Fast Inversion of Logging-While-Drilling (LWD) Resistivity Measurements

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Surface measurements on the sea.

Marine seismic measurements.
Surface measurements on the sea.

Marine controlled source electromagnetic (CSEM) measurements.
Surface measurements on land.

Seismic measurements.
Surface measurements on land.

*Magnetotelluric (MT) measurements.*
Logging measurements

Logging while drilling in a deviated well.

Multiphysics
Logging measurements

Multiphysics
Dip Angle

Logging while drilling in a deviated well.
Logging measurements

Multiphysics
Dip Angle
Borehole eccentricity

Logging while drilling in a deviated well.
Logging measurements

Multiphysics
Dip Angle
Borehole eccentricity
Invasion

Logging while drilling in a deviated well.
Logging measurements

- Multiphysics
- Dip Angle
- Borehole eccentricity
- Invasion
- Anisotropy

Logging while drilling in a deviated well.
Logging measurements

Multiphysics
Dip Angle
Borehole eccentricity
Invasion
Anisotropy
Fractures

Logging while drilling in a deviated well.
Logging measurements

Multiphysics
Dip Angle
Borehole eccentricity
Invasion
Anisotropy
Fractures
Different Logging Devices

Logging while drilling in a deviated well.
main areas of expertise

- **Resistivity Measurements:**
  - Marine CSEM measurements.
  - Magnetotelluric (MT) measurements.
  - Galvanic and induction devices.
  - Cased wells.
  - Cross-well and borehole-to-surface measurements.
  - Deviated wells.
  - Borehole eccentered tools.
  - Hydrofracture characterization.

- **Sonic Measurements:**
  - Wireline and logging-while-drilling.
  - Borehole-eccentered tools.

- **Inversion of Resistivity Measurements:**
  - One-dimensional model reduction.
  - Rapid inversion of logging-while-drilling measurements.
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  - One-dimensional model reduction.
  - Rapid inversion of logging-while-drilling measurements.
Motivation and objectives.

Assumptions.

Forward Problem.

Inverse Problem.

Numerical Results.

Conclusions.
Goal: Inversion of LWD resistivity measurements.
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- **Reliable.** It should provide error bars.
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- **Robust.** It should always converge to physically meaningful solutions.

- **Reliable.** It should provide error bars.

- **Useful.** It should work for any commercial LWD instrument with actual field measurements.
We assume a planarly TI layered media with piecewise constant resistivities.

We assume no borehole effects and no mandrel effects.

We know the bed boundaries a priori.

We know the dip and azimuthal angles of intersection a priori.
forward problem

Magnetic field $H$ produced by a magnetic dipole is obtained using a semi-analytical solution for a 1D planarly layered TI media (Kong, 1972).
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- C) Numerical inverse Hankel transform (integration).
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Result: Magnetic field $H$. 
CASE I: Triaxial Induction.

\[ H = \begin{pmatrix} H_{xx} & H_{xy} & H_{xz} \\ H_{yx} & H_{yy} & H_{yz} \\ H_{zx} & H_{zy} & H_{zz} \end{pmatrix}. \]
CASE II: Conventional LWD resistivity tool.

\[ H_q := \log \left| \frac{H_{zz}^{RX_1}}{H_{zz}^{RX_2}} \right| + i \left[ ph(H_{zz}^{RX_1}) - ph(H_{zz}^{RX_2}) \right] \]

- Attenuation
- Phase Difference

Graphs showing the relationship between resistivity and the logarithmic scale for attenuation and phase difference.
CASE II: Conventional LWD resistivity tool.

\[ \tilde{H}_q := \log \log \left| \frac{H_{zz}^{RX_1}}{H_{zz}^{RX_2}} \right| + i \log \left[ \frac{\text{ph}(H_{zz}^{RX_1}) - \text{ph}(H_{zz}^{RX_2})}{\text{ATTENUATION}} \right] \]

PHASE DIFFERENCE
To accelerate computations, we employ a WINDOWING system:
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forward problem

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- Forward problem
inverse problem (formulation)

Cost Functional:

\[ C_W(s) = \| H(s) - M \|_{W_M}^2, \]

where

- \( s \) is either the conductivity \( \sigma \), the resistivity \( \rho \), or \( \log \rho \),
- \( H(s) \) is the set of simulated measurement for \( s \),
- \( M \) is the set of actual (or synthetic) field measurements.

Goal: To find \( s^* := \arg \min_s C_W(s) \).
inverse problem (formulation)

Cost Functional:

\[
\mathcal{C}_W(s) = \| H(s) - M \|_{W_M}^2 + \lambda \| s - s_0 \|_{W_{s_0}}^2,
\]

where

- \( s \) is either the conductivity \( \sigma \), the resistivity \( \rho \), or \( \log \rho \),
- \( H(s) \) is the set of simulated measurement for \( s \),
- \( M \) is the set of actual (or synthetic) field measurements,
- \( \lambda \) is a regularization parameter, and
- \( s_0 \) is an \textit{a priori} distribution of \( s \).

Goal: To find \( s^* := \arg \min_s \mathcal{C}_W(s) \).
inverse problem (sol. method)

We select the following deterministic iterative scheme:

\[ s^{(n+1)} = s^{(n)} + \delta s^{(n)}. \]

Using a Taylor’s series expansion of first order of \( H \):

\[ H(s^{(n+1)}) \approx H(s^{(n)}) + \left( \frac{\partial H(s^{(n)})}{\partial s} \right) \delta s^{(n)}. \]

Solving \( \frac{\partial C_W(s^{(n+1)})}{\partial \delta s^{(n)}} = 0 \), we obtain Gauss-Newton’s method:

\[ \delta s^{(n)} := - \frac{Re(J, H(s^{(n)}) - M)_{L^2_{WM}} + \lambda (I, s^{(n)} - s_0)_{L^2_{W_0}}}{(J, J)_{L^2_{WM}} + \lambda (I, I)_{L^2_{W_0}}}. \]
To compute the Jacobian, we employ:

- The chain rule:

$$ J = \frac{\partial H(s)}{\partial s_j} = \frac{\partial H(s)}{\partial \rho_j} \frac{\partial \rho_j}{\partial s_j}. $$

- The definition of derivative:

$$ J_\rho = \frac{\partial H(s)}{\partial \rho_j} \approx \frac{H(\rho + h\delta \rho_j) - H(\rho)}{h} \quad (h \text{ small}). $$

Only one Jacobian matrix is computed for any variable $s$. 
inverse problem (jacobian)

Misfit( %) Dip Angle = $82^\circ$. Thinnest Bed: 0.37 m.

$\rho$ 11,35 %
Misfit ( %)  Dip Angle = $82^\circ$. Thinnest Bed: 0.37 m.

$\rho \quad 11,35 \%$
inverse problem (jacobian)

Misfit ( %)  Dip Angle = 82°. Thinnest Bed: 0.37 m.

\[ \rho \quad 11,35 \% \]

\[ \sigma \quad 11,32 \% \]
Inverse problem (jacobian)

Misfit (%) Dip Angle = 82°. Thinnest Bed: 0.37 m.

\[ \rho \] 11.35%

\[ \sigma \] 11.32%
Inverse problem (jacobian)

Dip Angle = \(82^\circ\). Thinnest Bed: 0.37 m.

\[ \begin{align*}
\rho & \quad 11.35\% \\
\sigma & \quad 11.32\% \\
\log \rho & \quad 7.87\%
\end{align*} \]
Dip Angle = 82°. Thinnest Bed: 0.37 m.

** Misfit (%) 

$\rho$ 11.35 %

$\sigma$ 11.32 %

$log\ \rho$ 7.87 %
inverse problem (jacobian)

Misfit (%)  Dip Angle = $82^\circ$. Thinnest Bed: 0.37 m.

$\rho$  11.35%

$\sigma$  11.32%

$\log \rho$  7.87%

Best  6.58%
Once we achieve convergence, we have:

\[ \delta s^{(n)} \approx 0. \]

Considering new noisy measurements of the type:

\[ \tilde{M} := M + N \]

and using these new measurements in our Gauss-Newton method, we obtain the following new correction \( \tilde{\delta s}^{(n)} \):

\[ \tilde{\delta s}^{(n)} := \frac{\text{Re}(J, N)_{L^2_{WM}}}{(J, J)_{L^2_{WM}} + \lambda(I, I)_{L^2_{Ws0}}} \]

Error bars: \[ [s^{(n)} - |\tilde{\delta s}^{(n)}|, s^{(n)} + |\tilde{\delta s}^{(n)}|]. \]
Dip Angle: $82^\circ$.

Thinnest bed: 0.37m
Dip Angle = 82°. Thinnest bed: 0.37m.
Dip Angle = 82°. Thinnest bed: 0.37m.
Sensitivity with respect to the Dip Angle. Thinnest bed: 0.37m.
Dip Angle: 82°.

Thinnest bed: 0.37m

Anisotropy.
numerical results (synthetic 1)

Vertical well $\rightarrow R_h$. 

Resistivity (Ohm-m)

HD (m)

1000 1002 1004 1006

100 10 1

30°
numerical results (synthetic 1)

Vertical well $\rightarrow R_h$.

Horizontal well $\rightarrow R_v$. 

Resistivity (Ohm-m)
Dip Angle: $82^\circ$. 

Thinnest bed: 0.05m.
numerical results (synthetic 2)
### Numerical Results (Synthetic 2)

<table>
<thead>
<tr>
<th>Tool 1</th>
<th>Triaxial No Noise</th>
<th>Triaxial 5% Noise</th>
<th>Triaxial 10% Noise</th>
<th>Triaxial No Noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistivity (Ohm-m)</td>
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The graphs depict the resistivity values over the true vertical depth for different noise levels. The x-axis represents the true vertical depth in meters, ranging from 0.0 to 1.4 meters. The y-axis represents the resistivity values in Ohm-meters, ranging from 1 to 100 Ohm-meters.
Almost horizontal.

Field data.
numerical results (field 1)

Almost horizontal.

Field data.
Almost horizontal.

Field data.

Zoom.
Dip Angle: 79.3°.

Field data.
Inversion results (blue) are similar to those obtained by Dr. Olabode Ijasan (red).
conclusions

- We have developed a library for the fast inversion of LWD resistivity measurements.
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- The library enables any well trajectory and any logging instrument. We assume a 1D planarly layered TI media.
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The library automatically selects the regularization parameter, stopping criteria, and inversion variable.
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It enables to first invert a subset of measurements and/or a subset of resistivities.
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The library automatically selects the regularization parameter, stopping criteria, and inversion variable.

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Numerical results illustrate the stability of the proposed inversion algorithm.
Computational cost of one forward simulation:

\[
COST = C \times N_{POSITIONS} \times N_{LAYERS} \times N_{FREQ.} \times N_{TX} \times N_{RX}
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Computational cost of building the Jacobian:

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Can we eliminate the factor \(N_{RX}\)? I think so!
Can we eliminate the factor \(N_{TX}\)? To some extend!
Can we eliminate the square on the factor \(N_{LAYERS}\)? Perhaps!
To employ a Model Reduction algorithm based on Cartesian (C) coordinates and obtain results for Borehole (B) coordinates, we employ:

\[ H_{BB} = J_{BC} \cdot H_{CC} \cdot J_{CB}, \]

where:

- \( H_{CC} \) and \( H_{BB} \) are the model reduction algorithms for the Cartesian and Borehole systems of coordinates, respectively,

- \( J_{CB} = \begin{pmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{pmatrix} \cdot \begin{pmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \)

- \( J_{BC} = J_{CB}^{-1} \), \( \theta \) is the dip angle, and \( \phi \) is the azimuthal angle.
We have:

\[ C^{(n)}_W(s) = \| H(s) - M \|_{WM}^2 + \lambda^{(n)} \| s - s_0 \|_{L^2_{W_0}}^2, \]

\text{MISFIT} + \text{REGULARIZATION}
We have:

\[ C_W^{(n)}(s) = \| H(s) - M \|^2_{W,M} + \lambda^{(n)} \| s - s_0 \|^2_{L_{W,0}} \]

We want the regularization term to contribute with 10% to the total cost functional.
inverse problem (reg. param.)

We have:

\[
C_W^{(n)}(s) = \| H(s) - M \|_{l_{WM}^2}^2 + \lambda^{(n)} \| s - s_0 \|_{L_{W_s^0}^2}^2,
\]

**MISFIT** 90 %  
**REGULARIZATION** 10 %

We want the regularization term to contribute with 10 % to the total cost functional. Then:

\[
\lambda^{(n)} := 0.1 * \frac{\| H(s^{(n)}) + J\delta s^{(n)}_{\lambda^{(n)}} - M \|_{l_{WM}^2}^2}{\| s^{(n)} + \delta s^{(n)}_{\lambda^{(n)}} - s_0 \|_{L_{W_s^0}^2}^2}
\]

We perform a fixed-point iteration to obtain the value of \( \lambda^{(n)} \).
We have:

\[
C_W(s_{\lambda(n)}^{(n+1)}) = \| H(s_{\lambda(n)}^{(n+1)}) - M \|_{L^2_{WM}}^2 + \lambda^{(n+1)} \| s_{\lambda(n)}^{(n+1)} - s_0 \|_{L^2_{W_{S_0}}}^2
\]

\[
\approx \| H(s^{(n)}) + J\delta s_{\lambda(n)}^{(n)} - M \|_{L^2_{WM}}^2 + \lambda^{(n)} \| s^{(n)} + \delta s_{\lambda(n)}^{(n)} - s_0 \|_{L^2_{W_{S_0}}}^2.
\]

We want the regularization term to contribute with 10% to the total cost functional. Then:

\[
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\]

We perform a fixed-point iteration to obtain the value of \( \lambda^{(n)} \).
We stop the inversion process when both the relative data misfit and regularization term do not vary significantly. Mathematically, we require the following two conditions to be satisfied:

\[
100 \* \frac{\| H(s^{(n+1)}) - M \|^2_{L^2_{WM}} - \| H(s^{(n)}) - M \|^2_{L^2_{WM}}}{\| M \|^2_{L^2_{WM}}} \leq 0.5 \%
\]

And:

\[
100\lambda^{(n)} \* \frac{\| s^{(n+1)} - s_0 \|^2_{L^2_{W_{s0}}} - \| s^{(n)} - s_0 \|^2_{L^2_{W_{s0}}}}{\| s_0 \|^2_{L^2_{W_{s0}}}} \leq 5 \%.
\]
inverse problem (formulation)

Cost Functional:

\[ C(s) = \| H(s) - M \|_{L^2}^2 + \lambda \| s - s_0 \|_{L^2}^2. \]

We want to weight all measurements and resistivities so equal relative errors will contribute equally to the cost functional.
Cost Functional:

\[ C(s) = \| H(s) - M \|_{L^2}^2 + \lambda \| s - s_0 \|_{L^2}^2. \]

We want to weight all measurements and resistivities so equal relative errors will contribute equally to the cost functional.

Weighted cost functional:

\[ C_W(s) = \| H(s) - M \|_{L^2}^2_{W_M} + \lambda \| s - s_0 \|_{L^2}^2_{W_{s_0}}. \]

Goal: To find \( s^* := \arg \min_s C_W(s) \).