-Receiver

Optimize Heliostat Field

Objective value improved?

Yes

Best Configuration

No

Stop
Receiver variables and feasible set:

- $\Theta = (h, r, \xi, \alpha)$,

- $\Theta = \{ \Theta \in \mathbb{R}^4 : r_{\text{min}} \leq r \leq \min(h, r_{\text{max}}) \leq h_{\text{max}} \}$. 

![Diagram of tower-receiver design with variables and coordinates labeled: x-North, y-West, z-axis, q_e, d_ap, r, h, ξ.](image)
**DESIGN TOWER-RECEIVER**

\[
\begin{align*}
(\mathcal{P}_S) \quad & S \text{ fixed} \\
& \max_{\Theta} \quad F(\Theta, S) = \frac{C(\Theta, S)}{E(\Theta, S)} \\
& \text{subject to} \quad \Theta \in \Theta \\
& \Pi_0 \leq \Pi_{T_d}(\Theta, S)
\end{align*}
\]

- Number of variables fixed and low.
- Non-convex objective function.
- Seems easy to solve (empirically unimodal).
**Design Tower-receiver**

\[ (P_S) \quad S \text{ fixed} \]

\[
\begin{align*}
\max_{\Theta} & \quad F(\Theta, S) = \frac{C(\Theta, S)}{E(\Theta, S)} \\
\text{subject to} & \quad \Theta \in \Theta \\
& \quad \Pi_0 \leq \Pi_{Td}(\Theta, S)
\end{align*}
\]

**Resolution**

Cyclic coordinate method and local searches for each variable.

<table>
<thead>
<tr>
<th>( \cdot \in [r_{\min}, r_{\max}] )</th>
<th>( h \in [h_{\min}, h_{\max}] )</th>
</tr>
</thead>
<tbody>
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<td>280</td>
<td>0.00 0.00 29.54 27.44 24.65 21.79 19.06 16.55 14.31</td>
</tr>
<tr>
<td>252.1</td>
<td>0.00 0.00 33.42 30.66 27.24 23.81 20.61 17.72 15.19</td>
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<tr>
<td>224.2</td>
<td>0.00 0.00 37.58 34.05 29.90 25.84 22.12 18.85 16.03</td>
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<tr>
<td>196.3</td>
<td>0.00 0.00 41.90 37.50 32.53 27.80 23.55 19.88 16.77</td>
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<tr>
<td>168.4</td>
<td>0.00 0.00 46.17 40.84 35.01 29.59 24.82 20.78 17.40</td>
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<td>140.5</td>
<td>0.00 0.00 50.14 43.85 37.18 31.10 25.86 21.48 17.88</td>
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<tr>
<td>112.6</td>
<td>0.00 0.00 53.49 46.32 38.90 32.24 26.61 21.96 18.18</td>
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<td>84.7</td>
<td>0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00</td>
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<tr>
<td>56.8</td>
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</tr>
<tr>
<td>28.9</td>
<td>0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00</td>
</tr>
<tr>
<td>1</td>
<td>0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00</td>
</tr>
</tbody>
</table>

| 1 | 3.375 | 5.75 | 8.125 | 10.5 | 12.875 | 15.25 | 17.625 | 20 |
DESIGN OF HELIOSTATS FIELD

- Heliostats positions

\[ S = \left\{ (x_i, y_i) \in \mathbb{R}^2 : i \in [1, N] \right\} \]

- Feasible set \( \mathcal{S} \)

\[ r_{\text{min}} \leq \sqrt{x_i^2 + y_i^2} \leq r_{\text{max}} \]
\[ \| (x_i, y_i) - (x_j, y_j) \| \geq \delta \]
Design of Heliostats Field

- **Heliostats positions**
  \[ S = \{(x_i, y_i) \in \mathbb{R}^2 : i \in [1, N]\} \]

- **Feasible set \( \mathcal{S} \)**
  \[ r_{\text{min}} \leq \sqrt{x_i^2 + y_i^2} \leq r_{\text{max}} \]
  \[ ||(x_i, y_i) - (x_j, y_j)|| \geq \delta \]
**Design of Heliostats Field**

\[(\mathcal{P}_\Theta) \quad \Theta \text{ fixed} \quad \left\{ \begin{array}{l}
\min_{\mathcal{S}} \\
\text{subject to}
\end{array} \right.
\]

\[
F(\Theta, \mathcal{S}) = \frac{C(\Theta, \mathcal{S})}{E(\Theta, \mathcal{S})}
\]

- Non-fixed number of variables (expected to be high).
- Non-convex objective function.
- Non-convex constraints.
- Many local optima.

“packing problem with interactions between circles”
DESIGN OF HELIOSTATS FIELD
STATE-OF-THE-ART METHODS: PATTERN-BASED

- Select a geometric pattern (radial-stagger, spiral, grid, etc.)
- Pattern described by a low number of parameters.
- Optimize the parameters with standard procedures (Powell, Genetic, etc.)

Fig: Source Abengoa
DESIGN OF HELIOSTATS FIELD
PATTERN-FREE

- Solve future challenges (flexible algorithm)

Fig: Multiple Receivers.
Source Abengoa (Patent US 2012/0125000)

Fig: Multi Tower Solar Array.
Source Schramek, 2013.
DESIGN OF HELIOSTATS FIELD
PATTERN-FREE

- Solve future challenges (flexible algorithm)

Greedy algorithm

- greedy-based heuristic
- pattern free to be flexible
- multi-start to avoid local optima
GREEDY ALGORITHM

- Heliostats located one by one, without fixed pattern.
GREEDY ALGORITHM

- Heliostats located one by one, without fixed pattern.
- Step $k$: optimization pb in 2 variables.
  - Locate heliostat number $k$
  - $k - 1$ heliostats in the field $S^{k-1}$
  - Constraints involving $S^{k-1}$

$$\max_{(x,y) \in F_S} \tilde{E}(x, y, S^{k-1})$$
subject to
$$\Pi_0 \leq \Pi_{T_d}(\Theta, S)$$
$$\|(x, y) - (x^i, y^i)\| \geq \delta \text{ for } i = 1, \ldots k - 1$$
GREEDY ALGORITHM

- Heliostats located one by one, without fixed pattern.
- Step $k$: optimization pb in 2 variables.
  - Locate heliostat number $k$.
  - $k-1$ heliostats in the field.
  - Constraints involving $S^{k-1}$.
- Multi-start strategy to avoid local optima.
  - $N_{ini}$ initial solutions.
  - Final solution with highest energy value.
  - Avoid local optima due to shadowing & blocking.

Annual Energy Values per unit area

![Annual Energy Values per unit area graph](image-url)
**Greedy Algorithm**

- Heliostats located one by one, without fixed pattern.
- Step $k$: optimization pb in 2 variables.
  - Locate heliostat number $k$.
  - $k - 1$ heliostats in the field.
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  - $N_{\text{ini}}$ initial solutions.
  - Final solution with highest energy value.
  - Avoid local optima due to shadowing & blocking.

---

Annual Energy Values per unit area with S and B
GREEDY ALGORITHM

- Heliostats located one by one, without fixed pattern.
- Step $k$: optimization pb in 2 variables.
  - Locate heliostat number $k$.
  - $k - 1$ heliostats in the field.
  - Constraints involving $S^{k-1}$.
- Multi-start strategy to avoid local optima.
  - $N_{\text{ini}}$ initial solutions.
  - Final solution with highest energy value.
  - Avoid local optima due to shadowing & blocking effects.
GREEDY ALGORITHM

- Heliostats located one by one, without fixed pattern.
- Step $k$: optimization pb in 2 variables.
  - Locate heliostat number $k$.
  - $k - 1$ heliostats in the field.
  - Constraints involving $S^{k-1}$.
- Multi-start strategy to avoid local optima.
  - $N_{ini}$ different random solutions.
  - Perform local search.
  - Final solution, the one with highest energy value.
- Determine the final number of heliostats.
  - $\Pi_0 \leq \Pi_{T_d}(x, y, S^{k-1}) \rightarrow$ feasible solution.
  - Continue locating heliostats.
  - Stop when the objective function $C/E$ does not improve.
  - Final $N$. 
GREEDY ALGORITHM

- Heliostats located one by one, without fixed pattern.
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**Greedy Algorithm**

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DESIGN OF HELIOSTATS FIELD
COMPARATIVE RESULTS

Radial-staggered
Heliostat Field Layout $N_{\text{hel}} = 624$

Spiral
Heliostat Field Layout $N_{\text{hel}} = 624$

Greedy
Heliostat Field Layout $N_{\text{hel}} = 624$

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## Design of Heliostats Field
### Comparative Results

<table>
<thead>
<tr>
<th>Field</th>
<th>$N$</th>
<th>$\Pi_{T_d}$</th>
<th>$E$</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PS10</td>
<td>624</td>
<td>0.43</td>
<td>0.12</td>
<td>0.0120</td>
</tr>
<tr>
<td>Spiral</td>
<td>624</td>
<td>0.42</td>
<td>0.12</td>
<td>0.0120</td>
</tr>
<tr>
<td>GPS10 Requirement Phase</td>
<td>624</td>
<td>0.43</td>
<td>0.12</td>
<td>0.0120</td>
</tr>
<tr>
<td>GPS10 Completion Phase</td>
<td>943</td>
<td>0.62</td>
<td>0.17</td>
<td><strong>0.0111</strong></td>
</tr>
</tbody>
</table>

Thermal power at $T_d$, $\Pi_{T_d}$ (MWth $10^{-2}$).

Annual thermal energy, $E$ (GWHth $10^{-3}$).

Cost function, $C$ (M€),

Cost per unit of annual thermal energy, $F = C/E$. 
DIFFERENT FEASIBLE REGIONS
RECTANGULAR, PERFORATED AND VALLEY REGIONS

Feasible Region 1

Feasible Region 2

Feasible Region 3
(d) PS10
(e) Requirement
(f) Completion
(g) RPS10
(h) Requirement
(i) Completion
**DIFFERENT FEASIBLE REGIONS**

*Rectangular, perforated and valley regions*

|     | Reg   | $S$ | $|S|$ | $\Pi_{T_d}$ | $\Pi^+$ | $E$  | $F$   | $F_{sep}$ |
|-----|-------|-----|------|-------------|--------|------|-------|-----------|
|     | Orig. |     | 420  | 0.29        | -      | 0.080| 0.01362| -         |
|     | Req.  |     | 419  | 0.29        | 0.31   | 0.079| **0.01362**| 1.5       |
|     | Compl. ($\Pi^+$) |   | -    | -           | 0.31   | -    | -     | 1.5       |
|     | Compl.|     | 425  | 0.30        | -      | 0.081| **0.01333**| 1.5       |
|     | Orig. |     | 611  | 0.42        | -      | 0.11 | 0.012 | -         |
|     | Req.  |     | 607  | 0.42        | 0.44   | 0.11 | **0.012** | 1.5       |
|     | Compl. ($\Pi^+$) |   | 639  | 0.44        | 0.44   | 0.12 | 0.012 | 1.5       |
|     | Compl.|     | 745  | 0.50        | -      | 0.14 | **0.01152**| 1.5       |
|     | Orig. |     | 565  | 0.38        | -      | 0.11 | 0.01224| -         |
|     | Req.  |     | 562  | 0.38        | 0.40   | 0.10 | **0.01248**| 1.6       |
|     | Compl. ($\Pi^+$) |   | 592  | 0.40        | 0.40   | 0.11 | 0.01224| 1.6       |
|     | Compl.|     | 856  | 0.56        | -      | 0.15 | **0.01134**| 1.6       |

Thermal power at $T_d$, $\Pi_{T_d}$ (MWth $10^{-2}$).

Annual thermal energy, $E$ (GWHth $10^{-3}$).

Cost function, $C$ (M€),

Cost per unit of annual thermal energy, $F = C/E$. 
MORE PROBLEMS

Heliostat pods \(^1\)

Multi-size-heliostats \(^2\)

Multiple receivers \(^3\)

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\(^1\) C. Domínguez-Bravo et al. “Field-design optimization with triangular heliostat pods”. In: Proceedings of SolarPaces 2015. 2015


**Multiple Receivers**

**One receiver**
(northern or southern field)

Source Abengoa (SP20)

**Circular receiver**
(surrounding field)

Source TorresolEnergy (Gemasolar)

**Multiple receivers**
(separate fields)

Source Abengoa
(Patent US 2012/0125000)
\[ \begin{align*}
(\mathcal{P}) & \quad \begin{cases}
\min_{\Theta, S} & F(\Theta, S) \\
\text{subject to} & \Theta \in \Theta \\
& S \in \mathcal{L} \\
& \Pi_i^- \leq \Pi_{Td}(\Theta_i, S) \leq \Pi_i^+ \quad i = 1, 2, 3
\end{cases}
\end{align*} \]

- \( F = C/E. \)
- \( i = 1, 2, 3, \) receivers.
- \( \Theta = (\Theta_1, \Theta_2, \Theta_3) \) with \( \Theta_i = (r_i, h_i, \xi_i, \alpha_i)^t \in \mathbb{R}^4 \quad \forall i. \)
MULTIPLE RECEIVERS

TOWER-RECEIVER(S): VARIABLES AND CONSTRAINTS

(j) height \( h \), tilt \( \xi \) and radius \( r \) (front-lateral)

(k) orientation \( \alpha \) (top)

(l) constraints (top)
Mathematical Optimization Problem

\[ \min_{S} F(\Theta, S) \]
subject to
\[ S \in \mathcal{I} \]
\[ \Pi_i^- \leq \Pi_{Td}(\Theta_i, S) \leq \Pi_i^+ \quad i = 1, 2, 3 \]

Assumptions:

1. **Separate fields**: each receiver have a separate field region.
   \[ S = S_1 \cup S_2 \cup S_3 \text{ and } S_1 \cap S_2 \cap S_3 = \emptyset \]
   \[ S_i = \{(x, y) \in S : \text{heliostat at } (x, y) \text{ aims at receiver } i\} \]

2. **Static aiming strategy**: heliostats always aim to the same receiver.
Multiple Receivers
State-of-the-art

Two integrated receivers (external and cavity)\(^5\)

Multiple apertures\(^6\)

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\(^6\) M. Schmitz et al. “Assessment of the potential improvement due to multiple apertures in central receiver systems with secondary concentrators”. In: *Solar Energy* 80 (2006), pp. 111–120
Algorithm steps

1. Calculate aiming regions: $S_1$, $S_2$, and $S_3$.

2. Locate the heliostats:
   2.1 Complete North region.
   2.2 Update boundaries.
   2.3 Complete West & East simultaneously.
**MULTIPLE RECEIVERS**

Calculate aiming regions:

1. Discretization of the feasible region.
2. At each point, calculate energy values for each receiver.
3. Select boundary points and obtain polynomial boundaries.
4. Different possibilities applying weights.
**Multiple Receivers**

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MULTIPLE RECEIVERS

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**MULTIPLE RECEIVERS**

Heliostats location: North, West & East.

1. Pattern-free location with greedy algorithm.
2. Until $\Pi^0_i$ is reached.
3. Complete the field while objective function improves.

![Heliostat Field Layout $N_{\text{hel}} = 624$](image)
MULTIPLE RECEIVERS

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Heliostat Field Layout $N_{\text{hel}} = 624$
MULTIPLE RECEIVERS

Heliostats location: North, West & East.

1. Pattern-free location with greedy algorithm.
2. Until $\Pi_i^0$ is reached.
3. Complete the field while objective function improves.

Heliostat Field Layout $N_{\text{hel}} = 1872$
## Multiple Receivers

**Alternating algorithm**

| Step | Pb | $|S|$ | $\Pi_{T_d}$ | $E$ | $C$ | $F$ |
|------|----|-----|------------|----|-----|-----|
| $k = 0$ | 1 : ($\Theta^0, S^0$) | 2009 | 118.7550 | 326.83 | 5.9984 | 0.01835 |
| $k = 1$ | 2 : ($\Theta^1, S^0$) | 2009 | 112.0731 | 310.62 | 5.3916 | 0.01736 |
| $k = 1$ | 3 : ($\Theta^1, S^1$) | 2033 | 115.0178 | 314.55 | 5.4445 | **0.01731** |
| $k = 2$ | 4 : ($\Theta^2, S^1$) | 2033 | 110.4583 | 306.45 | 5.3443 | 0.01744 |
| $k = 2$ | 5 : ($\Theta^2, S^2$) | 2084 | 114.8432 | 312.30 | 5.4567 | 0.01747 |

Thermal power at $T_d$, $\Pi_{T_d}$ (MWth).

Annual thermal energy, $E$ (GWHth).

Cost function, $C$ (M€).

Cost per unit of annual thermal energy, $F = C / E$. 
**MULTIPLE RECEIVERS**

**ALTERNATING ALGORITHM: RECEIVERS**

<table>
<thead>
<tr>
<th>Step</th>
<th>$h$ (m)</th>
<th>$\xi$ (grad)</th>
<th>$\alpha$ (grad)</th>
<th>$r$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Theta^0$</td>
<td>$\Theta_1$</td>
<td>100.50</td>
<td>12.50</td>
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</table>

Tower height, $h$ (m).

Receiver aperture tilt $\xi$ (grad), orientation $\alpha$ (grad) and radius $r$ (m).
MULTIPLE RECEIVERS

ALTERNATING ALGORITHM: FIELDS
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REFERENCES


