The scientific achievements of Vicent Caselles

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Univ. València

PARTIAL DIFFERENTIAL EQUATIONS, OPTIMAL DESIGN AND NUMERICS
DEDICATED TO THE MEMORY OF VICENT CASELLES
Benasque, August 27 2013
Vicent Caselles: A Life Devoted to Mathematics

Vicent (Gata, Alacant, 10-8-1960, Barcelona, 14-8-2013) was an outstanding mathematician, capable of the most rigorous mathematical formalism and, at the same time, eager to find challenging applications for him and his collaborators and to get excellent numerical results.

As his many collaborators and friends know, he was an excellent person, with an uncommon combination of sincerity, modesty and willingness to help, that made him so charming.

I hope this little tribute to his memory will serve to illustrate Vicent’s personal and scientific achievements.
1 Biography
   - Academic trajectory
   - Awards and prizes
   - Coauthors

2 Some selected topics
   - Gray image denoising
   - Total variation flow
   - Minimizing flow for linear growth functionals
   - Image restoration
   - Color image denoising
   - Geodesic active contours
   - Image inpainting
1989 - 1990: Research and Teaching Assistant (ATER) at Univ. Franche-Comté, Besançon.
1999 - 2013: Univ. Pompeu Fabra, Barcelona.

Data taken mainly from Vicent’s CV
142 papers in international journals.
64 other publications.
81 invited conferences.
4742 citations (two papers with more than 2000 citations)†
74 coauthors (48 with more than one paper)†
15 Ph.D. thesis

†Source: ISI-WOK
AWARDS AND PRIZES

- 2003: **Ferran Sunyer i Balaguer** Prize.
- 2009: **ICREA** Acadèmia (Gen. Cat.) prize for excellence in research.
- 2011: Invited plenary speaker at **ICIAM**, Vancouver, Canada.
- 2011: Test of Time Award at International Conference on Computer Vision (ICCV) for the **ICCV** paper “Geodesic Active Contours” in collaboration with R. Kimmel and G. Sapiro.
- 2012: Invited lecture at the **ECM**, Krakow, Poland.
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Catalina Sbert Juan, UIB, 1995. (J.M. Morel)
Manuel González Hidalgo, UIB, 1995. (J.M. Morel)
Andrés Solé Martínez, UPF, 2002.
José Salvador Moll Cebolla, UV, 2005. (J.M. Mazón, F. Andreu)
Gloria Haro Ortega, UPF, 2005. (R. Donat)
Marc Bernot, ENS Cachan, 2005. (J.M. Morel)
Laura Igual, UPF, 2006. (Luis Garrido)
François Alter, ENS Cachan, 2008. (J.M. Morel)
Gabriele Facciolo Furlan, UPF, 2010.
Sira Ferradans Ramonde, UPF, 2011. (M. Bertalmío).
PREVIOUS RESEARCH INTERESTS (1982-1990)
- Functional Analysis: geometry of Banach spaces and operator theory.

RESEARCH INTERESTS (1990-2013)
- Image processing
- Partial Differential Equations and their applications
- Differential geometry and its applications
- Computer vision
**IMAGE DENOISING**

- **Image (gray):** \( u : \Omega := (0, 1)^2 \rightarrow \mathbb{R} \) (\( \mathbb{R}^3 \equiv R \times G \times B \) for color),

  \( u(x, y) \equiv \) gray level at \((x, y)\):

- **Image acquisition introduces noise:** \( z = u + n \), \( z \) recorded image, \( n \) noise (unknown, up to some statistics):

Pictures taken from [Rudin, Osher, Fatemi, 92]
Goal: approximate $u$, preserving discontinuities (edges)

Assume we know $\|n\|_2 := (\int_{\Omega} n^2)^{\frac{1}{2}} = \sigma$; then solve

$$\min F(u), \quad \text{subject to } \|z - u\|_2 = \sigma$$

where $F : \mathcal{X} \to \mathbb{R}$ measures the regularity of $u$ in some sense.

This is equivalent to

$$\min F(u) + \frac{\lambda}{2} \|z - u\|_2^2$$

for suitable Lagrange multiplier $\lambda$, with Euler-Lagrange equation:

$$F'(u) + \lambda(u - z) = 0.$$
If 

\[ F(u) = \int_{\Omega} |\nabla u|^2 dx dy, \quad |\nabla u| = \sqrt{u_x^2 + u_y^2}, \]

for \( u \in \mathcal{X} = H^1(\Omega) \), then the E-L equation reads:

\[ -\Delta u + \lambda(u - z) = 0 \]

a linear elliptic equation with continuous solutions (no edge recovery).

[Rudin, Osher and Fatemi, 1992] use

\[ F(u) = TV(u) := \int_{\Omega} |\nabla u| dx dy \tag{1} \]

(Total Variation of \( u \)), \( u \in BV(\Omega) \), for then \( u \) can be discontinuous along curves. Of course, (1) is only valid for differentiable functions, a weak formulation applies otherwise.
The (formal) E-L equation is now:

\[-\text{div} \left( \frac{\nabla u}{|\nabla u|} \right) + \lambda (u - z) = 0\]

and the associated parabolic equation reads

\[u_t = \text{div} \left( \frac{\nabla u}{|\nabla u|} \right) - \lambda (u - z)\]

These equations do not make sense for non-differentiable functions or with extrema (where the denominator $|\nabla u|$ vanishes).
TOTAL VARIATION FLOW

- Of course, somebody had to give sense to all these equations. Vicent, together with F. Andreu and J.M. Mazón, studied the total variation flow in a series of papers starting at 2000:

\[ u_t = \text{div} \left( \frac{\nabla u}{|\nabla u|} \right), \]  

(2)

giving correct sense to all the terms involved in these formulation.

- This equation corresponds to the minimization of the total variation and can be used to denoise without prior knowledge of \( \sigma \):

\[ \min \int_{\Omega} |\nabla u|. \]  

(3)

- Related to anisotropic diffusion ([Perona, Malik, 1987], [Catte, Coll, Lions, Morel, 1992]).
Some selected topics

Total variation flow

2. The Dirichlet problem for the total variation flow, Andreu, F; Ballester, C; Caselles, V; Mazon, JM J. Functional Analysis, 2001.
4. Some qualitative properties for the total variation flow, Andreu, F; Caselles, V; Diaz, JI; Mazon, JM, J. Functional Analysis, 2002.
6. Evolution of characteristic functions of convex sets in the plane by the minimizing total variation flow, Alter, F; Caselles, V; Chambolle, A, Interfaces and Free Boundaries, 2005.
Vicent and coauthors then study generalizations of (2) and (3):\
\[
\min_u \int_{\Omega} f(x, \nabla u(x)) \, dx, \quad \Omega \text{ open subset of } \mathbb{R}^N
\]

for suitable convex functions $f(x, \xi)$ with linear growth (on $\xi \equiv \nabla u$), which give gradient flows:
\[
 u_t(x, t) = \text{div} [a(x, \nabla u(x, t))] , \quad a(x, \xi) = \nabla_\xi f(x, \xi).
\]

($f(x, \xi) = |\xi|$ gives the minimizing total variation flow)


Image acquisition introduces blur and noise: $z = Ku + n$, $z$ recorded image, $n$ noise and $Ku = k \ast u$, for known kernel $k$, with the goal of getting $\hat{u} \approx u$ (preserving edges).
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Image acquisition introduces blur and noise: $z = Ku + n$, $z$ recorded image, $n$ noise and $Ku = k \ast u$, for known kernel $k$, with the goal of getting $\hat{u} \approx u$ (preserving edges).
Assume that we know that $n = z - Ku$ is a Gaussian white noise with variance $\sigma^2$ (and 0 mean) uncorrelated to $Ku$; then

$$\Pr \left( \int_{\Omega'} n^2 \leq \sigma^2 \right) = 1$$

for any open $\Omega' \subseteq \Omega = (0, 1)^2$.

If we impose globally $\int_{\Omega} n^2 \leq \sigma^2$, then variational problem reads as:

$$\min TV(u), \quad \text{subject to } \frac{1}{2} \left( \int_{\Omega} (Ku - z)^2 - \sigma^2 \right) \leq 0 \quad (4)$$

The Euler-Lagrange equation is now:

$$-\text{div} \left( \frac{\nabla u}{|\nabla u|} \right) + \lambda K^*(Ku - z) = 0 \quad (K^* \equiv \text{adjoint of } K)$$

where $\lambda$ is the Lagrange multiplier for the (global) constraint.
It is observed that the solution $\hat{u}$ of (4) yields an approximated noise $\hat{n} = z - K\hat{u}$ which is correlated with $K\hat{u}$:

\[ \hat{n} \]

\[ \hat{n} \]
It is also observed that the approximated noise $\hat{n}$ does not verify

$$\int_{\Omega'} \hat{n}^2 \leq \sigma^2, \quad \forall \text{ open } \Omega' \subseteq \Omega$$

In [Almansa, Ballester, Caselles, Haro 2008] they propose a suitable discretization of the problem (with local constraints)

$$\min TV(u),$$

subject to $\int_{B_\delta(x,y)} (Ku - z)^2 \leq \sigma^2, \forall (x, y) \in \Omega$

for some $\delta > 0$.

They use a variant of Uzawa’s algorithm for the numerical solution.
$\hat{n}$ global constraint
Image restoration

\[ \hat{\eta} \text{ local constraints} \]
$\hat{u}$ global constraint
Some selected topics

**Image restoration:**

**IMAGE RESTORATION: RESULTS**

[Image of restored image using local constraints]

- The top image corresponds to the restored image obtained using functional with $\beta = 0$, $\sigma^2 = \sigma^2 = 1$ using a Gaussian window with $\sigma = 6.5$. In this case we have $\text{RMSE} = 9.0739$, mean $(G \ast (h \ast u - z))^2 = 0.9669$, and variance $(G \ast (h \ast u - z))^2 = 0.0012$.

- The bottom one is the function $\lambda(i, j)$ obtained the vectors $\xi_m(i, j), i, j \in \{1, \ldots, N\}$ and, for simplicity, write $\nabla u_m$ instead of $\nabla u_m(i, j)$.

Writing $v_{m+1} = \nabla u_m + \alpha \xi_m - \alpha \xi_{m+1}$, (42)

Restoration and zoom of irregularly sampled, blurred, and noisy images by accurate total variation minimization with local constraints, Almansa, A; Caselles, V; Haro, G; Rouge, B, Multiscale Modeling & Simulation, 2006.

Color image denoising

- Color image: \( u: \Omega := (0, 1)^2 \rightarrow \mathbb{R}^3 ((R, G, B)\text{-components}) \)
- Typical strategy for color image processing: apply gray-level procedure to each component \( \Rightarrow \) false colors.
- In [Tang, Sapiro, Caselles, 2001], \( u \) is decomposed in a different way:

\[
M = |u| \quad : \Omega \rightarrow \mathbb{R} \quad \text{brightness (magnitude)}
\]
\[
C = u/|u| \quad : \Omega \rightarrow S^2 \quad \text{chromaticity (direction)}
\]

- \( M \) is processed by a gray-level technique
- \( C \) is processed by solving variational problem:

\[
\min \int_{\Omega} \left( |C_x|^2 + |C_y|^2 \right)^{p/2}, \quad 1 < p < 2
\]
subject to \( |C(x, y)|^2 = 1, \quad \forall (x, y) \in \Omega \)
COLOR IMAGE DENOISING

Fig. 2. Examples of our algorithm and comparison with discrete approaches. See text for details.

Note how the proposed chroma diffusion removes the “color” noise while preserving the details in the image. The third column repeats this, but now the noise has been added to the full color image (original on the top). That is, both the chroma and brightness are noisy (middle row). To illustrate the effects of chroma diffusion alone, the bottom figure shows the results of the isotropic direction (chroma) flow, while the noisy magnitude (brightness) was kept without processing.

In Fig. 2 we show examples of our algorithm and compare with the approach and [22], where the discrete vector directional and magnitude median filters are combined. In all the columns, original is shown first, followed by the noisy one, the result of [22], and the result of our algorithm. In the first column we use Gaussian noise (second row), and the anisotropic chroma diffusion is combined with median filtering for the magnitude (last row). This is repeated in the second column. This time, scalar anisotropic diffusion for the brightness is combined with isotropic chroma diffusion.
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vector directional filter ([Karakos, Trahanias, 1997])
Vicent’s model
Goal: detect **object boundaries** in image.

Classical approach [Kass et. al., 1988] based on deforming an initial contour $C_0$ towards boundary of the object to be detected.

The deformation is obtained by trying to minimize a functional designed so that its (local) minimum is obtained at the boundary of the object.

Let $I : \Omega := (0, 1)^2 \rightarrow \mathbb{R}$ be the (given) image and $g : [0, \infty) \rightarrow (0, \infty)$ be strictly decreasing and $g(r) \rightarrow 0$, when $r \rightarrow \infty$, define energy of a closed curve $C : [0, 1] \rightarrow \Omega$ by:

$$E(C) = \int_0^1 |C'(q)|^2 \, dq + \beta \int_0^1 g(|\nabla I(C(q))|) \, dq$$
Classical **active contours** (snakes):

$$\min_C \int_0^1 |C'(q)|^2 dq + \beta \int_0^1 g(|\nabla I(C(q))|) dq$$

- **Internal energy** forces $C$ to be regular (i.e. $|C'| \downarrow 0$).
- **External energy** pushes $C$ to discontinuity of $I$
  $$(|\nabla I(C(q))| \to \infty \Rightarrow g(|\nabla I(C(q))|) \to 0).$$

But functional is not intrinsic (energy changes with reparametrization of the curves), depends on parameter $\beta$ and cannot deal with topological changes of the curve.
**Geodesic active contours:** [Caselles, Kimmel, Sapiro, 1997] define

\[ E(C) = \int_0^1 g(|\nabla I(C(q))|)|C'(q)|dq. \]

\( E \) is independent of parametrizations.

In fact, \( E(C) \) is the length of \( C \) when considering the Riemannian metric given by the first fundamental form \( g_{i,j} = g(|\nabla I|)\delta_{i,j}. \)

Therefore

\[ \min_C E(C) \]

is equivalent to finding a **geodesic** for the new metric.

Minimizing flow for \( C = C(t, q) \):

\[ C_t = (\tilde{g}(C)\kappa(C) - \nabla \tilde{g}(C) \cdot \mathcal{N})\mathcal{N}, \quad \tilde{g}(x) = g(|\nabla I(x)|), \quad (5) \]

where \( \kappa(C) \) is the curvature of \( C \) and \( \mathcal{N} \) is the inward unit normal to \( C \).
To deal with topological changes in $C$ a **level set** formulation can be applied: embed $C$ as 0-level set of unknown $u(x,t)$, so (5) is equivalent to:

$$u_t = \left( \tilde{g} \ \text{div} \left( \frac{\nabla u}{|\nabla u|} \right) + \nabla \tilde{g} \cdot \frac{\nabla u}{|\nabla u|} \right) |\nabla u|$$

$$u_t = \text{div} \left( g(|\nabla I|) \frac{\nabla u}{|\nabla u|} \right) |\nabla u|,$$

where we have taken into account that

$$\kappa(\text{level set of } u) = \text{div} \left( \frac{\nabla u}{|\nabla u|} \right)$$

$$\mathcal{N}(\text{level set of } u) = -\frac{\nabla u}{|\nabla u|}$$
Example of tumor detection in MRI
Example of tumor detection in MRI
Some selected topics

1. A geometric model for active contours in image-processing, CASELLES, V; CATTE, F; COLL, T; DIBOS, F, Numerische Mathematik, 1993.


Filling-in missing data (inpainting in art restoration) in digital images has a number of fundamental applications:

- Removal of scratches in old photographs and films,
- Removal of superimposed text like dates, subtitles, or publicity from a photograph,
- Recovery of pixel blocks corrupted during binary transmission.
Some selected topics

**Image inpainting**

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- Recovery of pixel blocks corrupted during binary transmission.

Fig. 7. Example of the proposed filling-in algorithm.

Fig. 8. Detail of the example in Fig. 7. Note the smooth continuation of the edges.

Fig. 9. Level-set corresponding to the region in Fig. 8. (a) Original and the results with (b) and (c).

This is imposed in the initialization of the level set and is maintained at each iteration of the algorithm by taking the supremum of the current solution with the characteristic function of . With this approach, we diminish the diffusive effects of the above algorithm and we better capture the shapes and discontinuities on the interpolated image.

The constraints on and can be introduced after each iteration of the above equations. We also comment that the constraint , which was introduced as a penalization term, could also be introduced by brute force after each time step iteration of the algorithm. Let us describe the experiments.

Fig. 10. Example of automatic text removal.

First, in Fig. 4 we display some experiment to illustrate functional (21). Fig. 4(a) displays the full image without the hole. Fig. 4(b) displays the image with the hole. The vector field has been computed on Fig. 4(a) and we see in Fig. 4(c) the result of interpolating the gray level knowing the vector field inside.

We see that the shape of the eye is recovered but not the gray level. This is not a surprise since the gray level inside the eye cannot be recovered from the gray level on the boundary of . The algorithm is able to capture the shapes inside the eye by integrating the vector field .

In the following experiments, we show the results of the joint interpolation of gray level and the vector field of directions using functional (10). The experiments have been done with and/or . The results are quite similar. Unless explicitly stated, we display the results obtained with .
Inpainting may be tackled by interpolatory techniques, e.g.

\[ \Delta u = 0 \quad \text{harmonic extension} \]

\[ \nabla^2 u(\nabla u, \nabla u) = 0 \quad \text{Absolute Minimal Lipschitz Extension} \]

with \( u|_{\partial \tilde{\Omega}} = u_0|_{\partial \tilde{\Omega}} \), where:

- \( \tilde{\Omega} \subset \subset \Omega \subset \subset (0, 1)^2 \) is the hole,
- \( B = \Omega \setminus \tilde{\Omega} \) is the band surrounding the hole \( \tilde{\Omega} \)
- \( u_0 : (0, 1)^2 \setminus \tilde{\Omega} \to \mathbb{R} \) is known.

This may work well for small \( \tilde{\Omega} \), specially AMLE (studied by Vicent and coauthors in a series of papers), but need more sophisticated techniques for larger \( \tilde{\Omega} \).

Idea: try to continue the level sets of \( u_0 \) affected by occlusion.
**Euler’s elastica**: to continue a partially occluded curve, knowing end points $p, q$ and tangent vectors $\tau_p, \tau_q$ at $p, q$

solve for given parameters $\alpha, \beta > 0$

$$\min_C \int_C \left( \alpha + \beta \kappa^2 \right) ds,$$

subject to $C(0) = p, C(1) = q, C'(0) = \tau_p, C'(1) = \tau_q$

for $ds$ the arc length measure and $\kappa$ the curvature.
[Ambrosio, Masnou, 2001] extend Euler’s elastica to join level sets of $u_0$ (defined on $(0, 1)^2 \setminus \tilde{\Omega}$):

$$\min_u \int_{\Omega} |\nabla u| (\alpha + \beta |\text{div} \left( \frac{\nabla u}{|\nabla u|} \right) |^p) ds, \quad p \geq 1$$

$u|_B = u_0|_B$. 
[Ballester, Bertalmio, Caselles, Sapiro, Verdera, 2001], [Ballester, Caselles, Verdera, 2003] relax a similar problem, introducing an auxiliary variable $\theta$ that should be in the limit $\frac{\nabla u}{|\nabla u|}$, i.e., the (outward) normal to the level set:

$$\min_{u,\theta} \int_{\Omega} |\text{div}(\theta)|^p (\gamma + \beta |\nabla k \ast u|)$$

$$|\theta| \leq 1, \nabla u - \theta |\nabla u| = 0$$

$$u|_{B} = u_0|_{B}$$

$$(\theta - \theta_0) \cdot \nu|_{\partial \Omega} = 0$$

where $u_0$ is the image known in $(0, 1)^2 \setminus \tilde{\Omega} \supseteq B$ and $\theta_0$ is any vector field in $B$ such that $(\nabla u_0 - \theta_0 |\nabla u_0|)|_{B} = 0$ and $\nu$ is the unit normal to $\partial \Omega$.

Convolution by kernel $k$ necessary for proving well-posedness.

Recent (and future!) work on video inpainting and stereo video inpainting (“3D” video) (got ERC advanced grant with these topics).
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(a) (b) (c)

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The constraints on and can be introduced after each iteration of the above equations. We also comment that the constraint , which was introduced as a penalization term, could also be introduced by brute force after each time step iteration of the algorithm.

Let us describe the experiments.

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Some selected topics: Image inpainting

**Image inpainting**

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First, in Fig. 4 we display some experiments to illustrate functional (21). Fig. 4(a) displays the full image without the hole. Fig. 4(b) displays the image with the hole. The vector field has been computed on Fig. 4(a) and we see in Fig. 4(c) the result of interpolating the gray level knowing the vector field inside.

We see that the shape of the eye is recovered but not the gray level. This is not a surprise since the gray level inside the eye cannot be recovered from the gray level on the boundary of . The algorithm is able to capture the shapes inside the eye by integrating the vector field .

In the following experiments, we show the results of the joint interpolation of gray level and the vector field of directions using functional (10). The experiments have been done with and/or . The results are quite similar. Unless explicitly stated, we display the results obtained with . Fig. 5(a) displays an image made of four circles covered by a square. In


Image inpainting

Some selected topics

Other Topics

- Image histogram equalization / contrast enhancement.
- Irrigation / transport problems.
- Image compression
- Flux limited equations / “relativistic” heat equation
- Optical flow
- Video editing / camera replay simulation
- and many more . . .
AN OUTSTANDING MATHEMATICIAN AND A BETTER PERSON

Pep Mulet (Univ. València)

Scientific achievements of Vicent Caselles