Progress Report (Baker-Atlas)

One Year in a Development of: “A Self-Adaptive Goal-Oriented hp-Finite Element Strategy for Simulation of Resistivity Logging Instruments”

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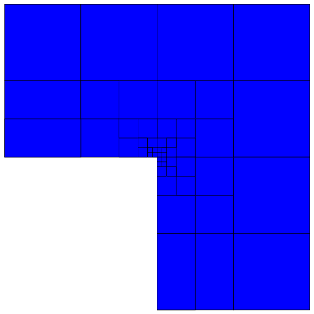
Institute for Computational Engineering and Sciences (ICES)
The University of Texas at Austin

OVERVIEW

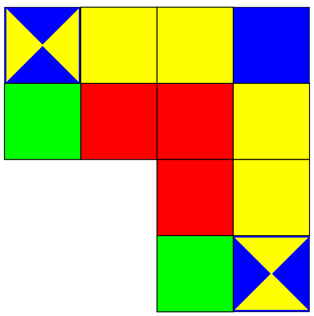
1. Overview
2. Main Idea of hp Goal-Oriented Adaptivity
3. Current Stage of the 2D High Performance FE Software
   - Flexibility
   - Reliability
   - Accuracy
   - Performance
4. Simulation of Logging Instruments
5. Accomplishments and Conclusions from the Project
6. Last Year Project Proposal: COMPLETED
7. Two New Project Proposals for: June 2005 - June 2006
   - 3D Direct Current Problems
   - 2D Multi-Physics Problems

The hp-FINITE ELEMENT METHOD



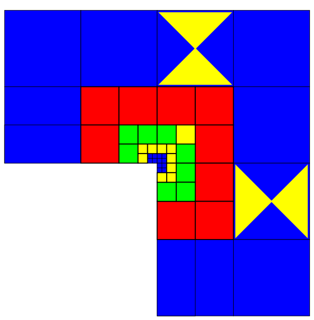
The h-Finite Element Method

1. Convergence limited by the polynomial degree, and large material contrasts.
2. Optimal *h*-grids do NOT converge exponentially in real applications.
3. They may “lock” (100% error).



The p-Finite Element Method

1. Exponential convergence feasible for analytical (“nice”) solutions.
2. Optimal *p*-grids do NOT converge exponentially in real applications.
3. If initial *h*-grid is not adequate, the *p*-method will fail miserably.



The hp-Finite Element Method

1. Exponential convergence feasible for ALL solutions.
2. Optimal *hp*-grids DO converge exponentially in real applications.
3. If initial *hp*-grid is not adequate, results will still be great.

GOAL-ORIENTED ADAPTIVITY

Mathematical Formulation (Goal-Oriented Adaptivity)

We consider the following problem (in variational form):

$$\begin{cases} \text{Find } L(\Psi)\text{, where } \Psi \in V \text{ such that :} \\ b(\Psi, \xi) = f(\xi) \quad \forall \xi \in V \ . \end{cases}$$

We define residual $r_e(\xi) = b(e, \xi)$. We seek for solution G of:

$$\begin{cases} \text{Find } G \in V'' \sim V \text{ such that :} \\ G(r_e) = L(e) \ . \end{cases}$$

This is necessarily solved if we find the solution of the *dual* problem:

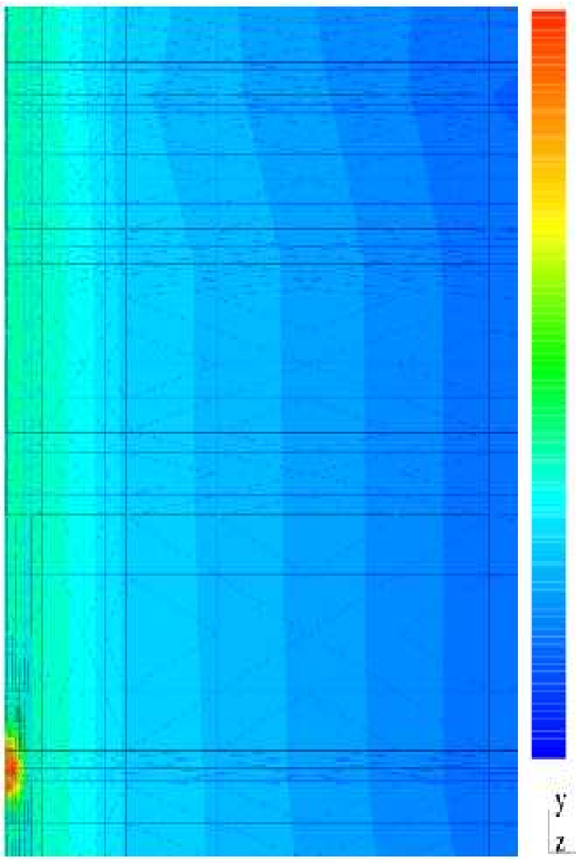
$$\begin{cases} \text{Find } G \in V \text{ such that :} \\ b(\Psi, G) = L(\Psi) \quad \forall \Psi \in V \ . \end{cases}$$

Notice that $L(e) = b(e, G)$.

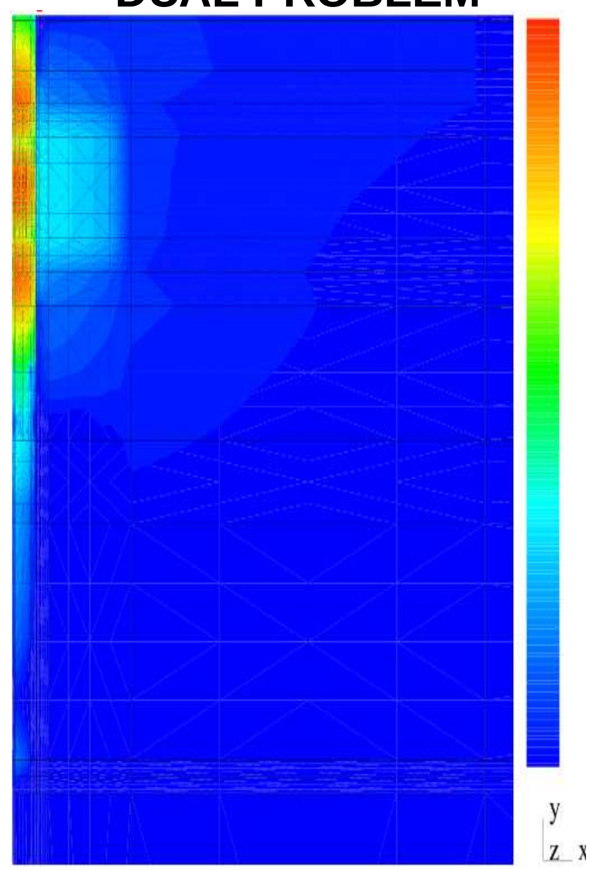
GOAL-ORIENTED ADAPTIVITY

Mathematical Formulation (Goal-Oriented Adaptivity)

DIRECT PROBLEM



DUAL PROBLEM

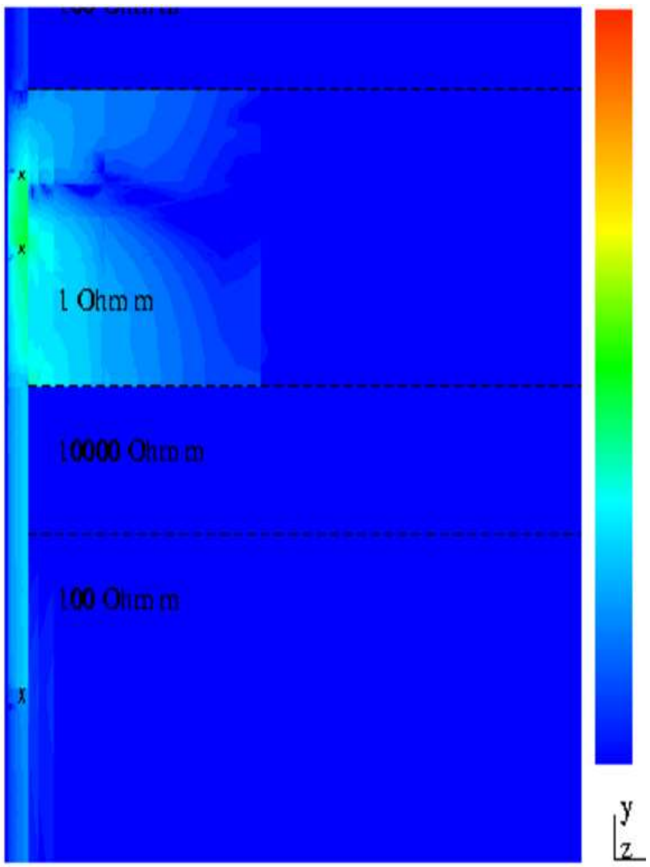


$$L(\Psi)=b(\Psi,G)$$

GOAL-ORIENTED ADAPTIVITY

Movie Presentation (Sensitivity Functions)

We want to study resolution and depth of investigation of a logging instrument.



We have:

$$|L(\Psi)| = |b(\Psi, G)| = |\int S\, dV| \leq \int |S|\, dV.$$

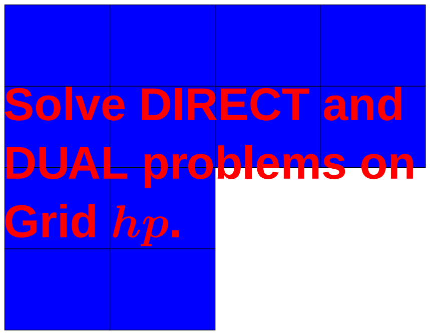
In the next movies, we display: $\log_{10}|S|$.

Scales:

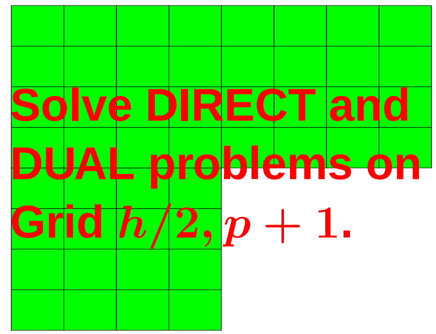
- **Red** $\longrightarrow |S| = |L(\Psi)| * 10^{4}$.
- **Blue** $\longrightarrow |S| = |L(\Psi)| * 10^{-2}$.

SELF-ADAPTIVE GOAL-ORIENTED hp-FEM

Algorithm for Goal-Oriented Adaptivity



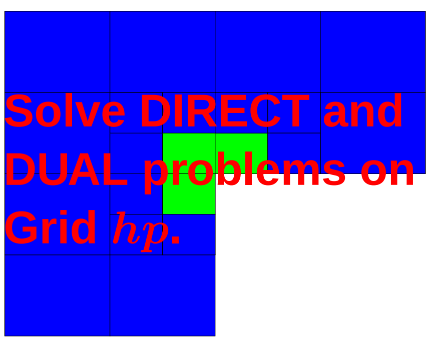
$\longrightarrow$



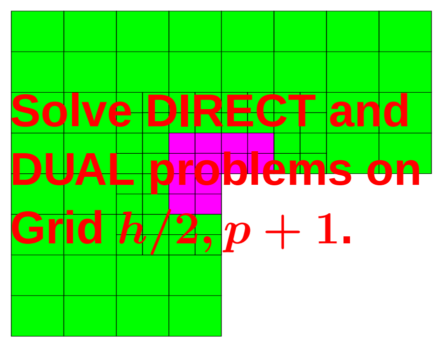
Compute $e = \Psi_{h/2,p+1} - \Psi_{hp}$, **and** $\tilde{e} = \Psi_{h/2,p+1} - \Pi_{hp}\Psi_{h/2,p+1}$.
Compute $\epsilon = G_{h/2,p+1} - G_{hp}$, **and** $\tilde{\epsilon} = G_{h/2,p+1} - \Pi_{hp}G_{h/2,p+1}$.

$$|L(e)| = |b(e,\epsilon)| \sim |b(\tilde{e},\tilde{\epsilon})| \leq \sum_K |b_K(\tilde{e},\tilde{\epsilon})| \leq \sum_K \parallel \tilde{e} \parallel_{E,K} \parallel \tilde{\epsilon} \parallel_{E,K}.$$

Apply the fully automatic hp-adaptive algorithm.

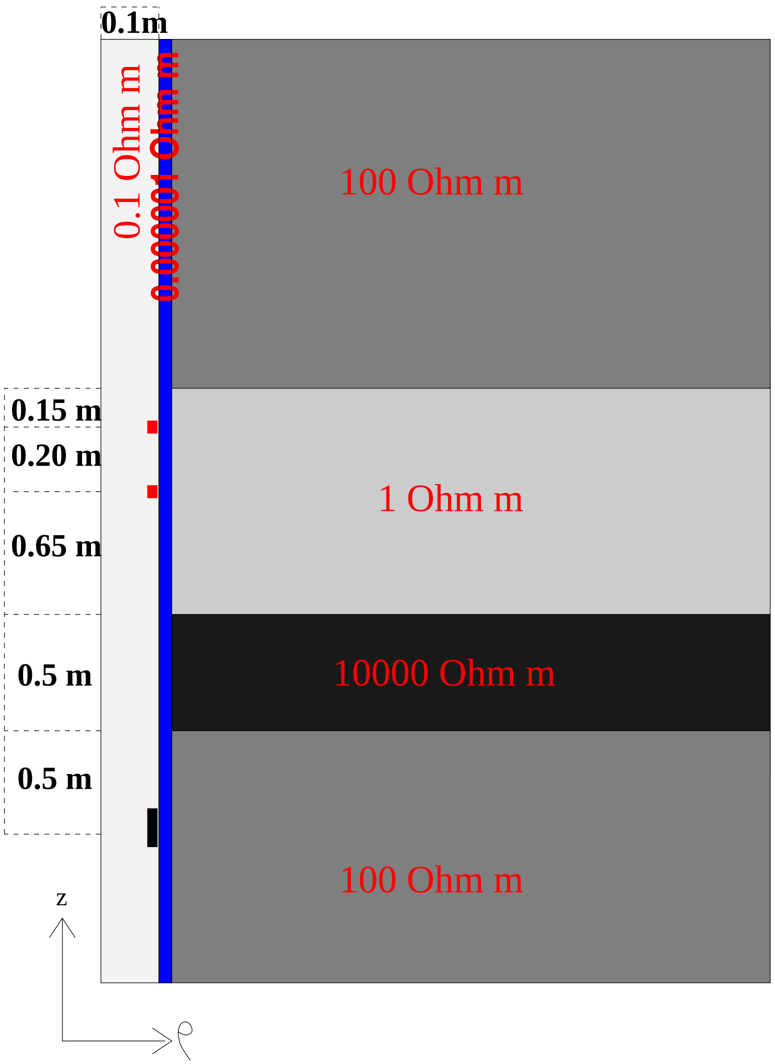


$\longrightarrow$



SELF-ADAPTIVE GOAL-ORIENTED hp-FEM

Movie Presentation (Visualization of Final hp-Grid)



Frequency: 10 Hz.

Casing resistivity: 10^{-6} Ohm $\cdot$ m.

Casing width: 0.01127 m

Discretization error $<$ 0.1 $\%$

Toroidal antennas

Size (domain): 500m x 4000m

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Flexibility (What Problems Can We Solve?)

Time-Harmonic Maxwell's Equations

$\nabla \times \mathrm{H} = (\bar{\bar{\sigma}} + j\omega\bar{\bar{\epsilon}})\mathrm{E} + \mathrm{J}^{imp}$	**Ampere's law**
$\nabla \times \mathrm{E} = -j\omega\bar{\bar{\mu}}\mathrm{H} - \mathrm{M}^{imp}$	**Faraday's law**
$\nabla \cdot (\bar{\bar{\epsilon}}\mathrm{E}) = \rho$	**Gauss' law of Electricity**
$\nabla \cdot (\bar{\bar{\mu}}\mathrm{H}) = 0$	**Gauss' law of Magnetism**

E-VARIATIONAL FORMULATION:

$$\left\{\begin{array}{l} \text{Find } \mathrm{E} \in \mathrm{E}_D + \boldsymbol{H}_D(\mathrm{curl};\Omega) \text{ such that:} \\ \displaystyle\int_\Omega (\bar{\bar{\mu}}^{-1}\nabla \times \mathrm{E}) \cdot (\nabla \times \bar{\mathrm{F}})\, dV - \int_\Omega (\bar{\bar{k}}^2\mathrm{E}) \cdot \bar{\mathrm{F}}\, dV = -j\omega \int_\Omega \mathrm{J}^{imp} \cdot \bar{\mathrm{F}}\, dV \\ \displaystyle +j\omega \int_{\Gamma_N} \mathrm{J}^{imp}_{\Gamma_N} \cdot \bar{\mathrm{F}}_t\, dS - \int_\Omega (\bar{\bar{\mu}}^{-1}\mathrm{M}^{imp}) \cdot (\nabla \times \bar{\mathrm{F}})\, dV \quad \forall\, \mathrm{F} \in \boldsymbol{H}_D(\mathrm{curl};\Omega) \end{array}\right.$$

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Flexibility (What Problems Can We Solve?)

AXISYMMETRIC PROBLEMS

E_ϕ **-Variational Formulation (Azimuthal)**

$$\begin{cases} \text{Find } E_\phi \in E_{\phi,D} + \tilde{H}^1_D(\Omega) \text{ such that:} \\ \int_\Omega (\bar{\bar{\mu}}^{-1}_{\rho,z} \nabla \times \mathrm{E}_\phi) \cdot (\nabla \times \bar{\mathrm{F}}_\phi)\, dV - \int_\Omega (\bar{\bar{k}}^2_\phi \mathrm{E}_\phi) \cdot \bar{\mathrm{F}}_\phi\, dV = -j\omega \int_\Omega J^{imp}_\phi\, \bar{F}_\phi\, dV \\ \qquad + j\omega \int_{\Gamma_N} J^{imp}_{\phi,\Gamma_N}\, \bar{F}_\phi\, dS - \int_\Omega (\bar{\bar{\mu}}^{-1}_{\rho,z} \mathrm{M}^{imp}_{\rho,z}) \cdot \bar{\mathrm{F}}_\phi\, dV \qquad \forall\, F_\phi \in \tilde{H}^1_D(\Omega) \end{cases}$$

$\mathrm{E}_{\rho,z}$ **-Variational Formulation (Meridian)**

$$\begin{cases} \text{Find } (E_\rho, E_z) \in \mathrm{E}_D + \tilde{H}_D(\mathrm{curl};\Omega) \text{ such that:} \\ \int_\Omega (\bar{\bar{\mu}}^{-1}_\phi \nabla \times \mathrm{E}_{\rho,z}) \cdot (\nabla \times \bar{\mathrm{F}}_{\rho,z})\, dV - \int_\Omega (\bar{\bar{k}}^2_{\rho,z} \mathrm{E}_{\rho,z}) \cdot \bar{\mathrm{F}}_{\rho,z}\, dV = \\ \qquad -j\omega \int_\Omega J^{imp}_\rho \bar{F}_\rho + J^{imp}_z \bar{F}_z\, dV + j\omega \int_{\Gamma_N} J^{imp}_{\rho,\Gamma_N} \bar{F}_\rho + J^{imp}_{z,\Gamma_N} \bar{F}_z\, dS \\ \qquad - \int_\Omega (\bar{\bar{\mu}}^{-1}_\phi \mathrm{M}^{imp}_\phi) \cdot \bar{\mathrm{F}}_{\rho,z}\, dV \qquad \forall\, (F_\rho, F_z) \in \tilde{H}_D(\mathrm{curl};\Omega) \end{cases}$$

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Flexibility (What Problems Can We Solve?)

- **Physical Devices: Casing, Casing Imperfections, Mandrel, Magnetic Buffers, Insulators, Displacement Currents, Combination of All, etc.**
- **Materials: Isotropic, Anisotropic*.**
- **Sources: Toroidal Antennas, Solenoidal Antennas, Dipoles in Any Direction, Electrodes, Finite Size Antennas, Combination of All, etc.**
- **Logging Instruments: Logging While Drilling (LWD), Laterolog, Normal, Induction, Dielectric Instruments, Cross-well, etc.**
- **Any Frequency (0-10 Ghz).**

ALL AXISYMMETRIC RESISTIVITY LOGGING PROBLEMS

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Reliability (Can We Trust the Solutions?)

- **Comparison Against Analytical Results.**
  1. Exact solution in a homogeneous media.
  2. Exact solution in a homogeneous media with a mandrel.
  3. Exact solution in a homogeneous media with casing.
- **Verification of Physical Properties.**
  1. Reciprocity principle (Gregory Itskovich).
  2. Discrete divergence free approximation for edge elements.
- **Numerical Verifications.**
  1. Different size of domain and antennas.
  2. Comparison against other numerical software (Yang Wei).
  3. Error control provided by the fine grid solution.
  4. Comparison between continuous elements vs. edge elements.

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Reliability (Can We Trust the Solutions?)

Problem with casing at 10 kHz.

Continuous Elements

Quantity of Interest	Real Part	Imag Part
COARSE GRID	0.1516098429E-08	-0.1456374493E-08
FINE GRID	0.1516094029E-08	-0.1456390824E-08

Edge Elements

Quantity of Interest	Real Part	Imag Part
COARSE GRID	0.1516060872E-08	-0.1456337248E-08
FINE GRID	0.1516093804E-08	-0.1456390864E-08

Error control provided by the fine grid solution.

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Reliability (Can We Trust the Solutions?)

Problem with casing at 10 kHz.

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Comparison between continuous elements vs. edge elements.

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Reliability (Can We Trust the Solutions?)

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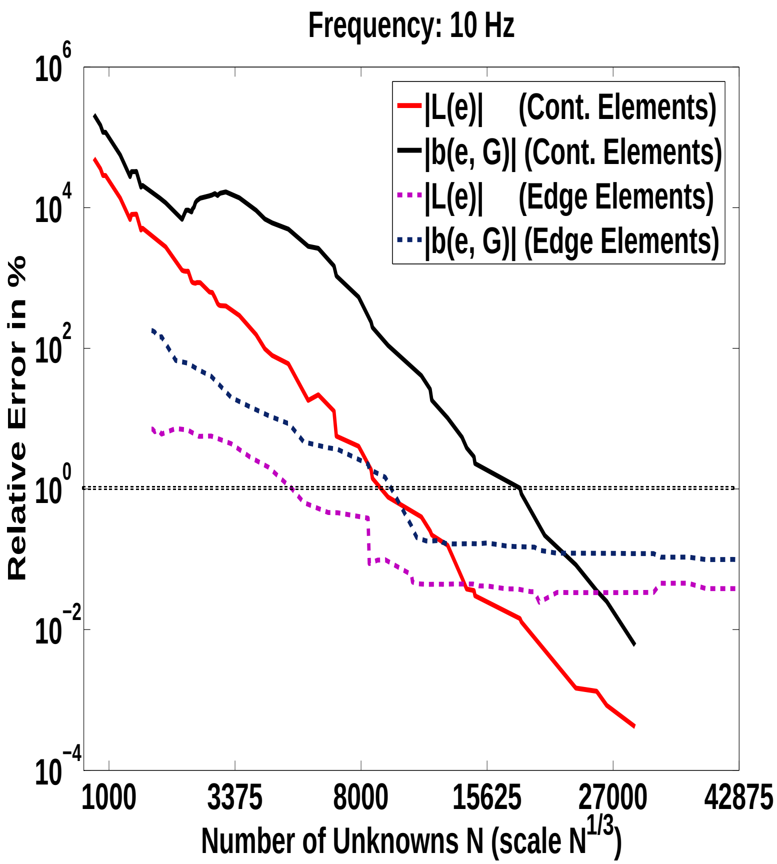
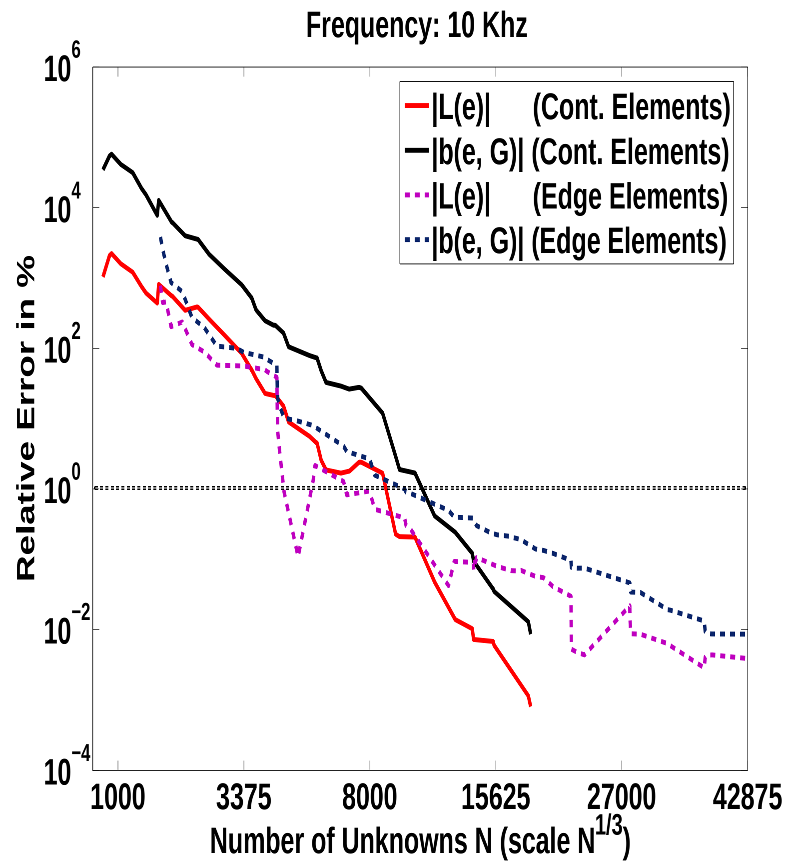
HIGHLY RELIABLE SOFTWARE

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Accuracy (Are the Solutions Accurate?)

Problem with Casing (Convergence Curve)





EXTREMELY ACCURATE SOFTWARE

CURRENT STAGE OF THE 2D hp-FE SOFTWARE

Performance (How Fast Can We Solve the Problems?)

80 Vert. Pos.	$10^{-6}\Omega \cdot m$	$10^{-5}\Omega \cdot m$
Toroid (10 Khz)	19' 46"	16' 28"
Ring of Vert. Dipoles (10 Khz)	22' 47"	17' 02"
Ring of Horiz. Dipoles (10 Khz)	19' 25"	13' 25"
Electrodes (0 Hz)	10' 10"	8' 35"

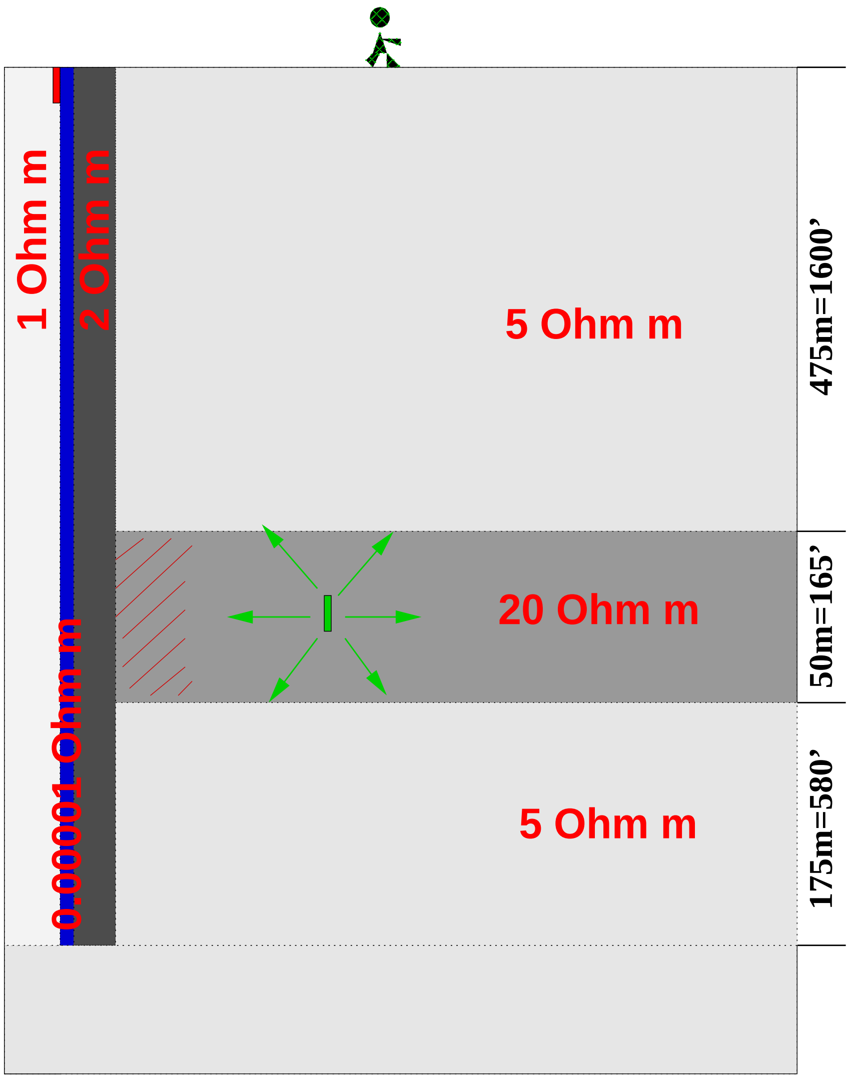
IBM Power 4 compiler 1.3 Ghz (4 years old)

Possible improvements in performance:

- **To use a 3.4 Ghz processor.**
- **To execute the code in 8 processors (10 positions per processor).**
- **To improve implementation.**

HIGH PERFORMANCE SOFTWARE

SIMULATION OF LOGGING INSTRUMENTS



5.5" Borehole radio ; 0.5" Casing ; 2" Cement

Axisymmetric 3D problem.

Five different materials.

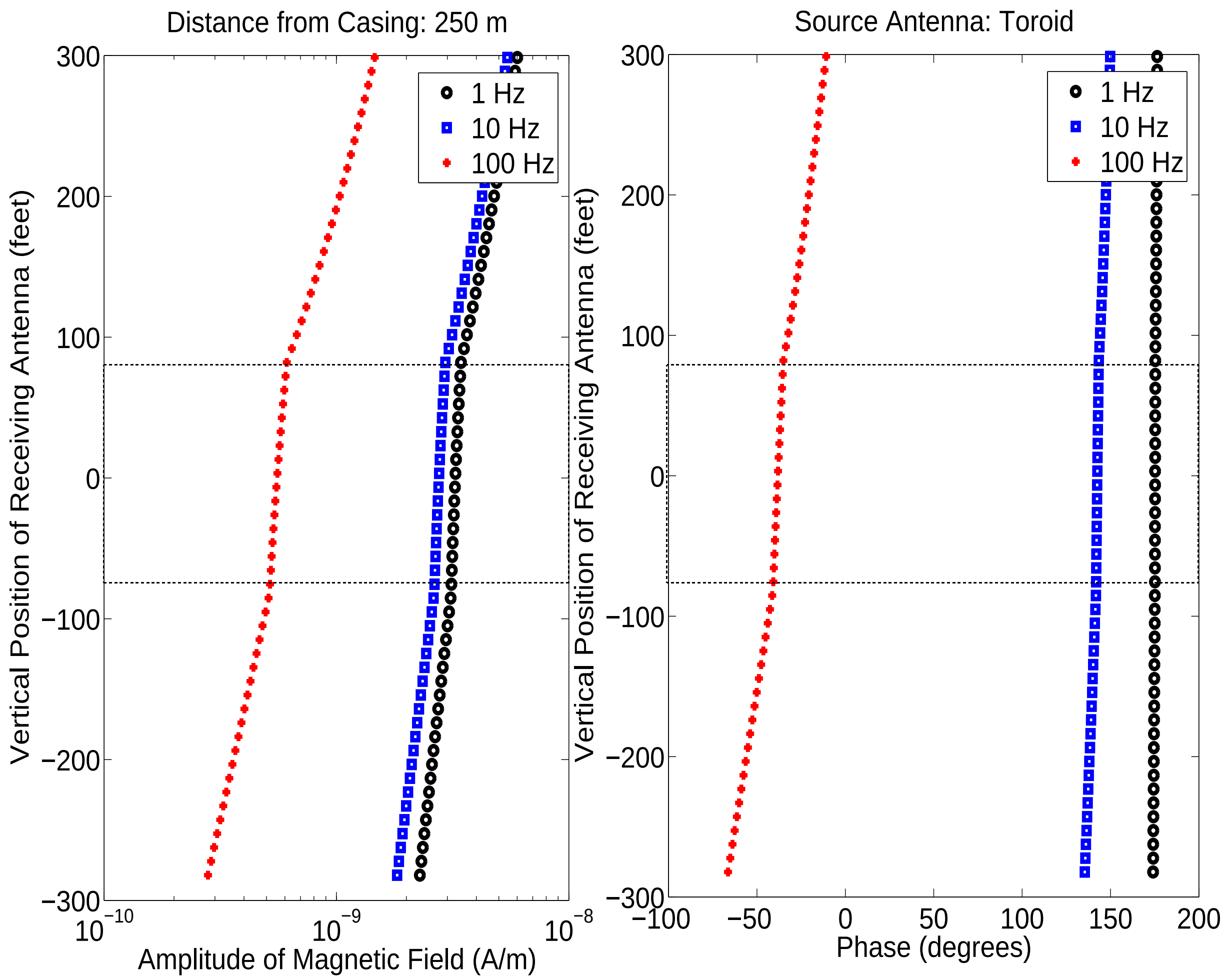
Different positions of receiving antenna.

Toroidal Transmitter and receiver separated by up to half a mile.

Objective: Determine First Difference of Potential Receiving Electrodes.

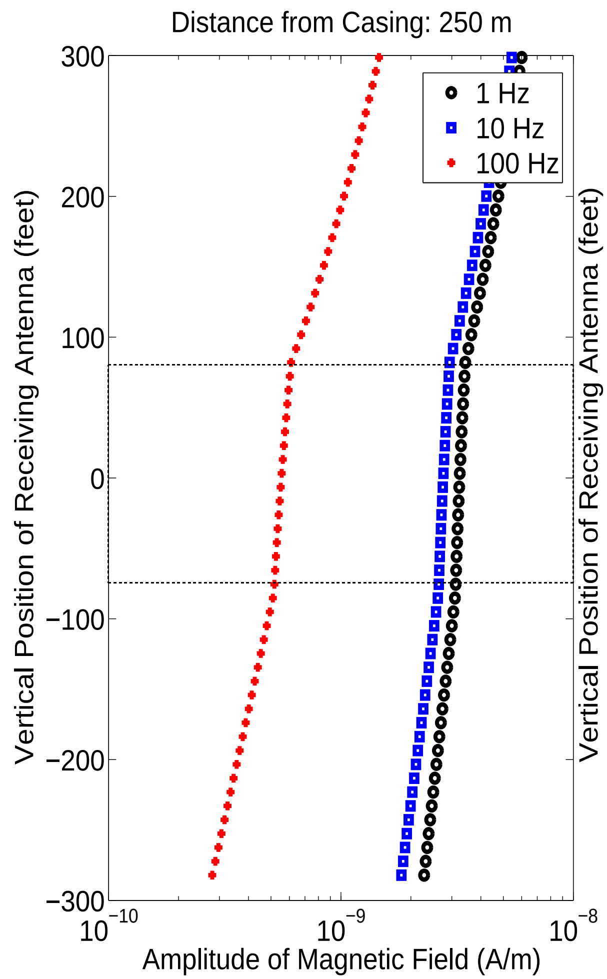
SIMULATION OF LOGGING INSTRUMENTS

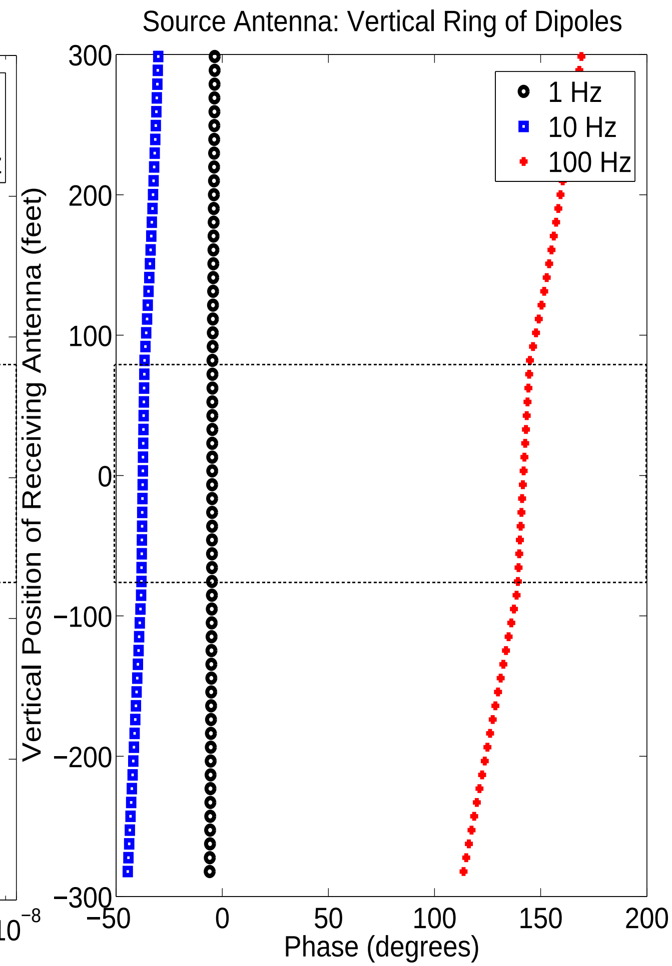
A Cross-Well Study with One Cased Well: Toroid Antennas



SIMULATION OF LOGGING INSTRUMENTS

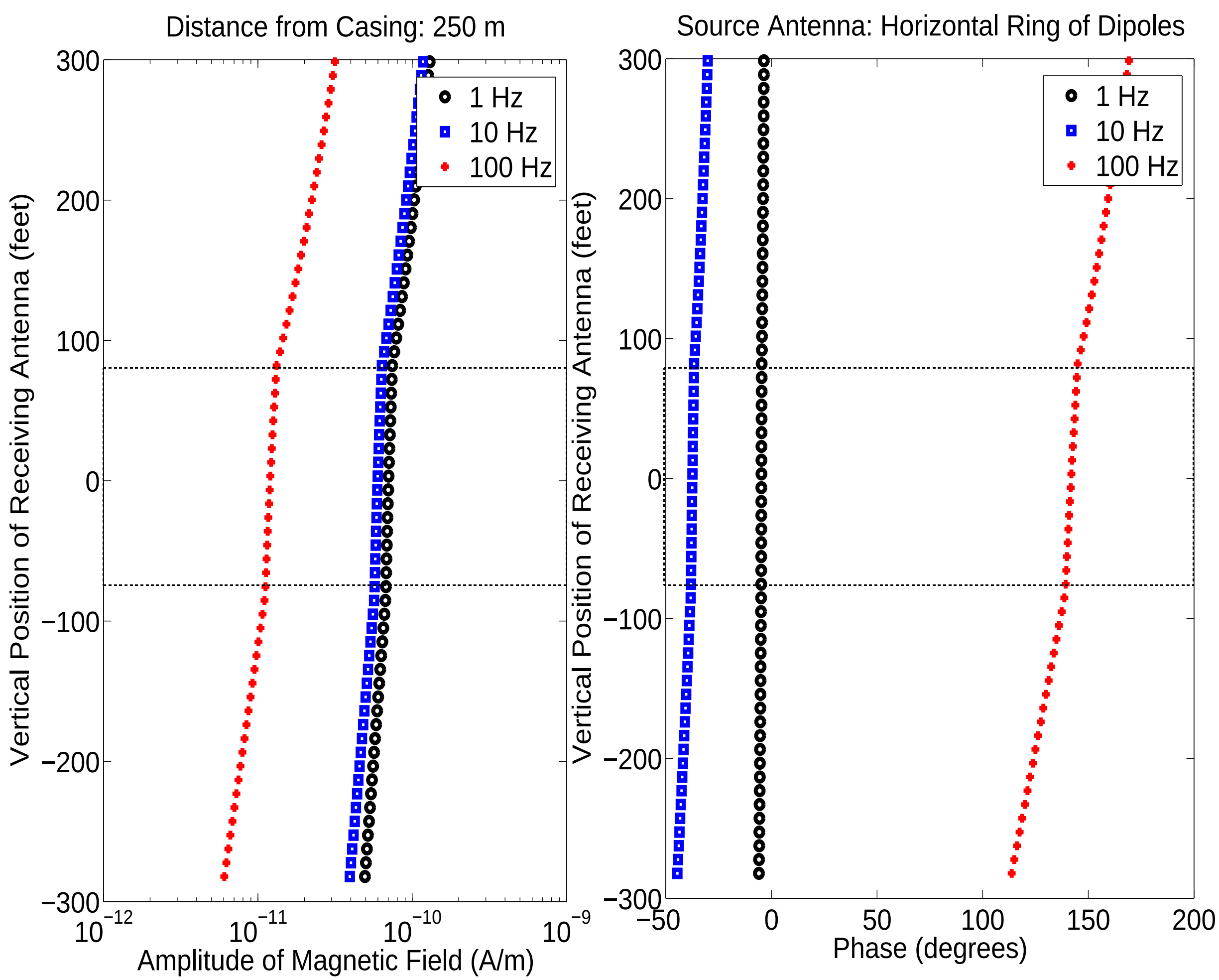
A Cross-Well Study: Vertical Dipoles





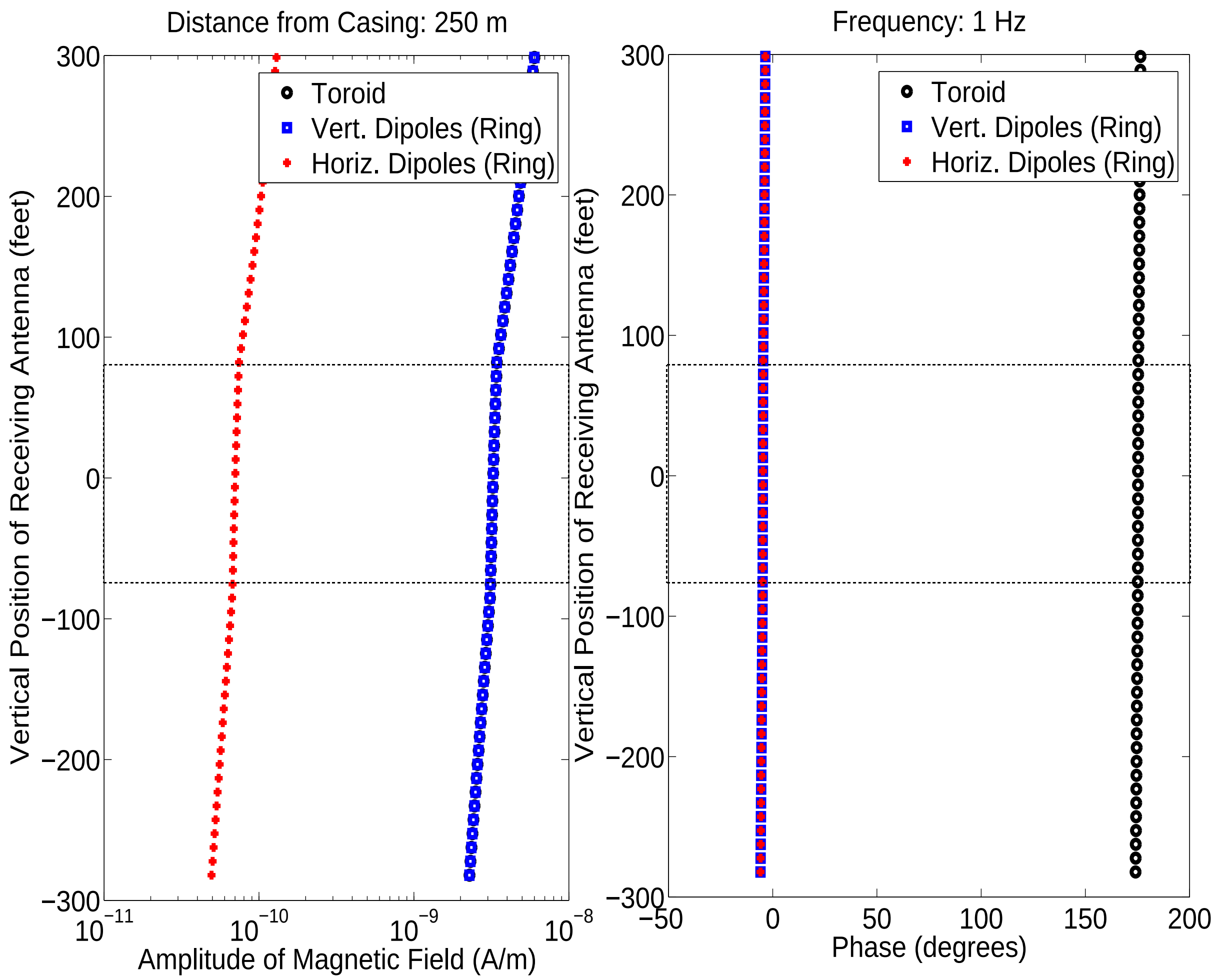
SIMULATION OF LOGGING INSTRUMENTS

A Cross-Well Study: Horizontal Dipoles



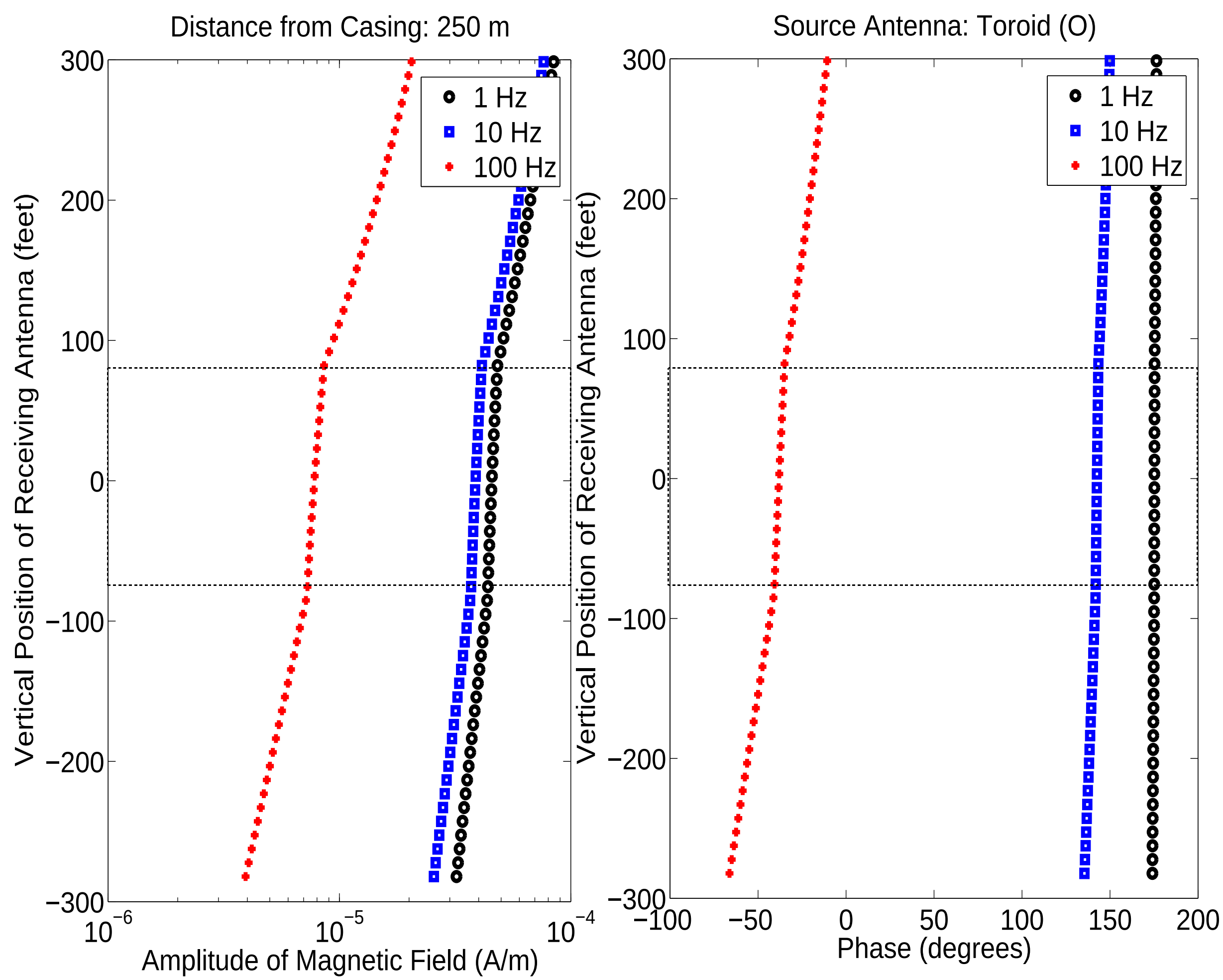
SIMULATION OF LOGGING INSTRUMENTS

A Cross-Well Study: Different Antennas



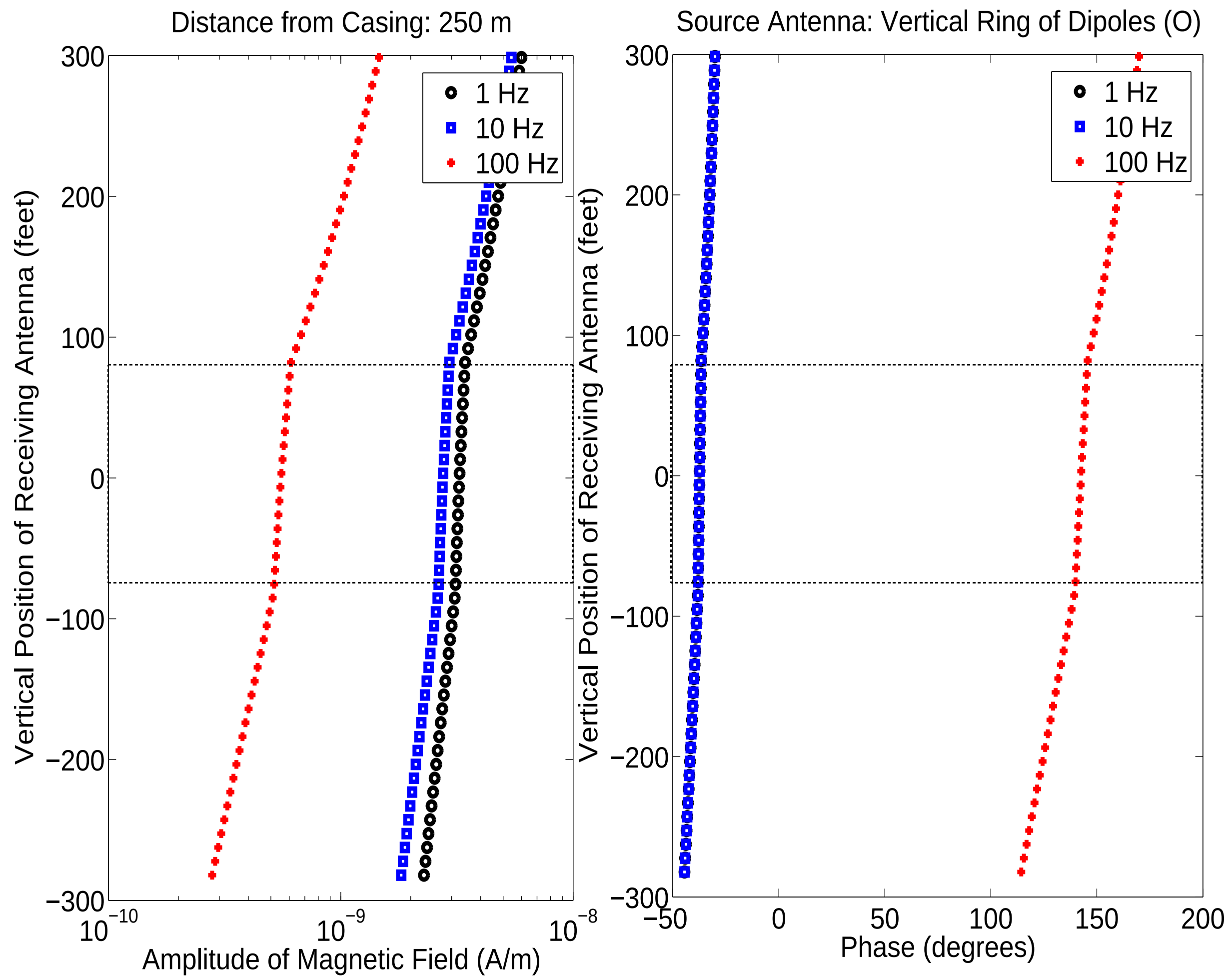
SIMULATION OF LOGGING INSTRUMENTS

A Cross-Well Study: Toroid Antennas (Outside Borehole)



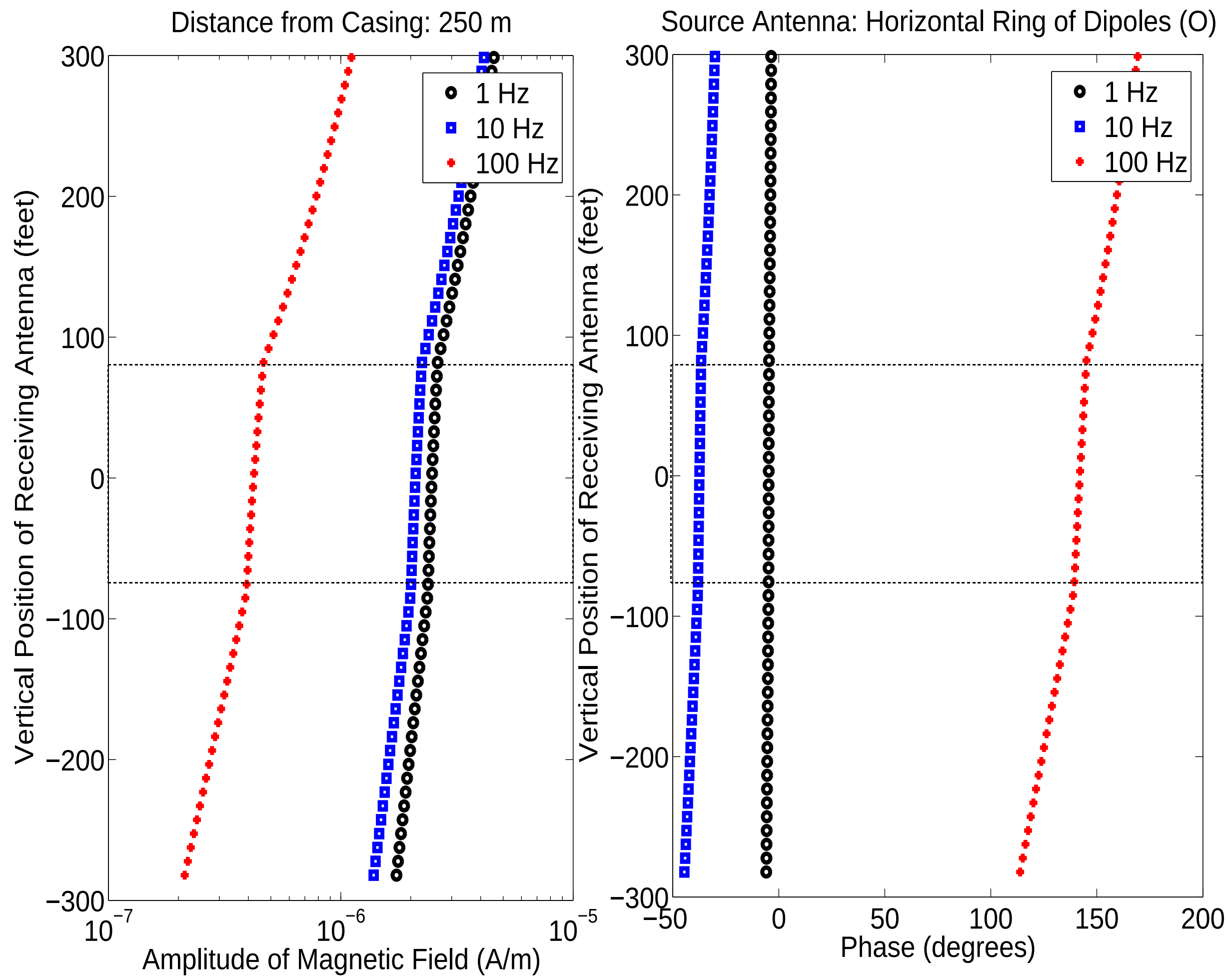
SIMULATION OF LOGGING INSTRUMENTS

A Cross-Well Study: Vertical Dipoles (Outside Borehole)



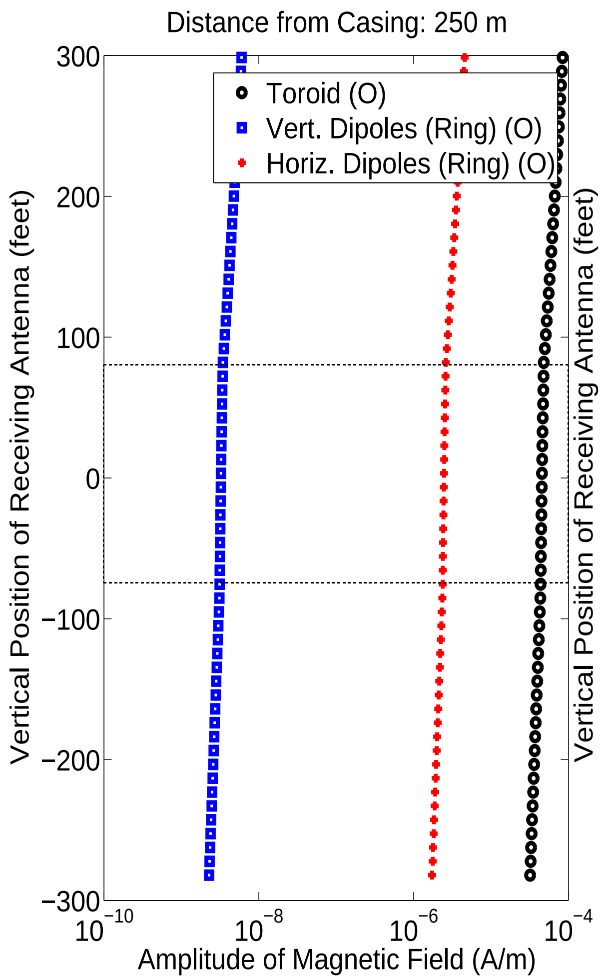
SIMULATION OF LOGGING INSTRUMENTS

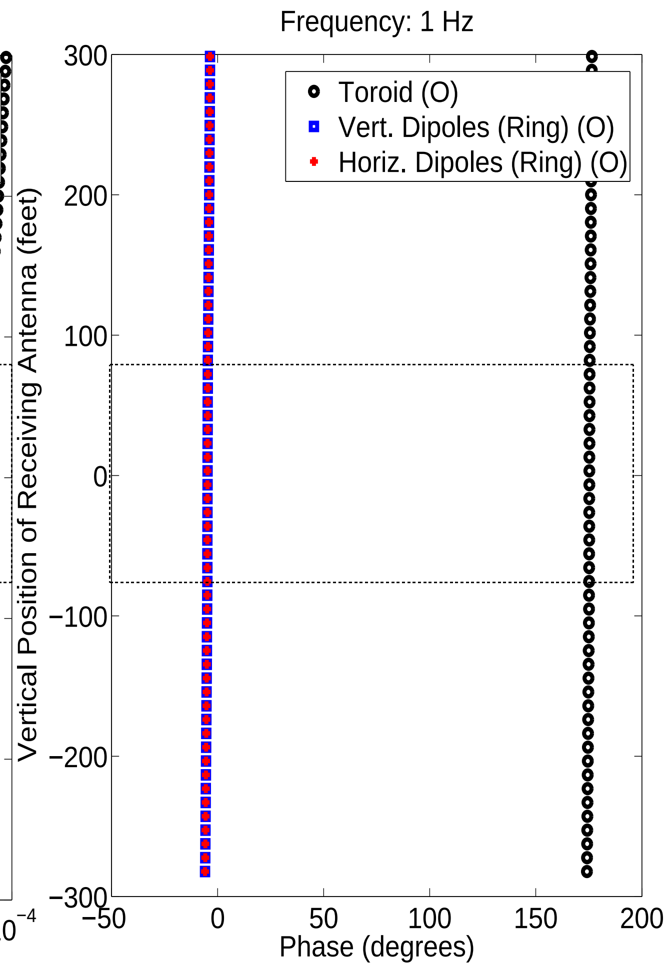
A Cross-Well Study: Horizontal Dipoles (Outside Borehole)



SIMULATION OF LOGGING INSTRUMENTS

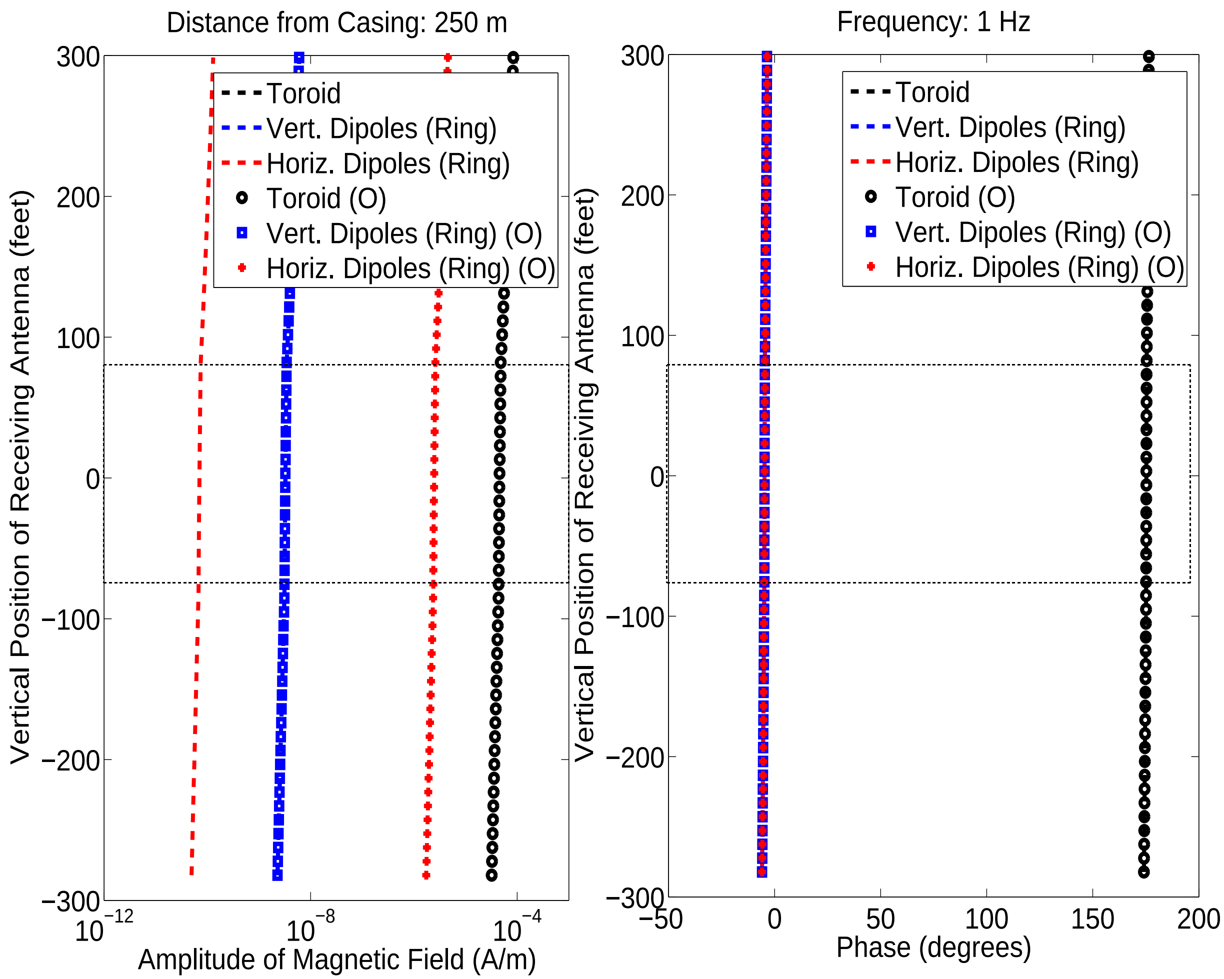
A Cross-Well Study: Different Antennas (Outside Borehole)





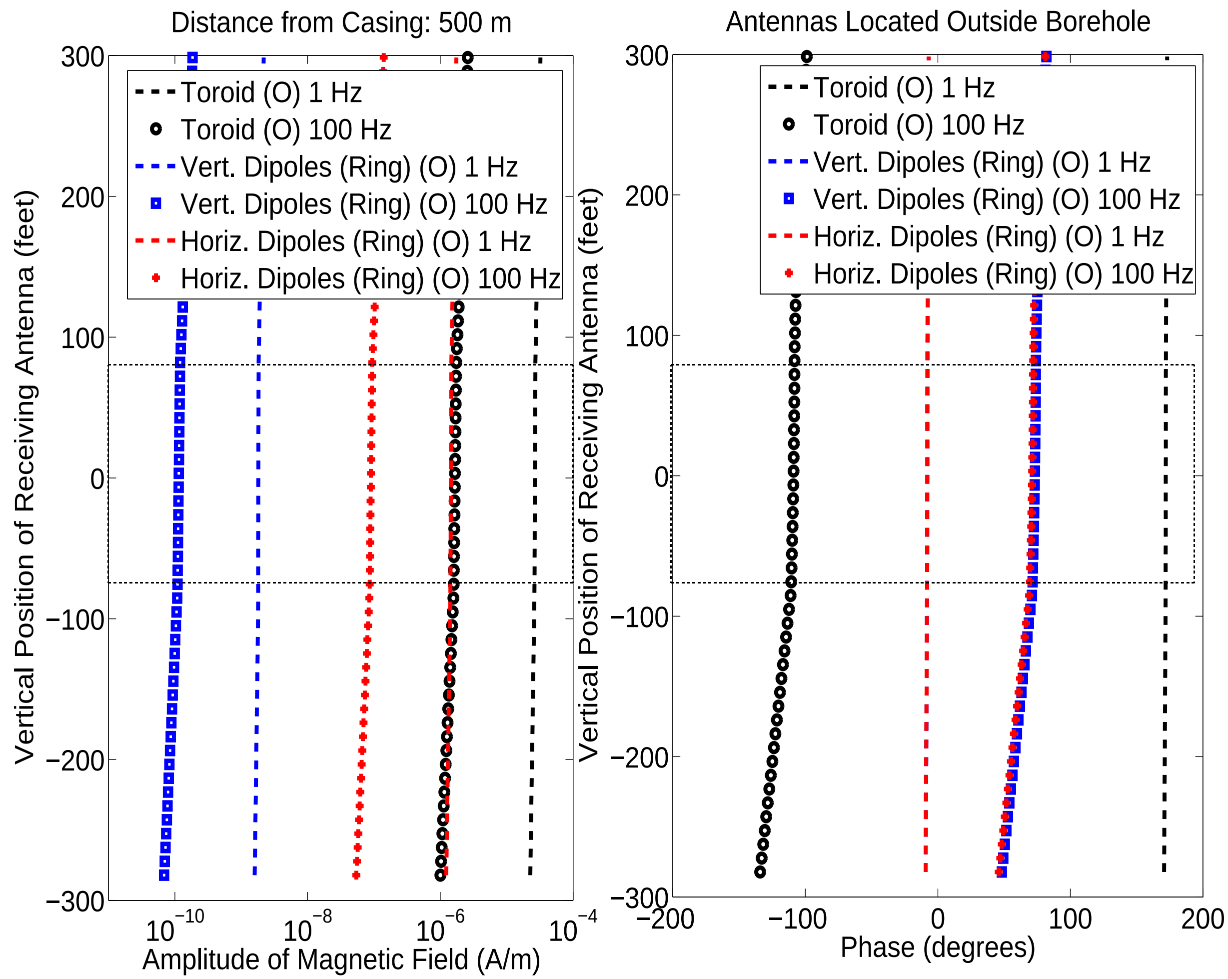
SIMULATION OF LOGGING INSTRUMENTS

A Cross-Well Study: Antennas Inside and Outside Borehole



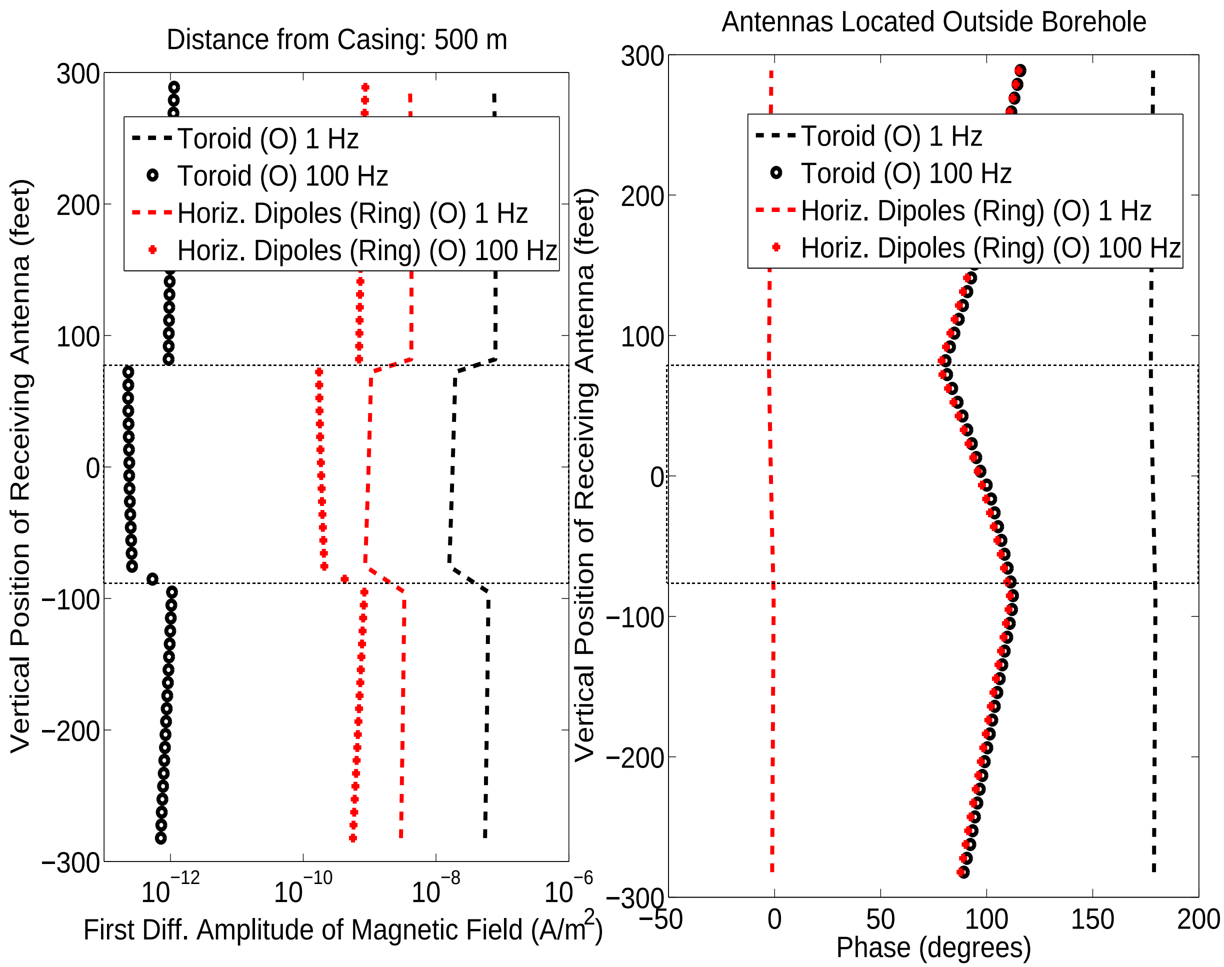
SIMULATION OF LOGGING INSTRUMENTS

A Cross-Well Study: Receivers at 500 m (Horizontal Distance)



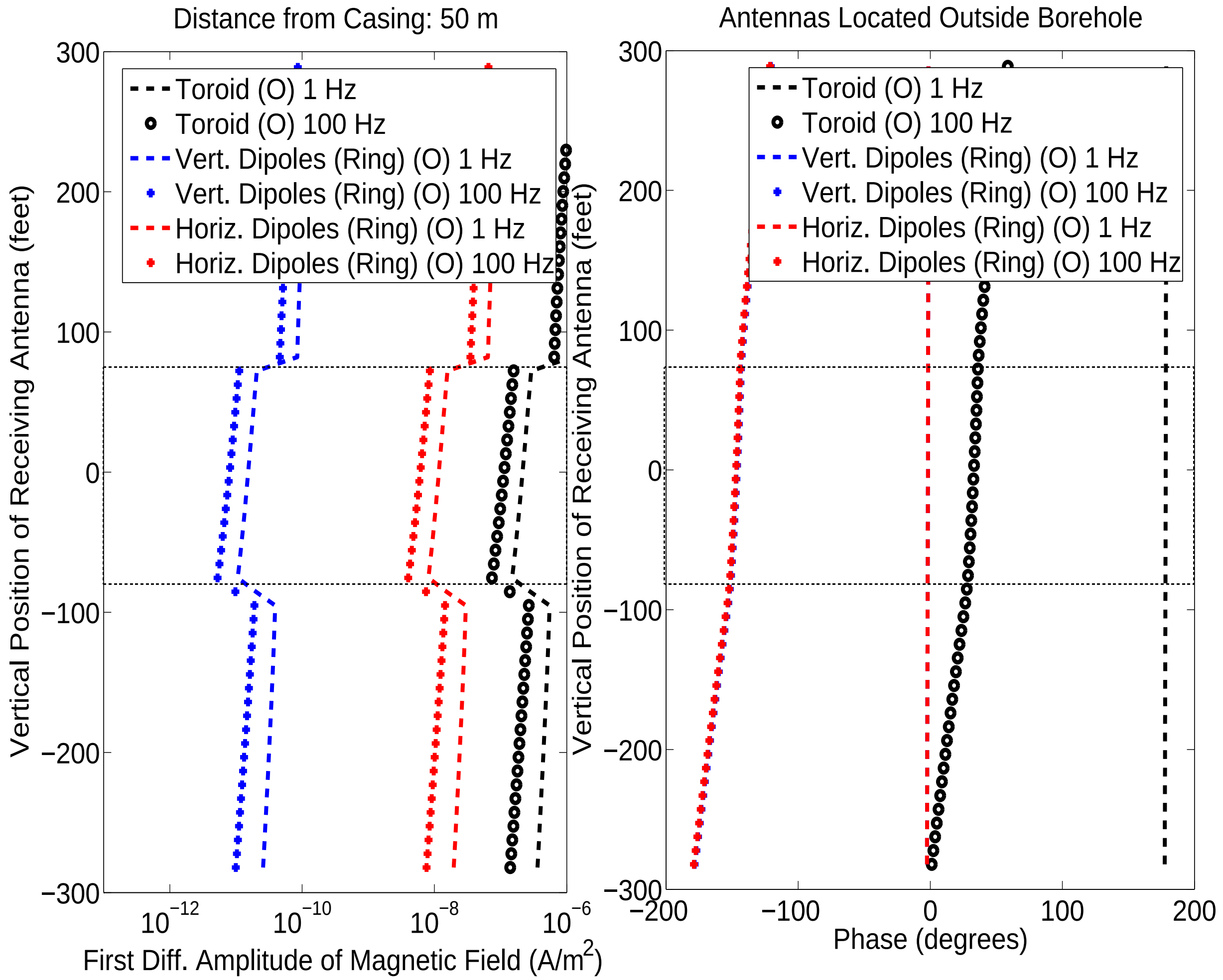
SIMULATION OF LOGGING INSTRUMENTS

A Cross-Well Study: First Vertical Diff. of Magnetic Field



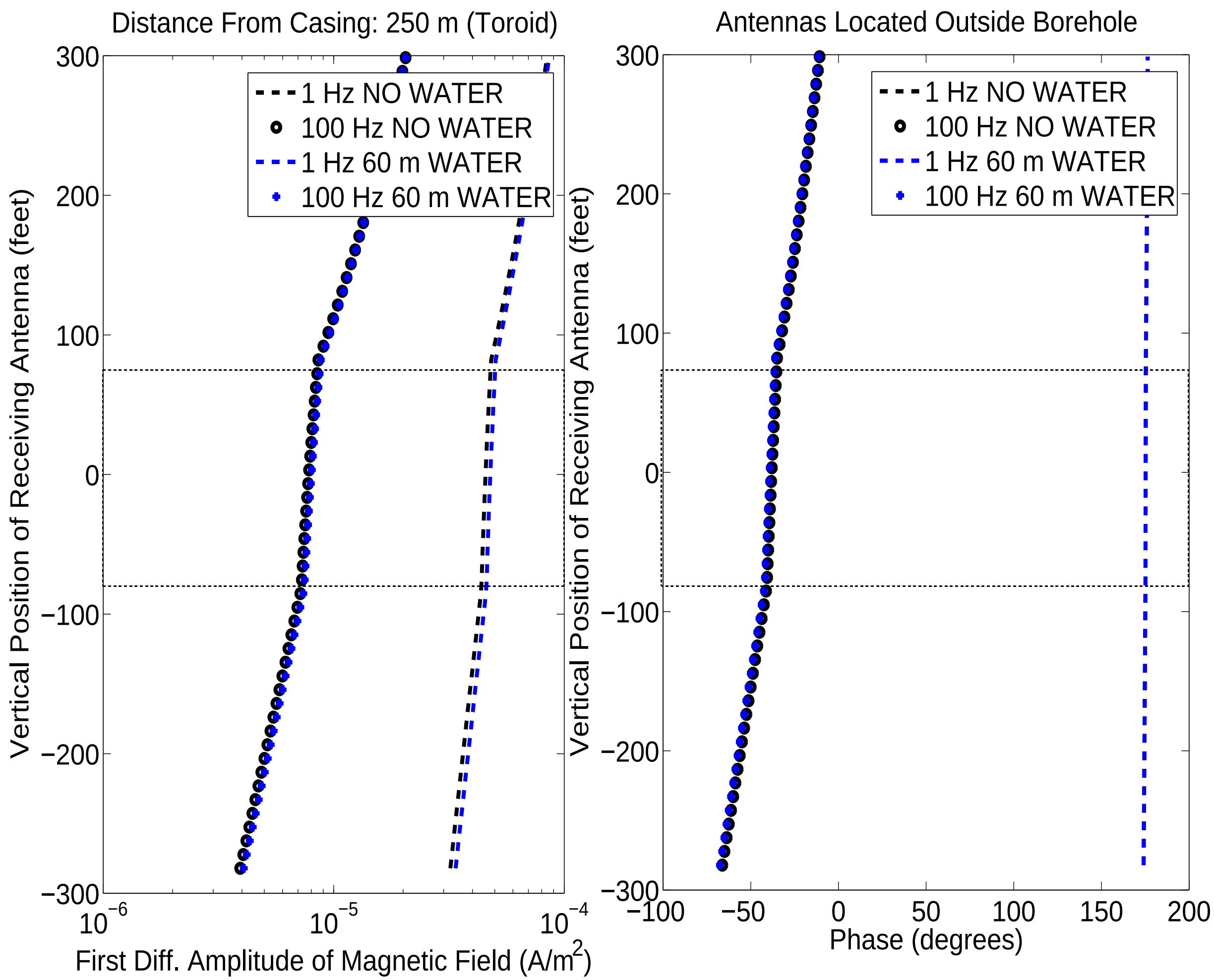
SIMULATION OF LOGGING INSTRUMENTS

A Cross-Well Study: First Vert. Diff. Magnetic Field (50 m)



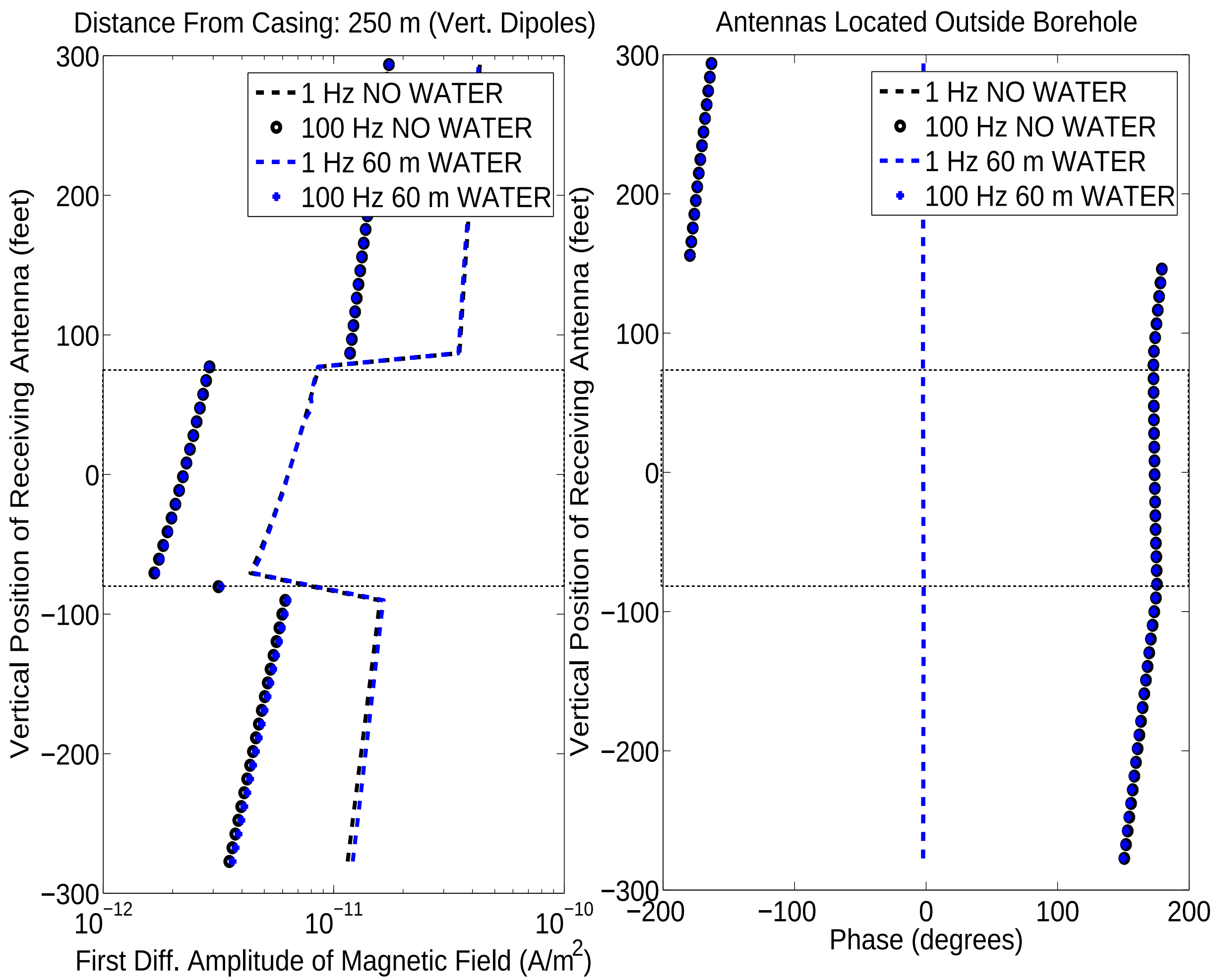
SIMULATION OF LOGGING INSTRUMENTS

A Cross-Well Study: Water Invasion with Toroids (250 m)



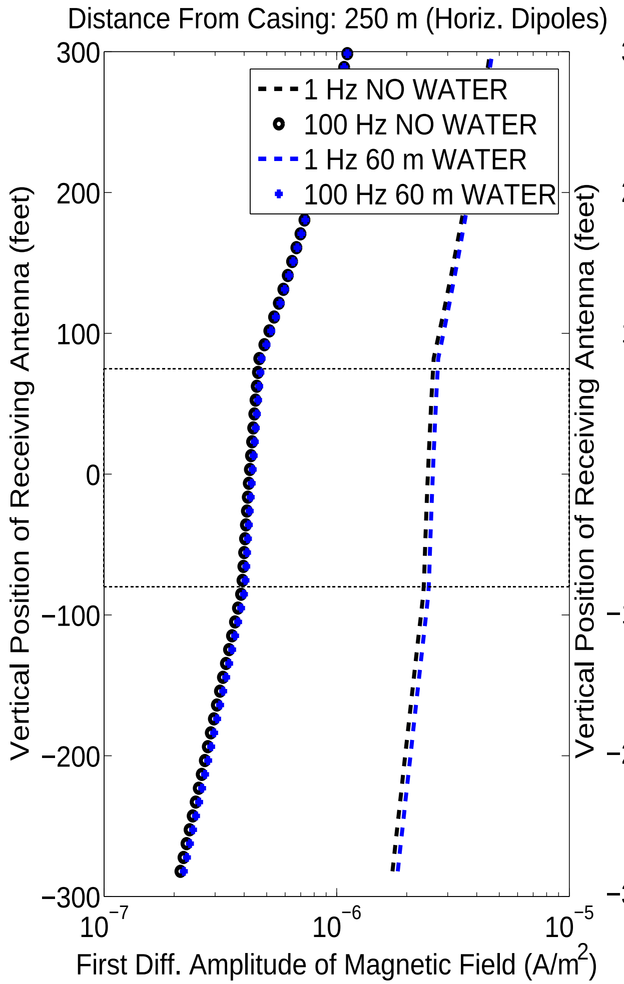
SIMULATION OF LOGGING INSTRUMENTS

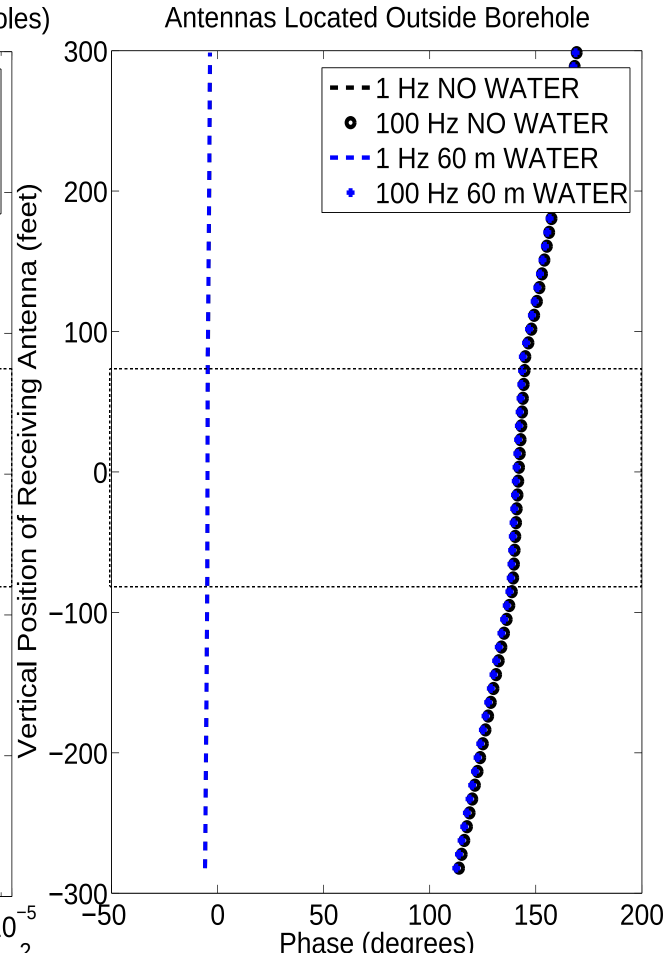
A Cross-Well Study: Water Invasion, Vert. Dipoles (250 m)



SIMULATION OF LOGGING INSTRUMENTS

A Cross-Well Study: Water Invasion, Horiz. Dipoles (250 m)





PROJECT ACCOMPLISHMENTS

The Main Accomplishment

Design,
implementation,
theoretical study,
and numerical study of a

flexible,
reliable,
accurate,
and efficient

self-adaptive goal-oriented hp-finite element
method for simulation of 2D axisymmetric
resistivity logging instruments.

PROJECT ACCOMPLISHMENTS

Detailed List of Accomplishments

- **SOFTWARE DEVELOPMENT:**
  - Developed a 2D code for DC axisymmetric EM problems.
  - Developed a 2D code for AC axisymmetric EM problems
    - using continuous elements, and
    - using edge elements.
  - Developed a self-adaptive goal-oriented algorithm.
  - Increased overall performance of the software.
  - Made software suitable for simulation of resistivity logging instruments.
- **RESISTIVITY LOGGING APPLICATIONS:**
  - Solved problems with (possibly damaged) casing.
  - Solved problems with (realistic) mandrel.
  - Simulated Laterolog/Normal instruments.
  - Simulated Induction instruments.
  - Simulated Cross-Well configurations with casing.

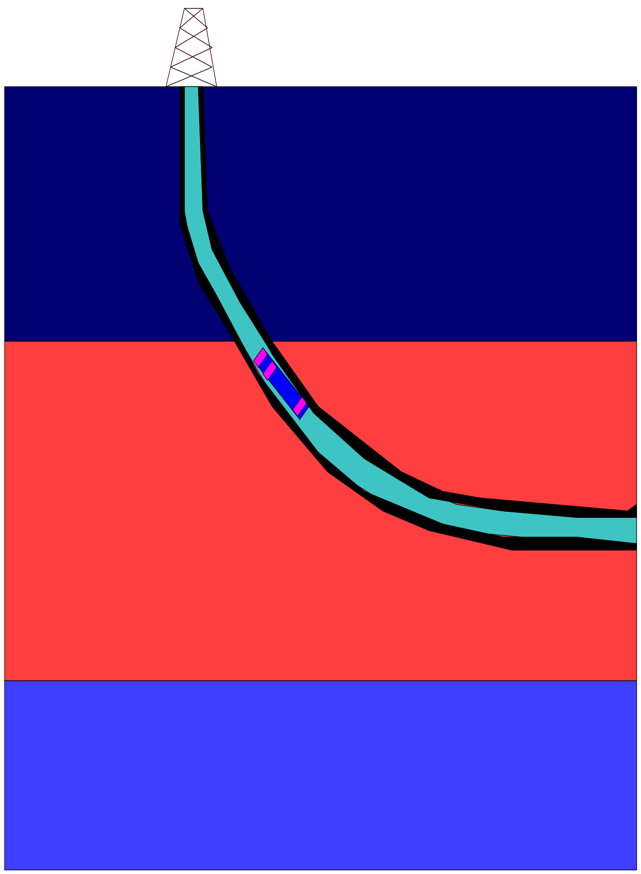
PROJECT ACCOMPLISHMENTS

Detailed List of Accomplishments

- **MATHEMATICAL STUDY:**
  - Derived adequate variational formulations for 2D and 3D EM problems.
  - Derived (and evaluated) exact solutions for several model problems.
  - Studied mathematical aspects related to the simulation of logging instruments using the FEM (use of Dirac's delta functions, etc.).
- **BENCHMARKING:**
  - Compared results against other codes.
  - Compared results against exact solutions.
  - Compared results among different electromagnetic formulations.
- **TRANSFER OF KNOWLEDGE:**
  - 5 reports given to Baker-Hughes (over 200 pages).
  - 5 presentations given to Baker-Hughes.
  - Several technical conversations (L. Tabarovsky, A. Bespalov, T. Wang, G. Itskovich, etc.)

TWO NEW PROJECT PROPOSALS

Project I: Simulation of 3D Resistivity Logging Problems



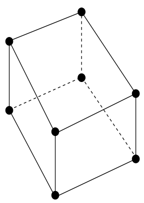
- **PART I: Simulate 3D DC Resistivity Logging Problems.**
  - Estimated completion time: 8-10 months (40 hours/week).
  - Main challenge: Speed.
  - Expected results: Similar results as in 2D.
- **PART II: Simulate 3D AC Resistivity Logging Problems.**
  - Estimated completion time: 8-10 months (40 hours/week).
  - Main challenge: Speed and Implementation.
  - Expected results: Similar results as in 2D.

TWO NEW PROJECT PROPOSALS

Project II: 2D Multi-Physics (Inversion)

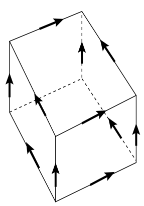
$$\mathbb{R} \longrightarrow H^1 \xrightarrow{\nabla} H(\text{curl}) \xrightarrow{\nabla\times} H(\text{div}) \xrightarrow{\nabla\circ} L^2 \longrightarrow 0$$

$H^1 = \{\psi : \int (|\nabla\psi|^2 + |\psi|^2) dV < \infty\}$ **(STANDARD ELEMENTS)**



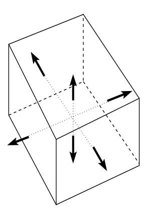
Scalar Potentials, Pressure, Temperature

$H(\text{curl}) = \{E : \int (|\nabla \times \mathrm{E}|^2 + |\mathrm{E}|^2) dV < \infty\}$ **(EDGE ELEMENTS)**



Electromagnetic Fields

$H(\text{div}) = \{\mathrm{u} : \int (|\nabla \cdot \mathrm{u}|^2 + |\mathrm{u}|^2) dV < \infty\}$ **(FACE ELEMENTS)**



Fluid Dynamics, Acoustics

TWO NEW PROJECT PROPOSALS

Project II: 2D Multi-Physics (Inversion)

- **PART I: Incorporate $H(div)$-elements into the 2D FE software, and simulate four acoustic logging instruments.**
  - **Estimated completion time: 7-10 months (40 hours/week).**
  - **Main challenge: Tedious implementation.**
  - **Expected results: Same as for electromagnetics.**
- **PART II: Solve inverse Electromagnetic - Acoustic problems.**
  - **Estimated completion time: 7-10 months (40 hours/week).**
  - **Main challenge: To manage different physical phenomena.**
  - **Expected results: Accurate inversion results.**
- **PART III: Solve inverse 2D Multi-physic problems.**
  - **Estimated completion time: 10+ months (40 hours/week).**
  - **Main challenge: To manage different physical phenomena.**
  - **Expected results: Better assesment of the petrophysical properties of the formation (saturation, porosity, and permeability).**