SELF-ADAPTIVE \( hp \) FINITE-ELEMENT SIMULATION OF MULTI-COMPONENT INDUCTION MEASUREMENTS ACQUIRED IN DIPPING, INVADED, AND ANISOTROPIC FORMATIONS

M. J. Nam\(^1\), D. Pardo\(^2\)*, and C. Torres-Verdín\(^1\),

\(^1\)The University of Texas at Austin, USA
\(^2\)Basque Center for Applied Mathematics, Spain
*Formerly, at The University of Texas at Austin, USA

Presentation at SIG meeting (SPWLA) Oct. 21, 2008.
Houston, TX, USA
Outline

• Main Features of Our Technology
  – A Self-Adaptive Goal-Oriented $hp$-FEM
  – Fourier Finite-Element Method

• Introduction to Tri-Axial Induction

• Numerical Results:
  – in Dipping, Invaded, Anisotropic Formations (Resistive Mandrel)
  – with Tool Eccentricity (Conductive/Resistive Mandrel)

• Conclusions
Self-Adaptive Goal-Oriented $hp$-FEM

We vary locally the element size $h$ and the polynomial order of approximation $p$ throughout the grid.

Optimal grids are automatically generated by the $hp$-algorithm.

The self-adaptive goal-oriented $hp$-FEM provides exponential convergence rates in terms of the CPU time vs. the error in a user prescribed quantity of interest.
3D Deviated Well

Cartesian system of coordinates: \((x_1, x_2, x_3)\)

New non-orthogonal system of coordinates: \((\zeta_1, \zeta_2, \zeta_3)\)

Subdomain 1
\[
\begin{align*}
    x_1 &= \zeta_1 \cos \zeta_2 \\
    x_2 &= \zeta_1 \sin \zeta_2 \\
    x_3 &= \zeta_3
\end{align*}
\]

Subdomain 2
\[
\begin{align*}
    x_1 &= \zeta_1 \cos \zeta_2 \\
    x_2 &= \zeta_1 \sin \zeta_2 \\
    x_3 &= \zeta_3 + \tan \theta \frac{\zeta_1 - \rho_1}{\rho_2 - \rho_1} \rho_2 \cos \zeta_2
\end{align*}
\]

Subdomain 3
\[
\begin{align*}
    x_1 &= \zeta_1 \cos \zeta_2 \\
    x_2 &= \zeta_1 \sin \zeta_2 \\
    x_3 &= \zeta_3 + \zeta_1 \tan \theta \cos \zeta_2
\end{align*}
\]
3D Deviated Well

Cartesian system of coordinates: \((x_1, x_2, x_3)\)

New non-orthogonal system of coordinates: \((\zeta_1, \zeta_2, \zeta_3)\)

Constant material coefficients in the quasi-azimuthal direction \(\zeta_2\) in the new non-orthogonal system of coordinates!!!!
Fourier Series Expansion in $\zeta_2$

Fourier Series Expansion of a Function $\omega$ in $\zeta_2$: 

$$\omega = \sum_{l=-\infty}^{l=\infty} \omega_l e^{j l \zeta_2} = \sum_{l=-\infty}^{l=\infty} F_l(\omega) e^{j l \zeta_2}$$

Final Variational Formulation of DC after Fourier Series Expansion in $\zeta_2$:

Find $F_l(u) \in F_l(u_D) + H^1_D(\Omega_{2D})$ such that:

$$\sum_{k=-\infty}^{k=\infty} \sum_{l=k-2}^{l=k+2} \left< F_k \left( \frac{\partial v}{\partial \zeta} \right), F_{k-l} \left( \sigma_{NEW} \right) F_l \left( \frac{\partial u}{\partial \zeta} \right) \right>_{L^2(\Omega_{2D})}$$

$$= \sum_{k=-\infty}^{k=\infty} \left[ \left< F_k(v), F_k(f_{NEW}) \right>_{L^2(\Omega_{2D})} + \left< F_k(v), F_k(g_{NEW}) \right>_{L^2(\Omega_{2D})} \right] \quad \forall F_k(v) \in H^1_D(\Omega),$$

because $F_{k-l}(\sigma_{NEW}) = 0$ for every $|k-l| > 2$.

Only Five Fourier Modes ($l$) are enough to represent $\sigma_{NEW}$ EXACTLY for each $k$.

Therefore, we need to truncate only Fourier Modes ($k$) for 3D solution.
**Eccentered Tool**

**Cartesian system of coordinates:** \((x_1, x_2, x_3)\)

**New non-orthogonal system of coordinates:** \((\zeta_1, \zeta_2, \zeta_3)\)

\[
\begin{align*}
\text{Subdomain 1} & : \\
x_1 &= \rho_0 + \zeta_1 \cos \zeta_2 \\
x_2 &= \zeta_1 \sin \zeta_2 \\
x_3 &= \zeta_3 \\
\end{align*}
\]

\[
\begin{align*}
\text{Subdomain 2} & : \\
x_1 &= \frac{\zeta_1 - \rho_2}{\rho_1 - \rho_2} \cdot \rho_0 + \zeta_1 \cos \zeta_2 \\
x_2 &= \zeta_1 \sin \zeta_2 \\
x_3 &= \zeta_3 \\
\end{align*}
\]

\[
\begin{align*}
\text{Subdomain 3} & : \\
x_1 &= \zeta_1 \cos \zeta_2 \\
x_2 &= \zeta_1 \sin \zeta_2 \\
x_3 &= \zeta_3 \\
\end{align*}
\]
Tri-Axial Induction Tool

\[ L = 1.016 \text{ m (40 in.)} \]

Operating frequency: 20 kHz

\( \theta \): dip angle

\( \alpha \): tool orientation angle
3D Source Implementation

1. Solenoidal Coil ($J_\phi$) for $M_z$
   \[ \rightarrow \text{becoming a 2D source in } (\rho, \phi, z) \]

2. Delta Function for 3D source $M_x$ or $M_y$
   \[ f(\phi) = \delta(\phi - \phi_0) \]
   $\phi_0$: the position of the center of the peak
   (0° for $M_x$; 90° for $M_y$)

$M_x$: Delta function

Gibb’s Phenomenon
3D Source and Receiver (Delta Functions)

Coupling between source and receiver: less Gibb’s phenomenon
Verification of 2.5D Simulation ($H_{xx} = H_{yy}$)

Real part of Hxx at 20 kHz

Imaginary part of Hxx at 20 kHz

Converged solutions with 3 Fourier modes

Verification of 2.5D Simulation ($H_{xy} = H_{yx}$)

![Graphs showing real and imaginary parts of $H_{xy}$ at 20 kHz](image)

Converged solutions with 5 Fourier modes

M. J. Nam, D. Pardo, C. Torres-Verdín
Verification of 2.5D Simulation \( (H_{xz} = H_{zx}) \)

The same solutions with 1 Fourier mode

M. J. Nam, D. Pardo, C. Torres-Verdín
Verification of 3D Simulation ($H_{xx} = H_{yy}$)

Real part of $H_{xx}$ at 20 kHz

- **em1d**
- hp with 3 modes
- hp with 7 modes
- hp with 9 modes

Imaginary part of $H_{xx}$ at 20 kHz

- **em1d**
- hp with 3 modes
- hp with 7 modes
- hp with 9 modes

Dip angle: 60 degrees

Converged solutions with 9 Fourier mode

M. J. Nam, D. Pardo, C. Torres-Verdín
Verification of 3D Simulation ($\mathbf{H}_{zz}$)

**Real part of Hzz at 20 kHz**

- $\text{Re}(H_{zz})$ field (A/m)

**Imaginary part of Hzz at 20 kHz**

- $\text{Im}(H_{zz})$ field (A/m)

Dip angle: 60 degrees

Converged solutions with 5 Fourier mode

M. J. Nam, D. Pardo, C. Torres-Verdín
Description of the Tri-Axial Tool

Operating frequency: 20 kHz

Transmitters

Y

M_y^T

M_x^T

X

1.016 m (40 in.)

Receivers

y

H_y^R

H_x^R

H_z^R

Z

1.016 m (40 in.)

Resistive mandrel (RM):
10^6 ohm-m μ₀

Conductive mandrel (CM):
10^-6 ohm-m 100μ₀

Finite size antenna

1 m

0.8 cm

1 cm

0.4 cm

0.09 m

0.1 m

Y

Transmitter

1.016 m (40 in.)

X

Z

Receiver
Verification of 2.5D Simulation ($H_{xx}$)

Relative errors of tri-axial induction solutions with respect to the solution with 9 Fourier modes

vertical well in a homogeneous formation

Relative Error (in %)

- real (without tool propt.)
- imaginary (without tool propt.)
Verification of 3D Simulation ($H_{xx}$)

$\theta = 60$ degrees

Relative errors of tri-axial Induction solutions with respect to the solution for the vertical well

- Real (without tool propt.)
- Imaginary (without tool propt.)

Number of Fourier Modes

Relative Error (in %)

60 degree deviated well in a homogeneous formation
Model for Experiments (Deviated Well)

Five layers: 100, 0.05, 10000, 1 and 20 ohm-m from top to bottom

Borehole: 0.1 m in radius
100 ohm-m in resistivity

$\theta = 0, 30$ and $60$ degrees

Resistive mandrel $(10^6 \text{ ohm-m}, \mu_0)$

Invasion in the third and fourth layers

Anisotropy in the second and fourth layers
Convergence History of $H_{xx}$ in Vertical Well

Real part of $H_{xx}$ at 20 kHz

Imaginary part of $H_{xx}$ at 20 kHz

Converged solutions with 5 Fourier modes

M. J. Nam, D. Pardo, C. Torres-Verdín
Convergence History of $H_{xx}$ in Deviated Well

M. J. Nam, D. Pardo, C. Torres-Verdín

θ = 60 degrees

Converged solutions with 9 Fourier modes
Deviated Wells (0, 30 & 60 degrees)

Dip angle has larger effects on tri-axial tools

M. J. Nam, D. Pardo, C. Torres-Verdín
$H_{zz}$ in Deviated Wells with Invasion (Im.)

Imaginary part of $H_{zz}$ at 20 kHz

- **No invasion**
- **With invasion**

**Vertical**

- $100 \text{ ohm-m}$
- $0.05 \text{ ohm-m}$
- $10000 \text{ ohm-m}$
- $1 \text{ ohm-m}$
- $20 \text{ ohm-m}$

**60 degrees**

- $10000 \text{ ohm-m}$
- $1 \text{ ohm-m}$
- $20 \text{ ohm-m}$

**Shallow invasion with $R = 0.1 \text{ m}$**

Almost no effects of invasion regardless of the dip angle
$H_{xx}$ in Deviated Wells with Invasion (Im.)

Shallow invasion with $R = 0.1$ m

Small effects of invasion

vertical

60 degrees
$H_{yy}$ in Deviated Wells with Invasion (Im.)

Imaginary part of $H_{yy}$ at 20 kHz

- No invasion
- With invasion

100 ohm-m

0.05 ohm-m

10000 ohm-m (500 ohm-m)

1 ohm-m (10 ohm-m)

20 ohm-m

Depth (m)

Im(Hyy) field (A/m)

Shallow invasion with $R = 0.1$ m

Small effects of invasion

vertical

60 degrees
$H_{zz}$ in Deviated Wells with Anisotropy (Im.)

Effects of anisotropy increase with increasing dip angle

vertical  30 degrees  60 degrees
\(H_{xx}\) in Deviated Wells with Anisotropy (Im.)

Effects of anisotropy decrease with increasing dip angle

vertical

30 degrees

60 degrees
$H_{yy}$ in Deviated Wells with Anisotropy (Im.)

**Effects of anisotropy decrease with increasing dip angle**

- **vertical**
- **30 degrees**
- **60 degrees**

M. J. Nam, D. Pardo, C. Torres-Verdín
$H_{xx}$ at 20 KHz and 2 MHz in Vertical Well

Larger variations at 2 MHz than at 20 kHz
Model for Experiments (Eccentered Tool)

Five layers: 100, 0.05, 10000, 1 and 20 ohm-m from top to bottom

Radius of borehole: 0.1 m
Model for Experiments (Eccentered Tool)

Five layers: 100, 0.05, 10000, 1 and 20 ohm-m from top to bottom

Radius of borehole: 0.1 m

Conductive borehole (CB): 1 ohm-m
Resistive borehole (RB): 1000 ohm-m

Conductive mandrel (CM): \(10^{-6}\) ohm-m, \(100\mu_0\)
Resistive mandrel (RM): \(10^6\) ohm-m, \(\mu_0\)

Eccentered distance \(\rho_0\): 0, 0.45, 2.25, 3.15 cm
$H_{zz} (\rho_0: 0, 0.45, 2.25, 3.15 \text{ cm})$

CM: Conductive Mandrel ($10^{-6}$ ohm-m, $100 \mu_0$)
RM: Resistive Mandrel ($10^6$ ohm-m)
CB: Conductive Borehole (1 ohm-m)
RB: Resistive Borehole ($10^3$ ohm-m)

No big difference between results with RM and CM
Slight deviations in results with RM
$H_{zz}$ ($\rho_0$: 0, 0.45, 2.25, 3.15 cm)

CM: Conductive Mandrel ($10^{-6}$ ohm-m, $100\mu_0$)
RM: Resistive Mandrel ($10^6$ ohm-m)
CB: Conductive Borehole ($1$ ohm-m)
RB: Resistive Borehole ($10^3$ ohm-m)

No big difference between results with RM and CM
Slight deviations in results with RM
$H_{xx}$ ($\rho_0$: 0, 0.45, 2.25, 3.15 cm)

- **CM**: Conductive Mandrel ($10^{-6}$ ohm-m, $100\mu_0$)
- **RM**: Resistive Mandrel ($10^6$ ohm-m)
- **CB**: Conductive Borehole (1 ohm-m)
- **RB**: Resistive Borehole ($10^3$ ohm-m)

**Different results between RM and CM**

**More deviations in results with RM**
$H_{xx}$ ($\rho_0$: 0, 0.45, 2.25, 3.15 cm)

- **CM**: Conductive Mandrel ($10^{-6}$ ohm-m, $100\mu_0$)
- **RM**: Resistive Mandrel ($10^6$ ohm-m)
- **CB**: Conductive Borehole (1 ohm-m)
- **RB**: Resistive Borehole ($10^3$ ohm-m)

Different results between RM and CM

More deviations in results with RM
Conclusions

• We successfully simulated 3D tri-axial induction measurements by combining the use of a Fourier series expansion in a non-orthogonal system of coordinates with a 2D high-order, self-adaptive hp finite-element method.

• Dip angle effects on tri-axial tools are larger than on more traditional induction logging instruments.

• Anisotropy effects on $H_{xx}$ and $H_{yy}$ decrease with increasing dip angle, while those on $H_{zz}$ increase.

• $H_{xx}$ at 20 kHz exhibits smaller variations than at 2 MHz.

• Differences in stability between conductive and resistive mandrels in the presence of tool eccentricity.
Acknowledgements

Sponsors of UT Austin’s consortium on Formation Evaluation:

- Anadarko Petroleum Corporation
- Aramco Saudi Aramco
- Baker Hughes Baker Atlas
- bhpbilliton
- bp
- Chevron
- ConocoPhillips
- Eni
- ExxonMobil
- Halliburton
- INI
- Marathon Oil Corporation
- Petrobras
- Schlumberger
- Shell
- StatoilHydro
- Total
- Weatherford