SIMULATION AND INTERPRETATION OF BOREHOLE GEOPHYSICAL MEASUREMENTS USING $hp$ FINITE ELEMENTS

$hp$-FEM team: D. Pardo, M. J. Nam, L. Demkowicz, C. Torres-Verdín, V. M. Calo, M. Paszynski, and P. J. Matuszyk
Overview

1. Main Lines of Research and Applications (D. Pardo)
   - Previous work
   - Main features of our technology

2. Application 1: Tri-Axial Induction Instruments (M. J. Nam)

3. Application 2: Dual-Laterolog Instruments (M. J. Nam)

4. Multi-Physics Inversion (D. Pardo)

5. Sonic Instruments (L. Demkowicz)
## Previous Work

### Type of Problems We Can Solve with our \textit{hp}-FEM software

<table>
<thead>
<tr>
<th>Applications</th>
<th>Borehole Measurements</th>
<th>Marine Controlled Source EM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spatial Dimensions</td>
<td>2D</td>
<td>3D</td>
</tr>
<tr>
<td>Well Type</td>
<td>Vertical Well</td>
<td>Deviated Well</td>
</tr>
<tr>
<td>Logging Instruments</td>
<td>LWD/MWD</td>
<td>Normal/Laterolog</td>
</tr>
<tr>
<td></td>
<td>Induction</td>
<td>Dielectric Instruments</td>
</tr>
<tr>
<td>Frequency</td>
<td>0 ~ 10 GHz</td>
<td></td>
</tr>
<tr>
<td>Materials</td>
<td>Isotropic</td>
<td>Anisotropic</td>
</tr>
<tr>
<td>Physical Devices</td>
<td>Magnetic Buffers</td>
<td>Insulators</td>
</tr>
<tr>
<td></td>
<td>Casing Imperfections</td>
<td>Displacement Currents</td>
</tr>
<tr>
<td>Sources</td>
<td>Finite Size Antennas</td>
<td>Dipoles in Any Direction</td>
</tr>
<tr>
<td></td>
<td>Toroidal Antennas</td>
<td>Electrodes</td>
</tr>
<tr>
<td>Invasion</td>
<td>Water</td>
<td>Oil</td>
</tr>
</tbody>
</table>

**MOST (OIL-INDUSTRY) GEOPHYSICAL PROBLEMS**
Main Features of Our Technology

1. Self-Adaptive Goal-Oriented $hp$-Refinements

2. Fourier Finite-Element Method

3. Parallel Implementation
Self-Adaptive Goal-Oriented $hp$-FEM

We vary locally the element size $h$ and the polynomial order of approximation $p$ throughout the grid.

Optimal grids are automatically generated by the $hp$-algorithm.

The self-adaptive goal-oriented $hp$-FEM provides exponential convergence rates in terms of the CPU time vs. the error in a user prescribed quantity of interest.
3D Deviated Well

Cartesian system of coordinates: \((x_1, x_2, x_3)\)

New non-orthogonal system of coordinates: \((\zeta_1, \zeta_2, \zeta_3)\)

\[
\begin{align*}
\text{Subdomain 1} & : & x_1 &= \zeta_1 \cos \zeta_2 \\
& & x_2 &= \zeta_1 \sin \zeta_2 \\
& & x_3 &= \zeta_3
\end{align*}
\]

\[
\begin{align*}
\text{Subdomain 2} & : & x_1 &= \zeta_1 \cos \zeta_2 \\
& & x_2 &= \zeta_1 \sin \zeta_2 \\
& & x_3 &= \zeta_3 + \tan \theta \frac{\zeta_1 - \rho_1}{\rho_2 - \rho_1} \rho_2 \cos \zeta_2
\end{align*}
\]

\[
\begin{align*}
\text{Subdomain 3} & : & x_1 &= \zeta_1 \cos \zeta_2 \\
& & x_2 &= \zeta_1 \sin \zeta_2 \\
& & x_3 &= \zeta_3 + \zeta_1 \tan \theta \cos \zeta_2
\end{align*}
\]
3D Deviated Well

Cartesian system of coordinates: \((x_1, x_2, x_3)\)

New non-orthogonal system of coordinates: \((\zeta_1, \zeta_2, \zeta_3)\)

Constant material coefficients in the quasi-azimuthal direction \(\zeta_2\) in the new non-orthogonal system of coordinates!!!
3D Deviated Well

For each Fourier mode, we obtain a 2D problem. Each 2D problem couples with up to five different 2D problems corresponding to different Fourier modes, therefore, constituting the resulting 3D problem.

<table>
<thead>
<tr>
<th>$A_{i,j}$</th>
<th>$A_{i,2}$</th>
<th>$A_{i,3}$</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{2,1}$</td>
<td>$A_{2,2}$</td>
<td>$A_{2,3}$</td>
<td>$A_{2,4}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$A_{3,1}$</td>
<td>$A_{3,2}$</td>
<td>$A_{3,3}$</td>
<td>$A_{3,4}$</td>
<td>$A_{3,5}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>$A_{4,2}$</td>
<td>$A_{4,3}$</td>
<td>$A_{4,4}$</td>
<td>$A_{4,5}$</td>
<td>$A_{4,6}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>$A_{5,3}$</td>
<td>$A_{5,4}$</td>
<td>$A_{5,5}$</td>
<td>$A_{5,6}$</td>
<td>$A_{5,7}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$A_{6,4}$</td>
<td>$A_{6,5}$</td>
<td>$A_{6,6}$</td>
<td>$A_{6,7}$</td>
<td>$A_{6,8}$</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$A_{7,5}$</td>
<td>$A_{7,6}$</td>
<td>$A_{7,7}$</td>
<td>$A_{7,8}$</td>
<td>$A_{7,9}$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$A_{8,6}$</td>
<td>$A_{8,7}$</td>
<td>$A_{8,8}$</td>
<td>$A_{8,9}$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$A_{9,7}$</td>
<td>$A_{9,8}$</td>
<td>$A_{9,9}$</td>
</tr>
</tbody>
</table>

When we use 9 Fourier Modes for the Solution:

$A_{i,j}$ : represents a full 2D problem for each Fourier basis function
For each Fourier mode, we obtain a 2D problem. Each 2D problem couples with up to five different 2D problems corresponding to different Fourier modes, therefore, constituting the resulting 3D problem.
3D Parallelization Implementation

Distributed Domain Decomposition

- Processor 1
- Processor 2
- Processor 3
- Processor 4
- Processor 5
- Processor 6
- Processor 7
- Processor 8

Shared Domain Decomposition!!

- Processor 1
- Processor 2
- Processor 3
- Processor 4
- Processor 5
- Processor 6
- Processor 7
- Processor 8
SELF-ADAPTIVE $hp$ FINITE-ELEMENT SIMULATION OF MULTI-COMPONENT INDUCTION MEASUREMENTS ACQUIRED IN DIPPING, INVADED, AND ANISOTROPIC FORMATIONS

M. J. Nam, D. Pardo, and C. Torres-Verdín,
The University of Texas at Austin

$hp$-FEM team: D. Pardo, M. J. Nam, L. Demkowicz, C. Torres-Verdín,
V. M. Calo, M. Paszynski, and P. J. Matuszik

8th Annual Formation Evaluation Research Consortium Meeting
August 14-15, 2008
Overview

1. Main Lines of Research and Applications (D. Pardo)
   - Previous work
   - Main features of our technology

2. Application 1: Tri-Axial Induction Instruments (M. J. Nam)

3. Application 2: Dual-Laterolog Instruments (M. J. Nam)

4. Multi-Physics Inversion (D. Pardo)

5. Sonic Instruments (L. Demkowicz)
Outline

• Introduction to Tri-Axial Induction

• Method

• Numerical Results:
  – Verification of 3D Method for Tri-Axial Induction Tool
  – Dipping, Invaded, Anisotropic Formations

• Conclusions
Tri-Axial Induction Tool

$L = 1.016 \text{ m (40 in.)}$

Operating frequency: $20 \text{ kHz}$

$\theta$: dip angle

$\alpha$: tool orientation angle
3D Source Implementation

1. Solenoidal Coil \( (J_\phi) \) for \( M_z \)
   ➔ becoming a 2D source in \((\rho, \phi, z)\)

2. Delta Function for 3D source \( M_x \) or \( M_y \)

\[
f(\phi) = \delta(\phi - \phi_0)
\]

\( \phi_0: \) the position of the center of the peak
(\( 0^\circ \) for \( M_x \); \( 90^\circ \) for \( M_y \))

\( M_x \): Delta function

Gibb’s Phenomenon
3D Source and Receiver (Delta Functions)

Coupling between source and receiver: less Gibb’s phenomenon
Method

Combination of:

1. A Self-Adaptive Goal-Oriented $hp$-FEM for AC problems

2. A Fourier Series Expansion in a Non-Orthogonal System of Coordinates

3. Parallel Implementation
Verification of 2.5D Simulation \( (H_{xx} = H_{yy}) \)

- Real part of \( H_{xx} \) at 20 kHz
- Imaginary part of \( H_{xx} \) at 20 kHz

**Converged solutions with 3 Fourier modes**

Verification of 2.5D Simulation ($H_{zz}$)

- Real part of $H_{zz}$ at 20 kHz
- Imaginary part of $H_{zz}$ at 20 kHz

The same solutions with 1 Fourier mode
Verification of 2.5D Simulation \( (H_{xy} = H_{yx}) \)

Real part of \( H_{xy} \) at 20 kHz

Imaginary part of \( H_{xy} \) at 20 kHz

Converged solutions with 5 Fourier modes
Verification of 2.5D Simulation \((H_{xz} = H_{zx})\)

The same solutions with 1 Fourier mode
Verification of 3D Simulation \((H_{xx} = H_{yy})\)

**Real part of Hxx at 20 kHz**

- **em1d**
- hp with 3 modes
- hp with 7 modes
- hp with 9 modes

**Imaginary part of Hxx at 20 kHz**

- **em1d**
- hp with 3 modes
- hp with 7 modes
- hp with 9 modes

**Dip angle: 60 degrees**

**Converged solutions with 9 Fourier mode**
Verification of 3D Simulation ($H_{zz}$)

Dip angle: 60 degrees

Converged solutions with 5 Fourier mode
Description of the Tri-Axial Tool

Operating frequency: 20 kHz

Finite size antenna

1 cm

0.09 m

1 m

0.1 m

1.016 m (40 in.)

Transmitter

Receiver

10^5 ohm-m
Verification of 2.5D Simulation ($H_{xx}$)

Relative errors of tri-axial induction solutions with respect to the solution with 9 Fourier modes

- vertical well in a homogeneous formation

![Graph showing relative errors with respect to the number of Fourier modes.](image-url)
Verification of 3D Simulation ($H_{xx}$)

θ = 60 degrees

Relative errors of tri-axial Induction solutions with respect to the solution for the vertical well

$60$ degree deviated well in a homogeneous formation

Relative Error (in %)

Number of Fourier Modes
Model for Numerical Experiments

Five layers: 100, 0.05, 10000, 1 and 20 ohm-m from top to bottom

Borehole: 0.1 m in radius
100 ohm-m in resistivity

Invasion in the third and fourth layers

Anisotropy in the second and fourth layers

$\theta = 0, 30$ and $60$ degrees
Convergence History of $\mathcal{H}_{xx}$ in Vertical Well

Real part of $\mathcal{H}_{xx}$ at 20 kHz

Imaginary part of $\mathcal{H}_{xx}$ at 20 kHz

Converged solutions with 5 Fourier modes
Convergence History of $H_{xx}$ in Deviated Well

$\theta = 60 \text{ degrees}$

Converged solutions with 9 Fourier modes
Deviated Wells (0, 30 & 60 degrees)

Dip angle has larger effects on tri-axial tools.
Deviated Wells (0, 30 & 60 degrees)

Dip angle has larger effects on tri-axial tools
$H_{zz}$ in Deviated Wells with Invasion (Re.)

Shallow invasion with $R = 0.1$ m

Real part of Hzz at 20 kHz

No invasion

With invasion

100 ohm-m

0.05 ohm-m

10000 ohm-m (500 ohm-m)

1 ohm-m (10 ohm-m)

20 ohm-m

Re(Hzz) field (A/m)

Depth (m)

60 degrees

vertical
Imaginary part of $H_{zz}$ at 20 kHz

No invasion

With invasion

Depth (m)

100 ohm-m

0.05 ohm-m

10000 ohm-m

1 ohm-m

(10 ohm-m)

20 ohm-m

Im(Hzz) field (A/m)

$10^{-5}$

$10^{-6}$

$10^{-7}$

100 ohm-m

0.05 ohm-m

10000 ohm-m

1 ohm-m

(10 ohm-m)

20 ohm-m

Almost no effects of invasion regardless of the dip angle

Shallow invasion with $R = 0.1$ m

60 degrees
$H_{xx}$ in Deviated Wells with Invasion (Re.)

**Shallow invasion with $R = 0.1\, \text{m}$**

**Horizontal cross-section**
- $100\, \text{ohm-m}$
- $0.05\, \text{ohm-m}$
- $10000\, \text{ohm-m}$
- $1\, \text{ohm-m}$
- $20\, \text{ohm-m}$

**Vertical cross-section**
- $1\, \text{ohm-m}$
- $20\, \text{ohm-m}$

**Angles**
- Vertical
- 60 degrees
$H_{xx}$ in Deviated Wells with Invasion (Im.)

- Imaginary part of $H_{xx}$ at 20 kHz for different resistivity layers:
  - No invasion
  - With invasion

- Depth (m) vs. Im($H_{xx}$) field (A/m) for:
  - 100 ohm-m
  - 0.05 ohm-m
  - 10 000 ohm-m (500 ohm-m)
  - 1 ohm-m (10 ohm-m)
  - 20 ohm-m

- Shallow invasion with $R = 0.1$ m:
  - Small effects of invasion

- 60 degrees vertical
$H_{yy}$ in Deviated Wells with Invasion (Re.)

**Real part of $H_{yy}$ at 20 kHz**

**Depth (m)**
- 0.05 ohm-m
- 100 ohm-m
- (500 ohm-m)
- 1 ohm-m (10 ohm-m)
- 20 ohm-m

**Re($H_{yy}$) field (A/m)**
- No invasion
- With invasion

---

**Shallow invasion with $R = 0.1$ m**

**60 degrees**

**vertical**
$H_{yy}$ in Deviated Wells with Invasion (Im.)

**Imaginary part of $H_{yy}$ at 20 kHz**

- **No invasion**
- **with invasion**

**Depth (m)**
- 0.05 ohm-m
- 100 ohm-m
- 1000 ohm-m (500 ohm-m)
- 1 ohm-m (10 ohm-m)
- 20 ohm-m

**Im(Hyy) field (A/m)**
- -6 to 0

**Shallow invasion with $R = 0.1$ m**

**Small effects of invasion**
$H_{zz}$ in Deviated Wells with Anisotropy (Re.)

Effects of anisotropy increase with increasing dip angle

- **vertical**
- **30 degrees**
- **60 degrees**
$H_{zz}$ in Deviated Wells with Anisotropy (Im.)

Imaginary part of Hzz at 20 kHz

Effects of anisotropy increase with increasing dip angle

vertical

30 degrees

60 degrees
$H_{xx}$ in Deviated Wells with Anisotropy (Re.)

Effects of anisotropy decrease with increasing dip angle

vertical

30 degrees

60 degrees
$H_{xx}$ in Deviated Wells with Anisotropy (Im.)

Effects of anisotropy decrease with increasing dip angle

vertical

30 degrees

60 degrees
$H_{yy}$ in Deviated Wells with Anisotropy (Re.)

Effects of anisotropy decrease with increasing dip angle

vertical
30 degrees
60 degrees
$H_{yy}$ in Deviated Wells with Anisotropy (Im.)

Effects of anisotropy decrease with increasing dip angle

vertical

30 degrees

60 degrees
$H_{xx}$ at 20 KHz and 2 MHz in Vertical Well

Larger variations at 2 MHz than at 20 kHz
Conclusions

• We successfully simulated 3D tri-axial induction measurements by combining the use of a Fourier series expansion in a non-orthogonal system of coordinates with a 2D high-order, self-adaptive $hp$ finite-element method.

• Dip angle effects on tri-axial tools are larger than on more traditional induction logging instruments.

• Anisotropy effects on $H_{xx}$ and $H_{yy}$ decrease with increasing dip angle, while those on $H_{zz}$ increase.

• $H_{xx}$ at 20 kHz exhibits smaller variations than at 2 MHz.
Acknowledgements

Sponsors of UT Austin’s consortium on Formation Evaluation: