Joint Industry Research Consortium on Formation Evaluation

Self-Adaptive hp-Finite Element Simulations of Multi-Component Induction Measurements Acquired in Dipping, Invaded and Anisotropic Formations.

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Department of Petroleum and Geosystems Engineering
OVERVIEW

1. 3D Methodology and Formulation (D. Pardo).

2. Numerical Results:
   - Through-Casing Instruments (D. Pardo).
   - Induction Instruments (D. Pardo + M. Paszynski).

3. Parallel Implementation (M. Paszynski).


6. Conclusions and Future Work (Ch. Michler).

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SUMMARY OF PREVIOUS YEARS

Goal: To Study the Effect of Invasion, Anisotropy, and Magnetic Permeability.
### SUMMARY OF PREVIOUS YEARS

**Type of Problems We Can Solve with 2Dhp90**

<table>
<thead>
<tr>
<th>Physical Devices</th>
<th>Magnetic Buffers</th>
<th>Insulators</th>
<th>Displacement Currents</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Casing</td>
<td>Casing Imperfections</td>
<td>Combination of all</td>
</tr>
<tr>
<td>Materials</td>
<td>Isotropic</td>
<td>Anisotropic</td>
<td></td>
</tr>
<tr>
<td>Sources</td>
<td>Toroidal Antennas</td>
<td>Solenoidal Antennas</td>
<td>Dipoles in Any Direction</td>
</tr>
<tr>
<td></td>
<td>Electrodes</td>
<td>Finite Size Antennas</td>
<td>Combination of All</td>
</tr>
<tr>
<td>Logging Instruments</td>
<td>LWD/MWD</td>
<td>Laterolog</td>
<td>Normal</td>
</tr>
<tr>
<td></td>
<td>Induction</td>
<td>Dielectric Instruments</td>
<td>Cross-well</td>
</tr>
<tr>
<td>Frequency</td>
<td>0-10 Ghz.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Invasion</td>
<td>Water</td>
<td>Oil</td>
<td>etc.</td>
</tr>
</tbody>
</table>

**ALL AXISYMMETRIC RESISTIVITY LOGGING PROBLEMS**

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MOTIVATION (APPLICATIONS)

Deviated Wells (Forward Problem)

Objective: Find solution at the receiver antennas.
MOTIVATION (APPLICATIONS)

Example: Solution in a 60 degrees deviated well \((-\nabla \sigma \nabla u = f)\)

Several hours to obtain one solution (3D forward simulation).
Several months needed to solve the inverse problem.
METHODOLOGY: MAIN IDEA

Non-Orthogonal System of Coordinates

Material coefficients are constant with respect to the quasi-azimuthal direction $\zeta_2$

Fourier Series Expansion in $\zeta_2$

DC Problems: $-\nabla \sigma \nabla u = f$

$$u(\zeta_1, \zeta_2, \zeta_3) = \sum_{l=-\infty}^{l=\infty} u_l(\zeta_1, \zeta_3) e^{jl\zeta_2}$$

$$\sigma(\zeta_1, \zeta_2, \zeta_3) = \sum_{m=-\infty}^{m=\infty} \sigma_m(\zeta_1, \zeta_3) e^{jm\zeta_2}$$

$$f(\zeta_1, \zeta_2, \zeta_3) = \sum_{n=-\infty}^{n=\infty} f_n(\zeta_1, \zeta_3) e^{jn\zeta_2}$$

Fourier modes $e^{jl\zeta_2}$ are orthogonal high-order basis functions that are (almost) invariant with respect to the gradient operator.
METHODOLOGY: NEW SYSTEM OF COORDINATES

Cartesian system of coordinates: \( x = (x_1, x_2, x_3) \).
New non-orthogonal system of coordinates: \( \zeta = (\zeta_1, \zeta_2, \zeta_3) \).

\[
\begin{align*}
    x_1 &= \zeta_1 \cos \zeta_2 \\
    x_2 &= \zeta_1 \sin \zeta_2 \\
    x_3 &= \zeta_3
\end{align*}
\]

\[
\begin{align*}
    x_1 &= \zeta_1 \cos \zeta_2 \\
    x_2 &= \zeta_1 \sin \zeta_2 \\
    x_3 &= \zeta_3 + \tan \theta_0 \frac{\zeta_1 - \rho_1}{\rho_2 - \rho_1} \rho_2
\end{align*}
\]

\[
\begin{align*}
    x_1 &= \zeta_1 \cos \zeta_2 \\
    x_2 &= \zeta_1 \sin \zeta_2 \\
    x_3 &= \zeta_3 + \tan \theta_0 \zeta_1
\end{align*}
\]
METHODOLOGY: NEW SYSTEM OF COORDINATES

Final Variational Formulation

We define the Jacobian matrix \( \mathcal{J} = \frac{\partial (x_1, x_2, x_3)}{\partial (\zeta_1, \zeta_2, \zeta_3)} \) and its determinant \( |\mathcal{J}| = \det(\mathcal{J}) \).

Variational formulation in the new system of coordinates:

\[
\begin{cases}
\text{Find } u \in u_D + H^1_D(\Omega) \text{ such that:} \\
\left\langle \frac{\partial v}{\partial \zeta}, \tilde{\sigma} \frac{\partial u}{\partial \zeta} \right\rangle_{L^2(\Omega)} = \left\langle v, \tilde{f} \right\rangle_{L^2(\Omega)} \quad \forall v \in H^1_D(\Omega) ,
\end{cases}
\]

where:

\[
\tilde{\sigma} := \mathcal{J}^{-1} \sigma \mathcal{J}^{-1}^T |\mathcal{J}| ; \quad \tilde{f} := f |\mathcal{J}| .
\]

Same variational formulation with new materials and load data.
METHODOLOGY: VARIATIONAL FORMULATION

Five Fourier modes are enough to represent EXACTLY the new material coefficients.

Direct Current:

\[
\begin{aligned}
\text{Find } u \in u_D + H_1^1(D) \text{ such that:} \\
\sum_{n=k-2}^{n=k+2} \left\langle \left( \frac{\partial v}{\partial \zeta} \right)_k, \tilde{\sigma}_{k-n} \left( \frac{\partial u}{\partial \zeta} \right)_n \right\rangle_{L^2(\Omega_2D)} = \left\langle v_k, \tilde{f}_k \right\rangle_{L^2(\Omega_2D)} \\
\forall v_k
\end{aligned}
\]

Alternate Current:

\[
\begin{aligned}
\text{Find } (E)_s \in H_\Gamma E(\text{curl}; \Omega) \text{ such that:} \\
\sum_{n=s-2}^{n=s+2} \left\langle (\nabla^\zeta \times F)_s, (\tilde{\mu}^{-1})_{s-n} (\nabla^\zeta \times E)_l \right\rangle_{L^2(\Omega_2D)} - \left\langle F_s, (\tilde{k}^2)_{s-n} E_l \right\rangle_{L^2(\Omega_2D)} = -j\omega \left\langle F_s, (\tilde{J}^{imp})_s \right\rangle_{L^2(\Omega_2D)} \forall F_s
\end{aligned}
\]
Example (7 Fourier Modes)

\[
\sum_{n=k-2}^{n=k+2} \left< \left( \frac{\partial v}{\partial \zeta} \right)_k, \tilde{\sigma}_{k-n} \left( \frac{\partial u}{\partial \zeta} \right)_n \right>_{L^2(\Omega_{2D})} = \left< v_k, \tilde{f}_k \right>_{L^2(\Omega_{2D})}
\]

Stiffness Matrix:

\[
\begin{pmatrix}
(3,0,-3) & (3,-1,-2) & (3,-2,-1) & 0 & 0 & 0 & 0 \\
(2,1,-3) & (2,0,-2) & (2,-1,-1) & (2,-2,0) & 0 & 0 & 0 \\
(1,2,-3) & (1,1,-2) & (1,0,-1) & (1,-1,0) & (1,-2,1) & 0 & 0 \\
0 & (0,2,-2) & (0,1,-1) & (0,0,0) & (0,-1,1) & (0,-2,2) & 0 \\
0 & 0 & (1,2,-1) & (1,1,0) & (1,0,1) & (1,-1,2) & (1,-2,3) \\
0 & 0 & 0 & (2,2,0) & (2,1,1) & (2,0,2) & (2,-1,3) \\
0 & 0 & 0 & 0 & (3,2,1) & (3,1,2) & (3,0,3)
\end{pmatrix}
\]
METHODOLOGY: 2D $h_p$-FEM

A Self-Adaptive Goal-Oriented $h_p$-FEM

Optimal 2D Grid (Through Casing Resistivity Problem)

We vary locally the element size $h$ and the polynomial order of approximation $p$ throughout the grid.

Optimal grids are automatically generated by the computer.

The self-adaptive goal-oriented $h_p$-FEM provides exponential convergence rates in terms of the CPU time vs. the error in a user prescribed quantity of interest.
NUMERICAL RESULTS: VERIFICATION

Three Model Problems

Problem I (Uniform Materials)
- Material 1: Material 1
- Material 2: Material 2
- Material 3: Material 3

Problem II
- Material 1: Material 1
- Material 2: Material 2
- Material 3: Material 3

Problem III
- Material 1: Material 1
- Material 2: Material 2
- Material 3: Material 3
NUMERICAL RESULTS: VERIFICATION

Three Model Problems (DC)

Exponential Convergence in terms of the Number of Fourier Modes
NUMERICAL RESULTS: DC RESULTS

Simulation of Through Casing Resistivity Measurements

Casing resistivity: $10^{-5} - 10^{-7} \Omega \cdot m$
Casing thickness: 0.0127 m

Left Figure:
Axial-symmetric model
One current electrode (emitter)
Three voltage electrodes (collectors)

Objective:
Compute second diff. of potential for various depth angles and possibly with water invasion

Method of solution:
Fourier series expansion + change of coordinates + 2D goal-oriented hp-FEM
## NUMERICAL RESULTS: DC RESULTS

Simulation of Through Casing Resistivity Measurements

<table>
<thead>
<tr>
<th>Algorithm (Case) Number</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Fourier mode used for adaptivity</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 Fourier modes used for adaptivity</td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Final $hp$-grid NOT $p$-enriched</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Final $hp$-grid globally $p$-enriched</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9 Fourier modes used for the final solution</td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15 Fourier modes used for the final solution</td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Different algorithms provide different levels of accuracy
NUMERICAL RESULTS: DC RESULTS

Through Casing Resistivity Measurements (60-Degree Deviated Well)

<table>
<thead>
<tr>
<th>Case Number</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Time (minutes)</td>
<td>21’</td>
<td>40’</td>
<td>39’</td>
<td>109’</td>
<td>244’</td>
<td>290’</td>
<td>286’</td>
<td>432’</td>
</tr>
</tbody>
</table>
NUMERICAL RESULTS: DC RESULTS

Through Casing Resistivity Measurements (60-Degree Deviated Well)

Results with the new methodology seem more accurate than those obtained with the 3D software. In addition, with the new methodology we reduce the CPU time from several days to two hours.
NUMERICAL RESULTS: DC RESULTS

Through Casing Resistivity Measurements (Casing Conductivity)

Casing Resistivity = $10^{-5} \Omega \cdot m$

Casing Resistivity = $2.3 \times 10^{-7} \Omega \cdot m$

Qualitatively, results for various casing conductivities are similar even for deviated wells.
NUMERICAL RESULTS: DC RESULTS

Through Casing Resistivity Measurements (Invasion)

Second difference of potential (V)

Vertical position of voltage electrodes (m)

30 degrees

5 Ω·m
10000 Ω·m
1 Ω·m → 0.01 Ω·m

60 degrees

5 Ω·m
10000 Ω·m
1 Ω·m → 0.01 Ω·m
NUMERICAL RESULTS: DC RESULTS

Through Casing Resistivity Measurements (Invasion)

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NUMERICAL RESULTS: AC RESULTS

Model Problem

- 150 kHz (Wireline)
- Finite Size Loop Antenna
- Fiber Glass Mandrel: $10^5 \Omega \cdot m$
- Mandrel Radius: 0.04 m

Anisotropy (10:1)

1 $\Omega \cdot m$

100 $\Omega \cdot m$

0.1 $\Omega \cdot m$

0.05 $\Omega \cdot m$

20 $\Omega \cdot m$

3 $\Omega \cdot m$

1 $\Omega \cdot m$

SHALE

OIL

SAND

Oil-Based Mud ($1000 \Omega \cdot m$)
NUMERICAL RESULTS: AC RESULTS

Verification

Logging Instrument in a Homogeneous Formation

Number of Fourier Modes vs. Relative Error (in %)

- 20 KHz (30 degrees)
- 20 KHz (60 degrees)
- 150 KHz (30 degrees)
- 150 KHz (60 degrees)
NUMERICAL RESULTS: AC RESULTS

Verification

Logging Instrument in a Homogeneous Formation

Wireline, 150 Khz
NUMERICAL RESULTS: AC RESULTS

Verification

Logging Instrument in a Homogeneous Formation

Wireline, 150 Khz

Graph showing resistivity, real part, and imaginary part against depth.
NUMERICAL RESULTS: AC RESULTS

Verification

Logging Instrument in a Homogeneous Formation

Wireline, 150 Khz

Resistivity (Ω-m)

Real Part (V/m)

Imag Part (V/m)
NUMERICAL RESULTS: AC RESULTS

Verification

Logging Instrument in a Homogeneous Formation

Wireline, 150 Khz
NUMERICAL RESULTS: AC RESULTS

Verification

Logging Instrument in a Homogeneous Formation

Wireline, 150 Khz

Resistivity ($\Omega$-m)

Real Part (V/m)

Imag Part (V/m)
NUMERICAL RESULTS: AC RESULTS

Dip Angle

Wireline, 150 Khz

Resistivity ($\Omega \cdot m$)
-2 -1 0 1 2

Real Part (V/m) x 10^{-3}
-2 -1 0 1

Imag Part (V/m) x 10^{-3}
-3 -2 -1 0

0 degrees 30 degrees 60 degrees

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NUMERICAL RESULTS: AC RESULTS

Dip Angle + Invasion

Wireline, 150 KHz

Resistivity (Ω−m)

Real Part (V/m) $\times 10^{-3}$

Imag Part (V/m) $\times 10^{-3}$

NO INVASION

INVASION
NUMERICAL RESULTS: AC RESULTS

Dip Angle + Anisotropy

Wireline, 150 Khz

- Resistivity ($\Omega$-m)
- Real Part (V/m)
- Imag Part (V/m)

0 degrees
30 degrees
60 degrees
NUMERICAL RESULTS: AC RESULTS

Dip Angle + Invasion + Anisotropy

Wireline, 150 Khz

Resistivity ($\Omega \cdot m$)

Real Part (V/m)

Imag Part (V/m)
NUMERICAL RESULTS: AC RESULTS

60-Degree Deviated Well

Wireline, 150 Khz
NUMERICAL RESULTS: AC RESULTS

Vertical Well with 0.03 m Eccentricity

Wireline, 150 Khz

- NO INV.
- NO INV + ECC.
- INV
- INV + ECC
NUMERICAL RESULTS: LWD

Model Problem and Verification

Magnetic Buffer:
\( \rho = 10000 \ \Omega \cdot m \)
\( \mu_r = 10000 \)

Metallic Mandrel:
\( \rho = 10^{-5} \Omega \cdot m \)
\( \mu_r = 50 \)

Finite Size Loop Antenna
NUMERICAL RESULTS: LWD

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NUMERICAL RESULTS: LWD

Dip Angle + Invasion

LWD, 2 Mhz

Resistivity ($\Omega \cdot m$)

Real Part (V/m)

Imag Part (V/m)

INVASION

NO INVASION
NUMERICAL RESULTS: LWD

Dip Angle + Anisotropy

LWD, 2 Mhz

Resistivity (Ω·m)

Real Part (V/m)

Imag Part (V/m)
NUMERICAL RESULTS: LWD

Dip Angle + Invasion + Anisotropy

LWD, 2 Mhz

Resistivity (Ω·m)

Real Part (V/m)

Imag Part (V/m)

0 degrees
30 degrees
60 degrees

NO INVASION
HORIZ. ρ
VERT. ρ

INVASION

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NUMERICAL RESULTS: LWD

60-Degree Deviated Well

LWD, 2 Mhz

- Resistivity (Ω·m)
- Real Part (V/m)
- Imag Part (V/m)

NO INV.  NO INV + ANI.
INV  INV + ANI

INVASION
NO INVASION
VERT.ρ  HORIZ.ρ

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