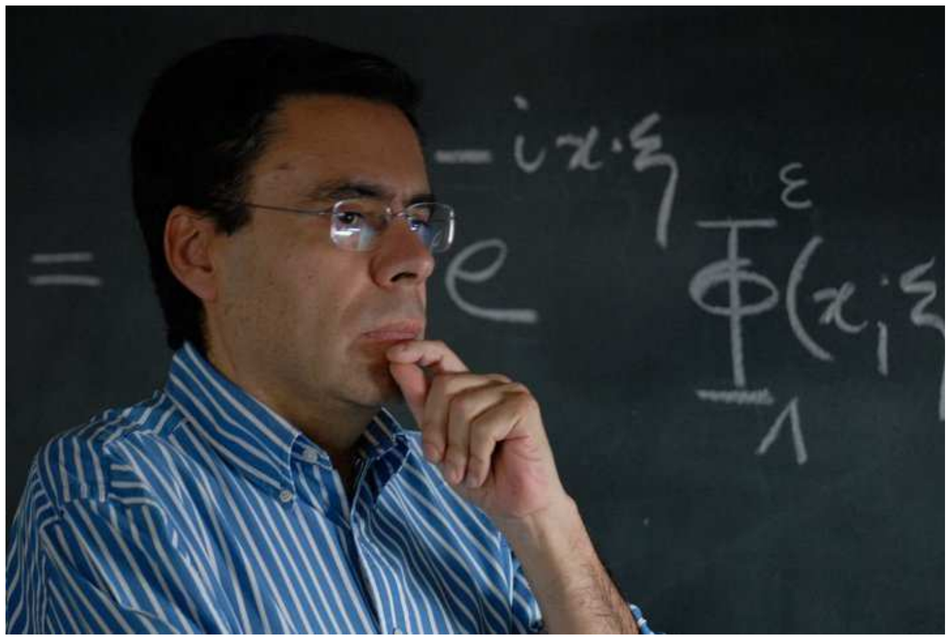
A glimpse at the scientific works of CARLOS CONCA on the occasion of his 60th birthday



Grégoire Allaire, Ecole Polytechnique. BCAM, Bilbao, December 12th, 2014.

His achievements

- ☞ 107 articles
- ☞ 2 books
- ☞ 31 book chapters
- ☞ 4 edited books
- ☞ more than 750 citations in MathSciNet
- ☞ ...
- ☞ 1 wife (Lola)
- ☞ 1 child (Maite)
- ☞ many, many friends !

But let's not focus on numbers...

His main fields of research

1. Homogenization in fluid mechanics and reactive flows
2. Boundary condition for Navier-Stokes equations
3. Bloch wave theory
4. Fluid-structure interaction
5. Inverse problems (detecting a moving obstacle in a fluid)
6. Bio-mathematics (bio-films, chemotaxis)
7. Optimal design

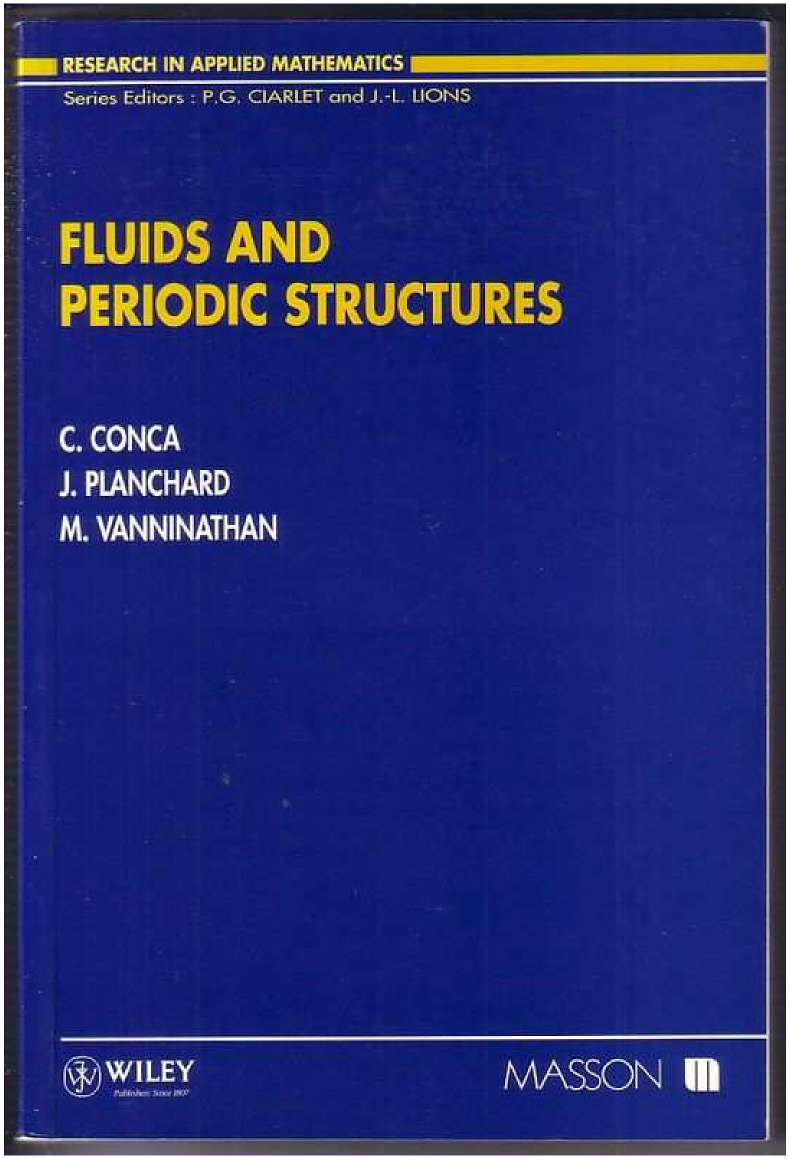
His work on periodic fluid-structure interactions

Let me focus on just one exemplary work: periodic fluid-structure interactions.

It all started with his collaboration with the late Jacques Planchard (engineer at EDF).

It is a perfect mixture of

- clear physical and industrial motivation,
- sound mechanical modelling,
- homogenization and Bloch wave analysis,
- surprising theoretical and numerical results,
- inspiration for further research.


RESEARCH IN APPLIED MATHEMATICS
Series Editors : P.G. CIARLET and J.-L. LIONS
FLUIDS AND PERIODIC STRUCTURES
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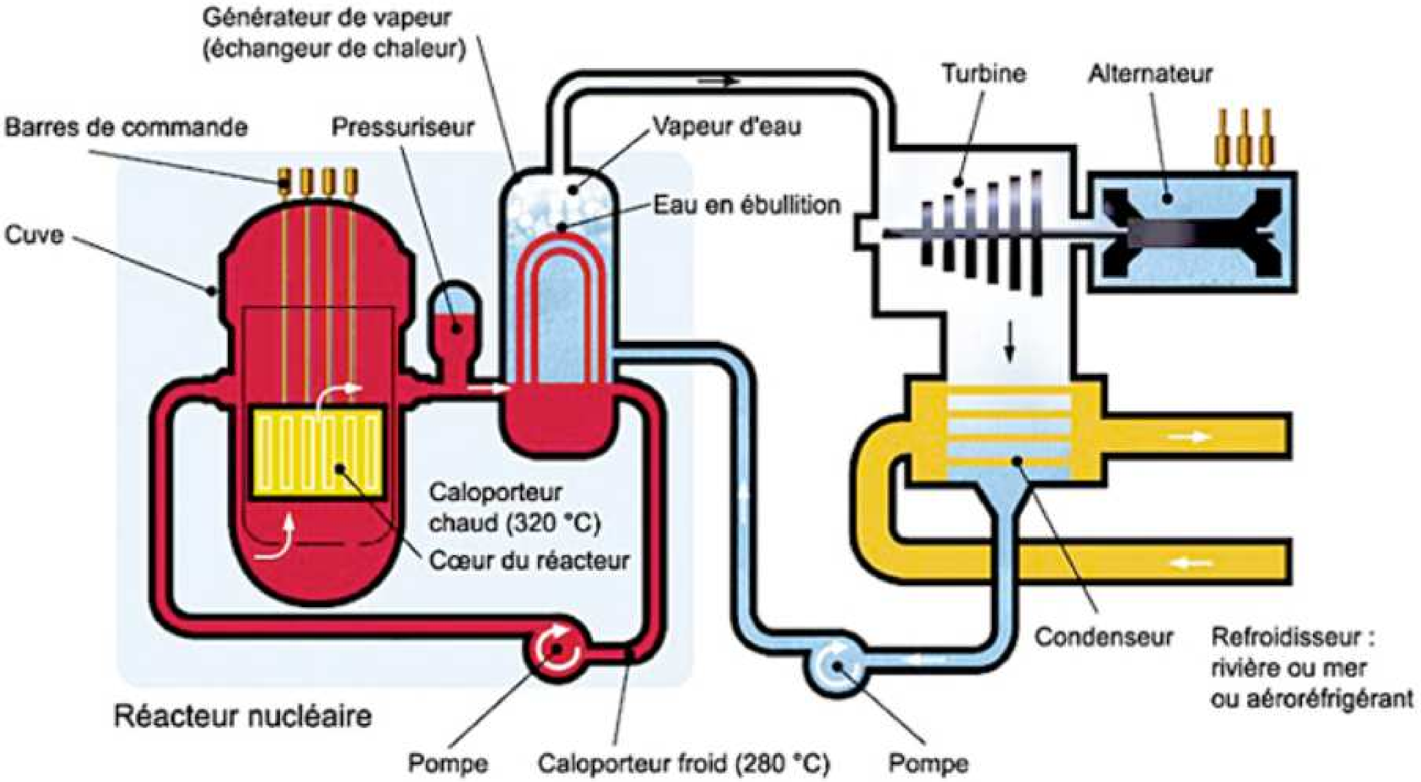
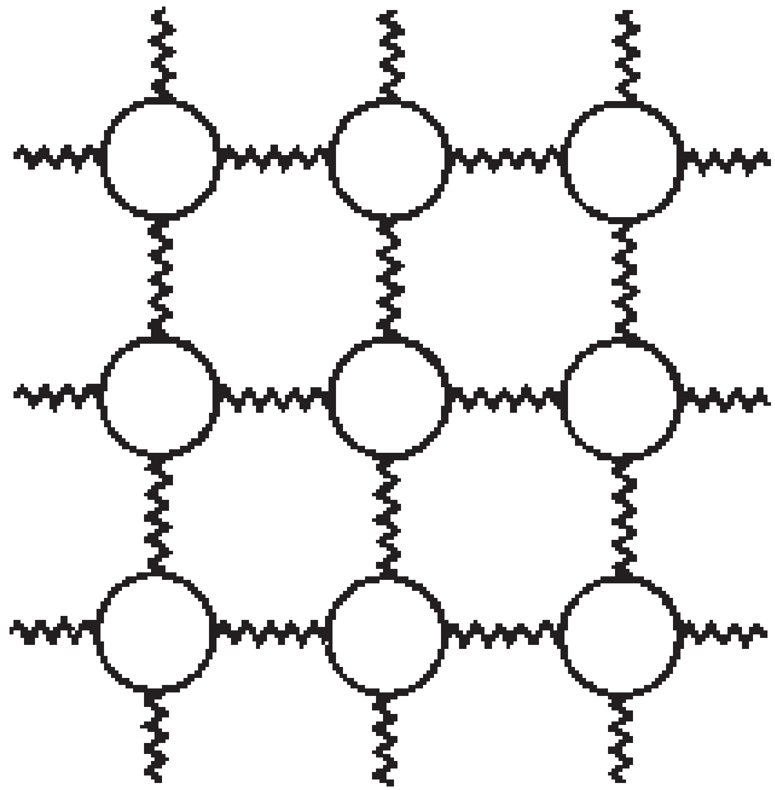
Motivation from nuclear reactor safety



Assemblage combustible

Mechanical modelling of a 2-d cross-section

Eigenfrequencies of a Tube Bundle Immersed in a Fluid



etc...

Figure 1.4. The spring system.

Mechanical modelling of a 2-d cross-section (Ctd.)

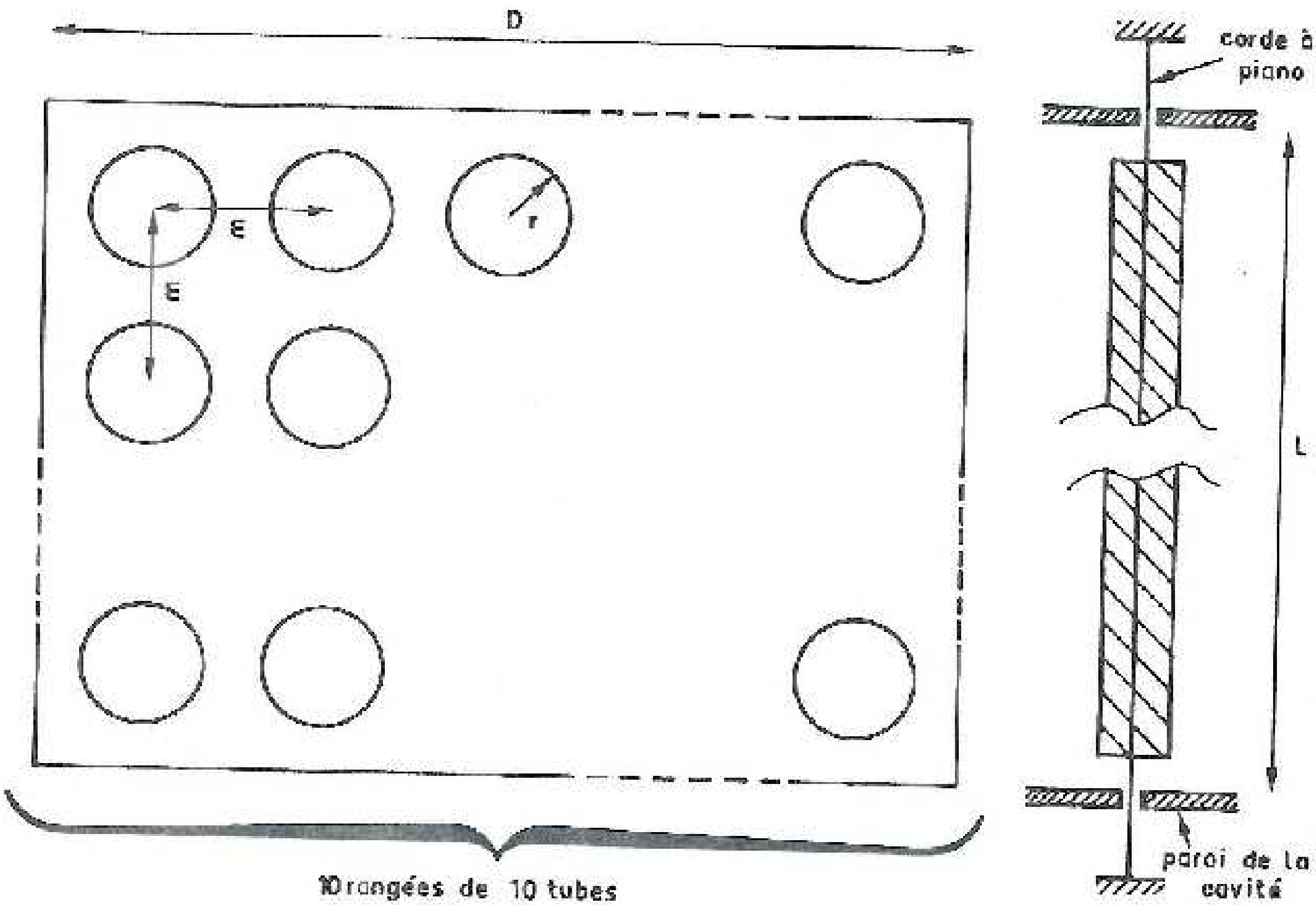


Fig. 1

Mathematical modelling

Fluid domain $\Omega_\epsilon = \Omega \setminus \bigcup_{p=1}^{n(\epsilon)} T_p^\epsilon$

Potential flow equation

$$\Delta u_{\epsilon 0} = 0 \text{ in } \Omega_\epsilon$$

Boundary condition at the fluid/solid interface

$$\frac{\partial u_{\epsilon 0}}{\partial n} = \frac{d\vec{r}_{0p}}{dt} \cdot \vec{n} \quad \text{on } \Gamma_p^\epsilon, \quad p = 1, \ldots, n(\epsilon),$$

Rigid tube motion

$$m\frac{d^2\vec{r}_{0p}}{dt^2} + k\vec{r}_{0p} = \int_{\Gamma_p^\epsilon} p_{\epsilon 0}(x,t)\vec{n}ds \quad \text{for } p = 1, \ldots, n(\epsilon),$$

Bernoulli relationship for the pressure

$$p_{\epsilon 0} = -\rho\frac{\partial u_{\epsilon 0}}{\partial t} + c(t),$$

Mathematical modelling (Ctd.)

One looks for vibrating solutions of the form

$$u_{\epsilon 0}(x,t) = u_\epsilon e^{\imath \omega_\epsilon t}, \quad \vec{r}_{0p}(t) = \vec{r}_p e^{\imath \omega_\epsilon t},$$

One can solve explicitly the ode for $\vec{r}_{0p}$

$$\vec{r}_{0p}(t) = -\frac{\imath \rho \omega_\epsilon e^{\imath \omega_\epsilon t}}{k - m\omega_\epsilon^2} \int_{\Gamma_p} u_\epsilon \vec{n} ds$$

To simplify the notations, we define a rescaled frequency

$$\lambda_\epsilon = \frac{k - m\omega_\epsilon^2}{\epsilon^N \rho \omega_\epsilon^2}.$$

Eigenvalue problem:

$$\begin{cases} -\Delta u_\epsilon = 0 & \text{in } \Omega_\epsilon \\ \lambda_\epsilon \frac{\partial u_\epsilon}{\partial n} = \frac{1}{|Y_p^\epsilon|} \vec{n} \cdot \int_{\Gamma_p^\epsilon} u_\epsilon \vec{n} ds & \text{on } \Gamma_p^\epsilon, \text{ for } 1 \le p \le n(\epsilon) \\ u_\epsilon = 0 & \text{on } \partial\Omega, \end{cases}$$

Added mass

Compute a finite number of flow configurations, for $i = 1, 2$ and $1 \leq p \leq n(\epsilon)$

$$\begin{cases} -\Delta \phi_q^i = 0 & \text{in } \Omega_\epsilon \\ \frac{\partial \phi_q^i}{\partial n} = \delta_{pq} \vec{n} \cdot e_i & \text{on } \Gamma_p^\epsilon, \text{ for } 1 \leq p \leq n(\epsilon) \\ \phi_q^i = 0 & \text{on } \partial\Omega, \end{cases}$$

Define the added mass matrix

$$\mathcal{M}_{i,j,p,q} = \int_{\Omega_\epsilon} \nabla \phi_p^i \cdot \nabla \phi_q^j \, dx$$

Introduce the vector of tube displacements $\vec{s} = (\vec{s}_p)_{1 \leq p \leq n(\epsilon)}$ with each $\vec{s}_p \in \mathbb{R}^2$. Then

$$(\lambda_\epsilon \rho M + m \mathrm{Id}) \ddot{\vec{s}} + k \vec{s} = 0$$

The mass of the tubes is increased by the fluid interaction.

Goal

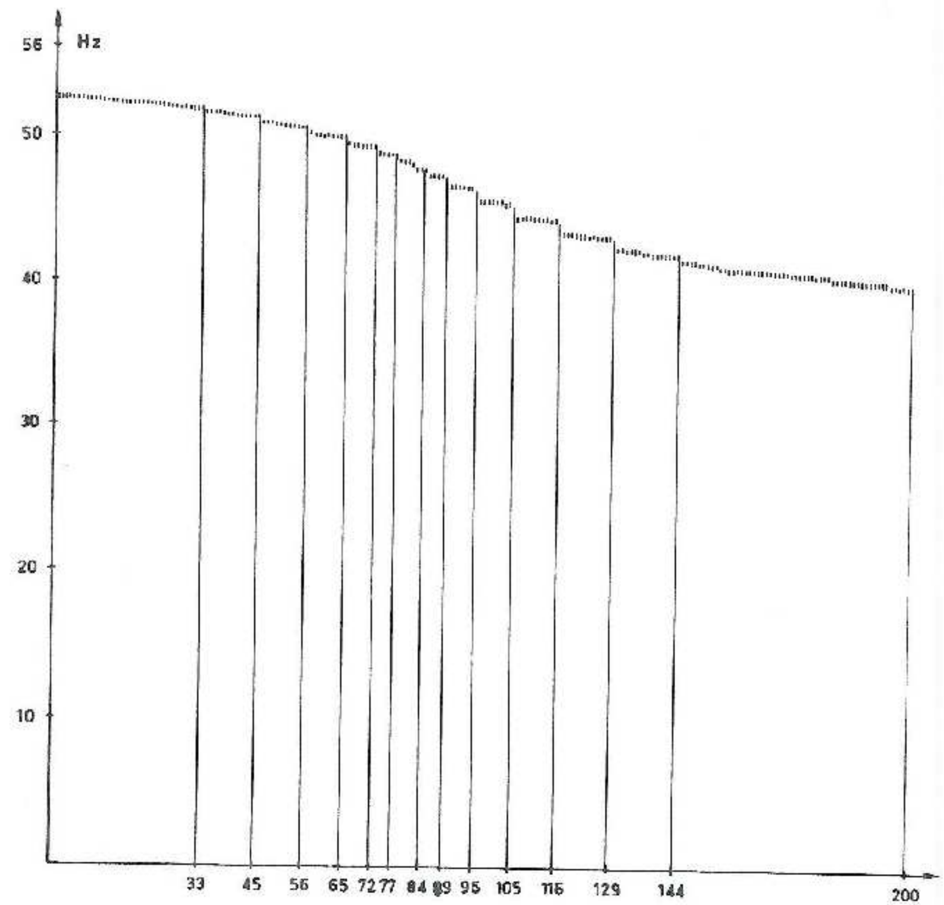
The fluid-structure vibrates with frequencies

$$\omega_\epsilon = \sqrt{\frac{k}{m + \rho\lambda_\epsilon}}$$

- ☞ Determine the homogenized limit of the spectrum $(\lambda_\epsilon, u_\epsilon)$.
- ☞ Find homogenization formulas for the limit spectrum.
- ☞ Try to avoid resonances with other vibrations like earthquakes. Optimize to have the larger possible first eigenfrequency.

To get some insight, let us look at some direct numerical simulations of Planchard for a 10 by 10 bundle.

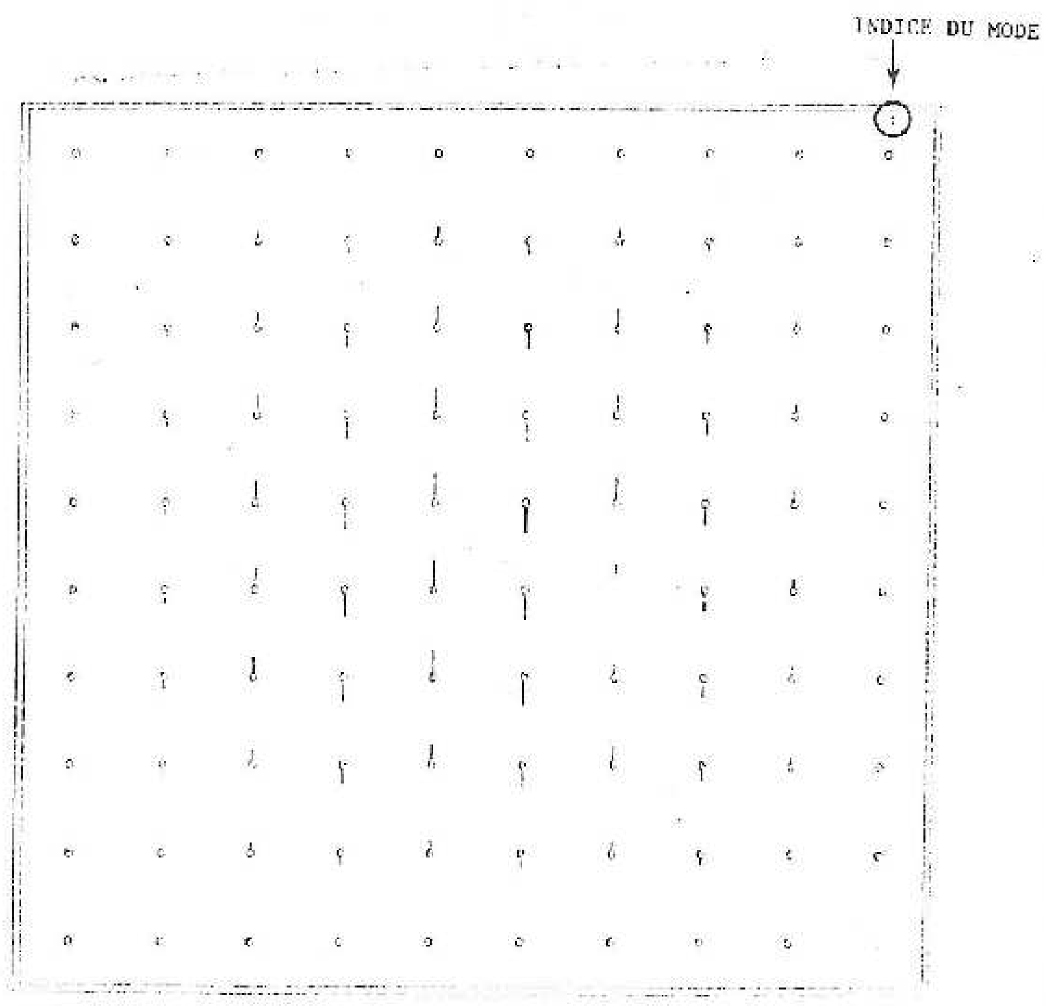
Bounded and continuous spectrum



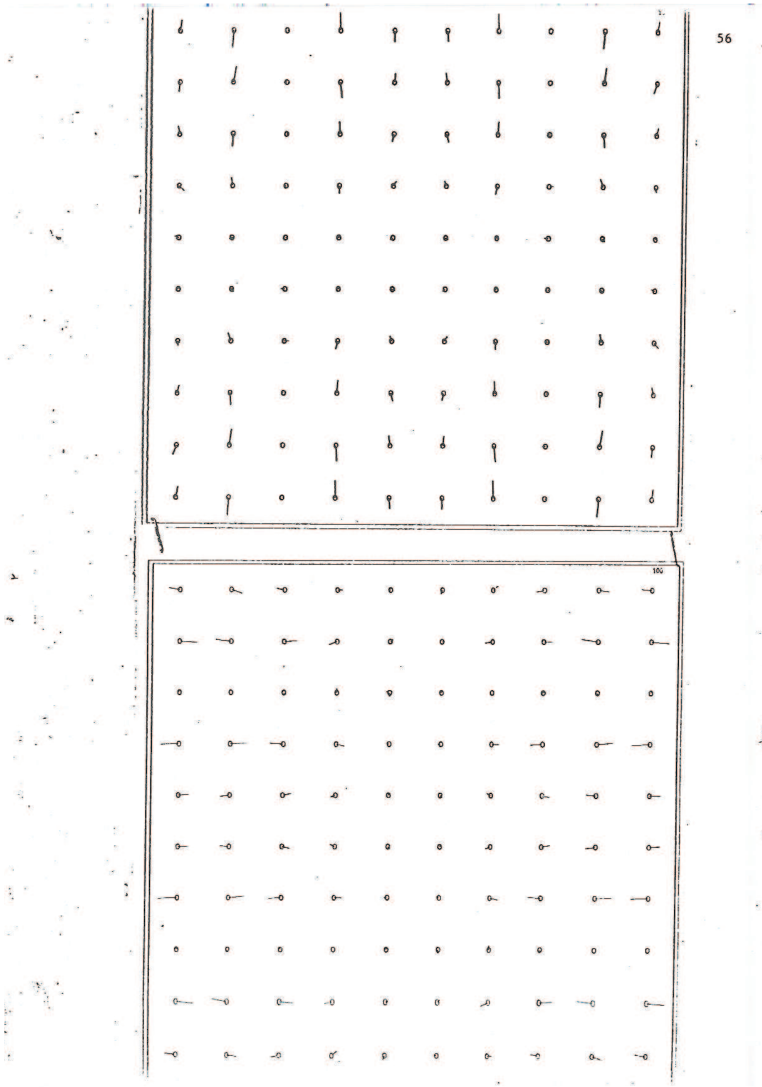
Répartition des fréquences propres
(en abscisse: indice des fréquences propres)

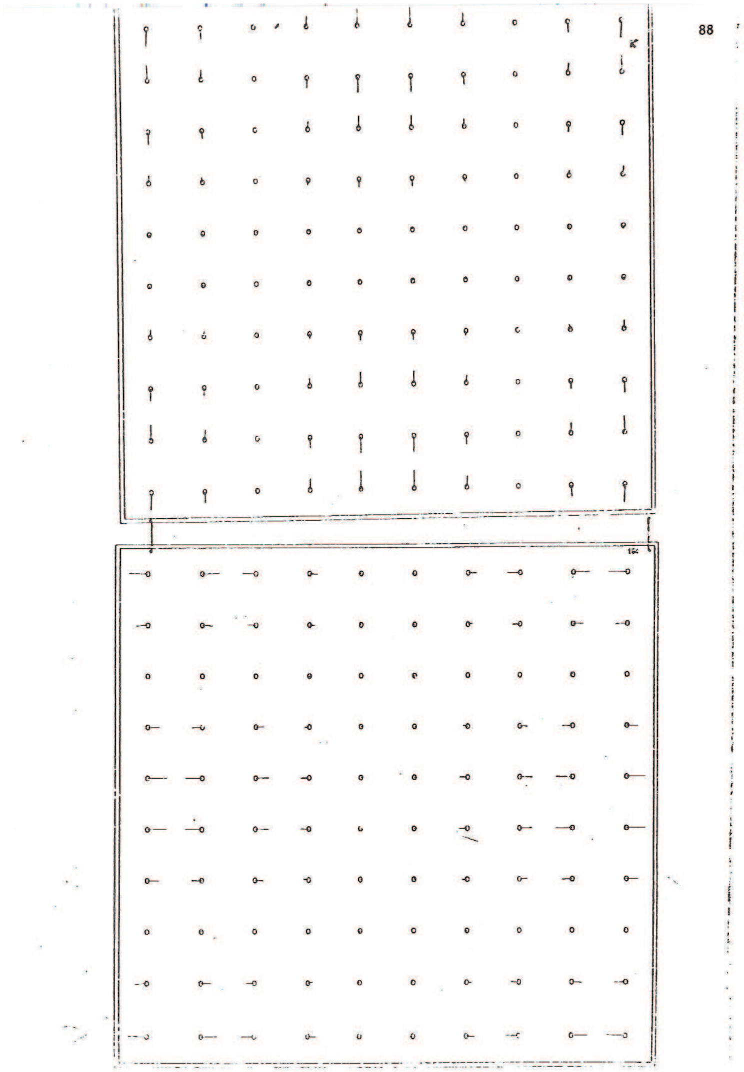
Fig. 2

Lowest frequency (not periodic !)



Boundary layer modes





Analysis

- When the domain Ω is a cube with periodic boundary conditions, Aguirre and Conca (1988) diagonalized the operator by using the Bloch wave decomposition.
- For a general domain, Conca and Vanninathan (1988) introduced a new asymptotic method, the so-called non-standard homogenization procedure.
- Conca and Allaire (1996) showed that standard homogenization (two-scale convergence) gives a different limit !
- How to explain the discrepancy between Bloch wave theory and homogenization ?
- The theoretical and numerical results are **very different** from the well-known elliptic case (typical of the Schrödinger equation).

Difference between homogenization and Bloch wave decomposition

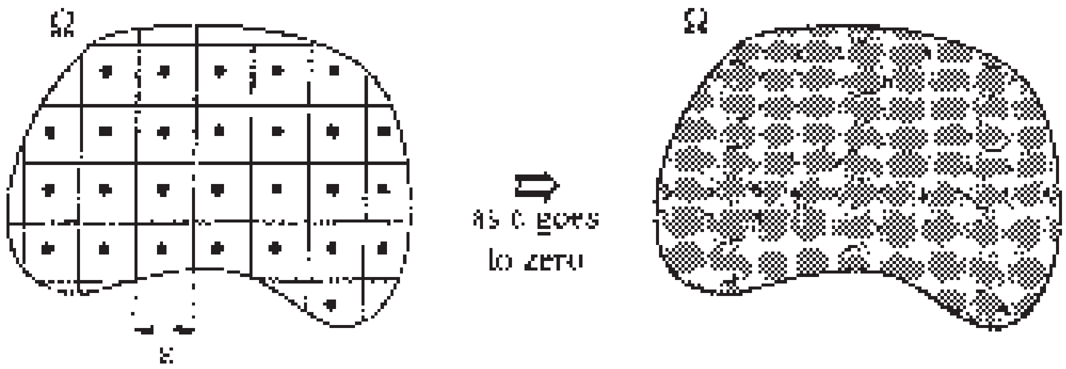
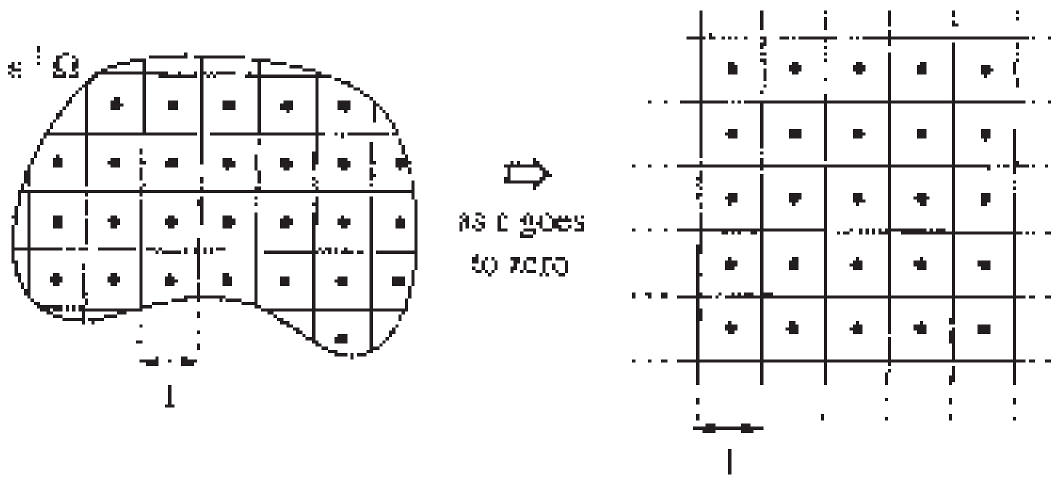


Fig. 1. Homogenization process



Definition of Bloch waves

Theorem. For any function $u(y) \in L^2(\mathbb{R}^N)$ there exists a unique function $\hat{u}(y,\theta) \in L^2(Y \times Y)$, with $Y = (0,1)^N$, such that

$$u(y) = \int_Y \hat{u}(y,\theta) e^{2i\pi\theta\cdot y} d\theta.$$

The function $y \to \hat{u}(y,\theta)$ is Y-periodic while the function $\theta \to e^{2i\pi\theta\cdot y}\hat{u}(y,\theta)$ is Y-periodic. Furthermore, the Bloch transform $u \to \hat{u}$ is an isometry from $L^2(\mathbb{R}^N)$ into $L^2(Y \times Y)$, i.e. Parseval formula holds for any $u, v \in L^2(\mathbb{R}^N)$

$$\int_{\mathbb{R}^N} u(y)\overline{v(y)}\, dy = \int_Y \int_Y \hat{u}(y,\theta)\overline{\hat{v}(y,\theta)}\, dy\, d\theta.$$

Each periodic function $\hat{u}(y,\theta)$ can be further decomposed on a basis of eigenfunctions.

The Bloch transform diagonalizes differential operators with periodic coefficients.

Main result of the Bloch wave method

Theorem. (Conca and Vanninathan) Let σ_ϵ be the spectrum made of the discrete eigenvalues λ_ϵ. Then

$$\sigma_{Bloch} \subset \lim_{\epsilon \to 0} \sigma_\epsilon \quad \text{with} \quad \sigma_{Bloch} = \cup_{\theta \in Y} \sigma(\theta)$$

where $\sigma(\theta)$ is the discrete spectrum of the Bloch cell problem

$$\begin{cases} -\Delta u_\theta = 0 & \text{in } Y \\ \lambda_\theta \frac{\partial u_\theta}{\partial n} = \vec{n} \cdot \int_\Gamma u_\theta \vec{n} ds & \text{on } \Gamma \\ y \to u_\theta(y) e^{-2i\pi\theta \cdot y} & \text{is } Y\text{-periodic} \end{cases}$$

The inclusion is possibly strict.

In 2-d $\sigma(\theta) = \{\lambda_\theta^1, \lambda_\theta^2\}$.

Numerical results of Aguirre-Conca (1988)

Eigenfrequencies of a Tube Bundle Immersed in a Fluid 37

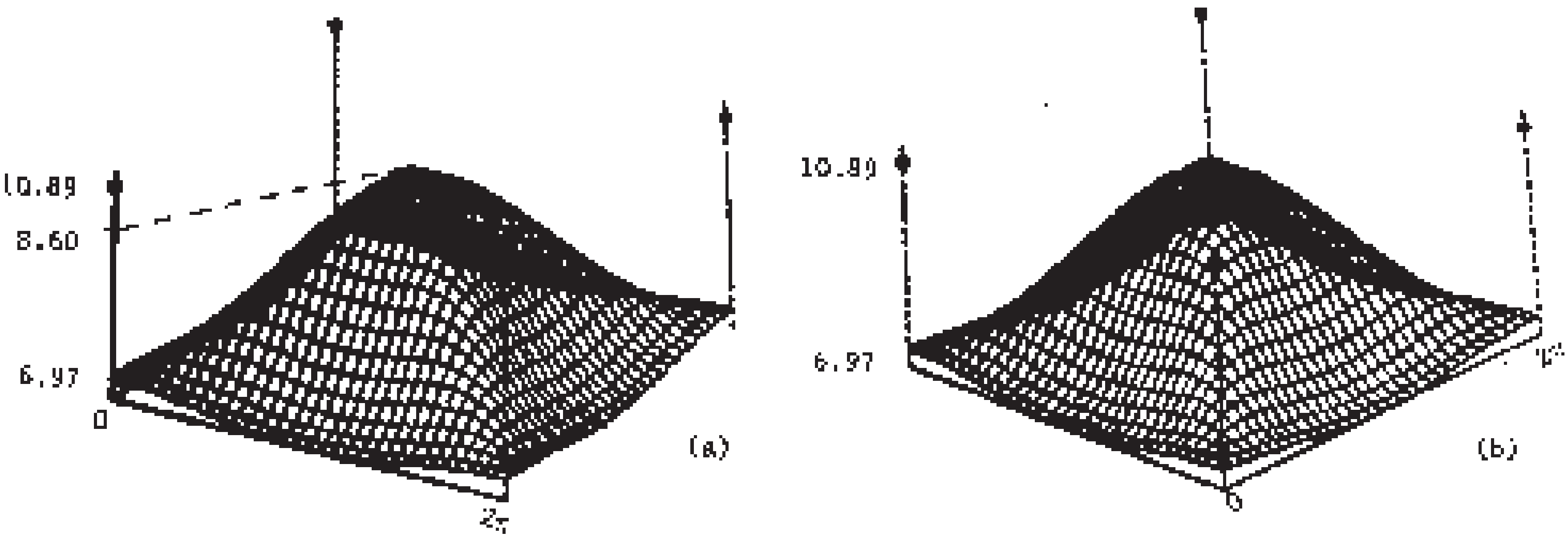
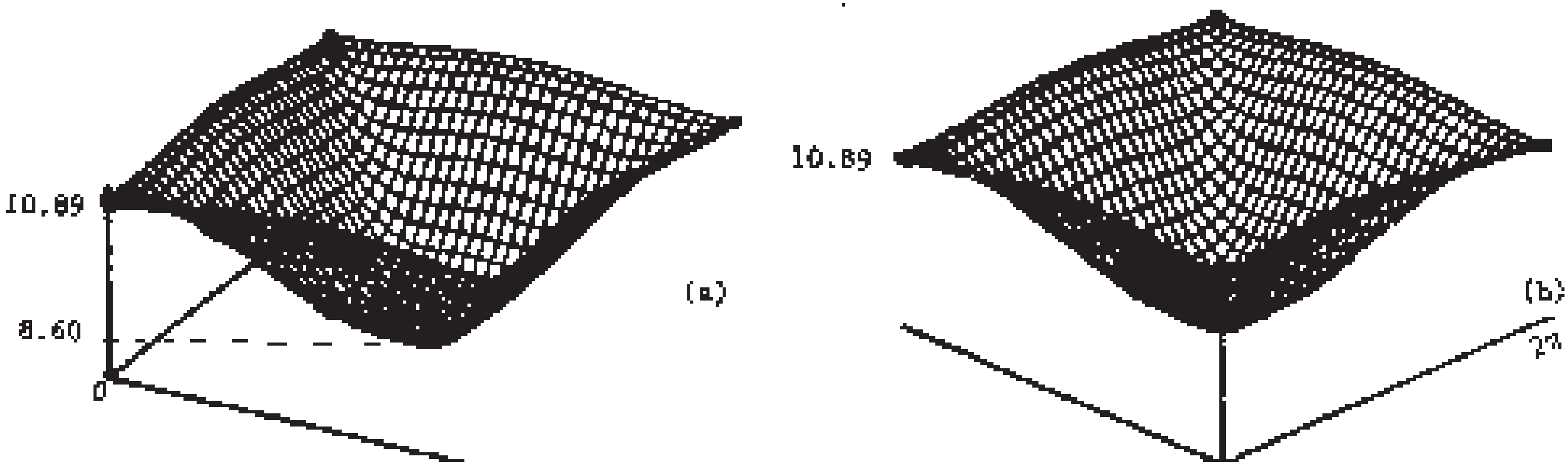


Figure 5.2. Graphical representation of $\beta_1(\cdot)$ under two different points of view.

Numerical results of Aguirre-Conca (1988)



Interpretation of the results

- The spectrum is continuous and bounded in the limit.
- The minimal eigenfrequency is obtained for $\theta = (0, 1/2)$. The ground state is not periodic !
- The eigenvalue λ_θ^1 is not continuous at 0 ! The long wave length modes do not behave like Y-periodic modes.
- Very different from the elliptic case (for which $A^* = \frac{1}{2}\nabla_\theta\nabla_\theta\lambda^1(\theta = 0)$).
- σ_{Bloch} is just a part of the limit spectrum.

Homogenization approach

- Replace the discrete vector displacements $\vec{s}$ by the average on each tube of a continuous function $\mathbf{s}(x)$ from Ω into $\mathbb{R}^2$.
- Locally the problem is Y-periodic.
- In the limit ones finds the influence of the domain shape and boundary condition.
- The homogenized spectrum σ_{hom} is essential (no point spectrum) and a priori different from the Bloch spectrum.

Homogenization result

Theorem. (Conca and Allaire) There exists an homogenized equation and

$$\sigma_{hom} \subset \lim_{\epsilon \to 0} \sigma_\epsilon \quad \text{with} \quad \sigma_{Bloch} \neq \sigma_{hom}$$

If one takes into account the "boundary layer spectrum" $\sigma_{boundary}$, then

$$\lim_{\epsilon \to 0} \sigma_\epsilon = \sigma_{hom} \cup \sigma_{Bloch} \cup \sigma_{boundary}$$

- The lowest frequency are not attained in σ_{hom}.
- The usual rule of homogenizing with a single cell does not work.
- The boundary layer spectrum $\sigma_{boundary}$ depends on the sequence ϵ (Moskow and Vogelius, Castro and Zuazua).

Truth about the applications and what remains to be done

- Vibrations are not critical for the core of PWR (pressurized water reactor).
- Vibrations are important in steam generator (exchanger) but shocks should be taken into account.
- In Fast Sodium Reactor vibrations are important because of coupling with neutronics. Work to be done...

CONCLUSION

Happy birthday Carlos !

Feliz cumpleaños!