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OVERVIEW


2. Methodology:
   - Fourier series expansion.
   - Non-orthogonal system of coordinates.
   - 2D goal-oriented self-adaptive $hp$-FEM.
   - Verification of the methodology.

3. Numerical results:
   - 3D DC through-casing measurements in deviated wells.
   - 3D DC dual laterolog measurements in deviated wells.
   - 3D AC wireline and LWD measurements in deviated wells.

4. Conclusions and future work.

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MOTIVATION (APPLICATIONS)

Deviated Wells (Forward Problem)

Dip Angle
Invasion
Anisotropy
Triaxial Induction
Eccentricity
Laterolog
Through-Casing
Induction-LWD
Induction-Wireline
Inverse Problems
Multi-Physics

Objective: Find solution at the receiver antennas.
METHODOLOGY: MAIN IDEA

Non-Orthogonal System of Coordinates

Fourier Series Expansion in $\zeta_2$

DC Problems: $-\nabla \sigma \nabla u = f$

$$u(\zeta_1, \zeta_2, \zeta_3) = \sum_{l=-\infty}^{l=\infty} u_l(\zeta_1, \zeta_3) e^{jl\zeta_2}$$

$$\sigma(\zeta_1, \zeta_2, \zeta_3) = \sum_{m=-\infty}^{m=\infty} \sigma_m(\zeta_1, \zeta_3) e^{jm\zeta_2}$$

$$f(\zeta_1, \zeta_2, \zeta_3) = \sum_{n=-\infty}^{n=\infty} f_n(\zeta_1, \zeta_3) e^{jn\zeta_2}$$

Material coefficients are constant with respect to the quasi-azimuthal direction $\zeta_2$

Fourier modes $e^{jl\zeta_2}$ are orthogonal high-order basis functions that are (almost) invariant with respect to the gradient operator.
METHODOLOGY: NEW SYSTEM OF COORDINATES

Cartesian system of coordinates: \( x = (x_1, x_2, x_3) \).

New non-orthogonal system of coordinates: \( \zeta = (\zeta_1, \zeta_2, \zeta_3) \).

Subdomain I
\[
\begin{align*}
    x_1 &= \zeta_1 \cos \zeta_2 \\
    x_2 &= \zeta_1 \sin \zeta_2 \\
    x_3 &= \zeta_3
\end{align*}
\]

Subdomain II
\[
\begin{align*}
    x_1 &= \zeta_1 \cos \zeta_2 \\
    x_2 &= \zeta_1 \sin \zeta_2 \\
    x_3 &= \zeta_3 + \tan \theta_0 \frac{\zeta_1 - \rho_1}{\rho_2 - \rho_1} \rho_2
\end{align*}
\]

Subdomain III
\[
\begin{align*}
    x_1 &= \zeta_1 \cos \zeta_2 \\
    x_2 &= \zeta_1 \sin \zeta_2 \\
    x_3 &= \zeta_3 + \tan \theta_0 \zeta_1
\end{align*}
\]
METHODOLOGY: NEW SYSTEM OF COORDINATES

Final Variational Formulation

We define the Jacobian matrix \( \mathcal{J} = \frac{\partial (x_1, x_2, x_3)}{\partial (\zeta_1, \zeta_2, \zeta_3)} \) and its determinant \( |\mathcal{J}| = \det(\mathcal{J}) \).

Variational formulation in the new system of coordinates:

\[
\begin{align*}
\text{Find } u &\in u_D + H^1_D(\Omega) \text{ such that:} \\
\left\langle \frac{\partial v}{\partial \zeta}, \tilde{\mathbf{\sigma}} \frac{\partial u}{\partial \zeta} \right\rangle_{L^2(\Omega)} &= \left\langle v, \tilde{f} \right\rangle_{L^2(\Omega)} \quad \forall v \in H^1_D(\Omega),
\end{align*}
\]

where:

\[
\tilde{\mathbf{\sigma}} := \mathcal{J}^{-1} \mathbf{\sigma} \mathcal{J}^{-1}^T |\mathcal{J}| \quad ; \quad \tilde{f} := f |\mathcal{J}|.
\]

Same variational formulation with new materials and load data
METHODOLOGY: FOURIER SERIES EXPANSION

For a mono-modal test function $v = v_k e^{j k \zeta_2}$, we have:

\[
\begin{align*}
\text{Find } u & \in u_D + H^1_D(\Omega) \text{ such that:} \\
\sum_{m,n} \left\langle \left( \frac{\partial v}{\partial \zeta} \right)_k e^{j k \zeta_2}, \tilde{\sigma}_m \left( \frac{\partial u}{\partial \zeta} \right)_n e^{j (m+n) \zeta_2} \right\rangle_{L^2(\Omega)} = \\
= \sum_l \left\langle v_k e^{j k \zeta_2}, \tilde{f}_l e^{j l \zeta_2} \right\rangle_{L^2(\Omega)} & \forall v_k e^{j k \zeta_2} \in H^1_D(\Omega)
\end{align*}
\]

Using the $L^2$-orthogonality of Fourier modes:

\[
\begin{align*}
\text{Find } u & \in u_D + H^1_D(\Omega) \text{ such that:} \\
\sum_n \left\langle \left( \frac{\partial v}{\partial \zeta} \right)_k , \tilde{\sigma}_{k-n} \left( \frac{\partial u}{\partial \zeta} \right)_n \right\rangle_{L^2(\Omega_{2D})} = \left\langle v_k , \tilde{f}_k \right\rangle_{L^2(\Omega_{2D})} & \forall v_k
\end{align*}
\]
METHODOLOGY: FOURIER SERIES EXPANSION

Five Fourier modes are enough to represent EXACTLY the new material coefficients.

\[ \tilde{\sigma}(\zeta_1, \zeta_2, \zeta_3) = \sum_{m=-2}^{m=2} \tilde{\sigma}_m(\zeta_1, \zeta_3)e^{jm\zeta_2} \]
METHODOLOGY: FOURIER SERIES EXPANSION

Five Fourier modes are enough to represent EXACTLY the new material coefficients.

\[ \tilde{\sigma}(\zeta_1, \zeta_2, \zeta_3) = \sum_{m=-2}^{m=2} \tilde{\sigma}_m(\zeta_1, \zeta_3) e^{jm\zeta_2} \]

Final Variational Formulation

Find \( u \in u_D + H^1_D(\Omega) \) such that:

\[
\begin{align*}
\sum_{n=k-2}^{n=k+2} \left\langle \left( \frac{\partial v}{\partial \zeta} \right)_k, \tilde{\sigma}_{k-n} \left( \frac{\partial u}{\partial \zeta} \right)_n \right\rangle_{L^2(\Omega_{2D})} &= \left\langle v_k, \tilde{f}_k \right\rangle_{L^2(\Omega_{2D})} \quad \forall v_k
\end{align*}
\]
METHODOLOGY: VARIATIONAL FORMULATION

Five Fourier modes are enough to represent EXACTLY the new material coefficients.

Direct Current:

\[
\begin{cases}
\text{Find } u \in u_D + H^1_D(\Omega) \text{ such that: } \\
\sum_{n=k-2}^{n=k+2} \left\langle \frac{\partial v}{\partial \zeta}, \tilde{\sigma}_{k-n} \frac{\partial u}{\partial \zeta} \right\rangle_{L^2(\Omega_2D)} = \left\langle v_k, \tilde{f}_k \right\rangle_{L^2(\Omega_2D)} \quad \forall v_k \\
\end{cases}
\]

Alternate Current:

\[
\begin{cases}
\text{Find } (E)_s \in H_{\Gamma_E}(\text{curl}; \Omega) \text{ such that: } \\
\sum_{n=s-2}^{n=s+2} \left\langle \nabla^\zeta \times F_s, (\tilde{\mu}^{-1})_{s-n} \nabla^\zeta \times E_l \right\rangle_{L^2(\Omega_2D)} - \left\langle F_s, (\tilde{k}^2)_{s-n} E_l \right\rangle_{L^2(\Omega_2D)} = -j\omega \left\langle F_s, (\tilde{J}^{\text{imp}})_s \right\rangle_{L^2(\Omega_2D)} \quad \forall F_s \\
\end{cases}
\]
Example (7 Fourier Modes)

\[
\sum_{n=k-2}^{n=k+2} \left\langle \left( \frac{\partial v}{\partial \zeta} \right)_k, \tilde{\sigma}_{k-n} \left( \frac{\partial u}{\partial \zeta} \right)_n \right\rangle_{L^2(\Omega_{2D})} = \left\langle v_k, \tilde{f}_k \right\rangle_{L^2(\Omega_{2D})}
\]

Stiffness Matrix:

\[
\begin{pmatrix}
(-3,0,-3) & (-3,-1,-2) & (-3,-2,-1) & 0 & 0 & 0 & 0 \\
(-2,1,-3) & (-2,0,-2) & (-2,-1,-1) & (-2,-2,0) & 0 & 0 & 0 \\
(-1,2,-3) & (-1,1,-2) & (-1,0,-1) & (-1,-1,0) & (-1,-2,1) & 0 & 0 \\
0 & (0,2,-2) & (0,1,-1) & (0,0,0) & (0,-1,1) & (0,-2,2) & 0 \\
0 & 0 & (1,2,-1) & (1,1,0) & (1,0,1) & (1,-1,2) & (1,-2,3) \\
0 & 0 & 0 & (2,2,0) & (2,1,1) & (2,0,2) & (2,-1,3) \\
0 & 0 & 0 & 0 & (3,2,1) & (3,1,2) & (3,0,3)
\end{pmatrix}
\]
METHODOLOGY: 2D $hp$-FEM

A Self-Adaptive Goal-Oriented $hp$-FEM

Optimal 2D Grid
(Through Casing Resistivity Problem)

We vary locally the element size $h$ and the polynomial order of approximation $p$ throughout the grid.

Optimal grids are automatically generated by the computer.

The self-adaptive goal-oriented $hp$-FEM provides exponential convergence rates in terms of the CPU time vs. the error in a user prescribed quantity of interest.
NUMERICAL RESULTS: VERIFICATION

Three Model Problems

**Problem I** (Uniform Materials)
- Material 1:1 $\Omega$–m
- Material 2:1 $\Omega$–m
- Material 3:1 $\Omega$–m

**Problem II**
- Material 1:0.00001 $\Omega$–m
- Material 2:10 $\Omega$–m
- Material 3:1 $\Omega$–m

**Problem III**
- Material 1:10000 $\Omega$–m
- Material 2: 0.2 $\Omega$–m
- Material 3:1 $\Omega$–m
NUMERICAL RESULTS: VERIFICATION

Three Model Problems (DC)

Exponential Convergence in terms of the Number of Fourier Modes

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NUMERICAL RESULTS: DC RESULTS

Simulation of Through Casing Resistivity Measurements

Casing resistivity: $10^{-5} - 10^{-7} \Omega \cdot m$
Casing thickness: 0.0127 m

Left Figure:
- Axial-symmetric model
- One current electrode (emitter)
- Three voltage electrodes (collectors)

Objective:
- Compute second diff. of potential
- for various depth angles and possibly with water invasion

Method of solution:
- Fourier series expansion +
- change of coordinates +
- 2D goal-oriented hp-FEM
NUMERICAL RESULTS: DC RESULTS

Simulation of Through Casing Resistivity Measurements

<table>
<thead>
<tr>
<th>Algorithm (Case) Number</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Fourier mode used for adaptivity</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 Fourier modes used for adaptivity</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Final $hp$-grid NOT $p$-enriched</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Final $hp$-grid globally $p$-enriched</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>9 Fourier modes used for the final solution</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15 Fourier modes used for the final solution</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

Different algorithms provide different levels of accuracy.
NUMERICAL RESULTS: DC RESULTS

Through Casing Resistivity Measurements (60-Degree Deviated Well)

<table>
<thead>
<tr>
<th>Case Number</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Time</td>
<td>21’</td>
<td>40’</td>
<td>39’</td>
<td>109’</td>
<td>244’</td>
<td>290’</td>
<td>286’</td>
<td>432’</td>
</tr>
</tbody>
</table>
NUMERICAL RESULTS: DC RESULTS

Through Casing Resistivity Measurements (60-Degree Deviated Well)

Results with the new methodology seem more accurate than those obtained with the 3D software. In addition, with the new methodology we reduce the CPU time from several days to two hours.
NUMERICAL RESULTS: DC RESULTS

Through Casing Resistivity Measurements (Casing Conductivity)

Casing Resistivity = $10^{-5} \Omega \cdot m$

Casing Resistivity = $2.3 \times 10^{-7} \Omega \cdot m$

Qualitatively, results for various casing conductivities are similar even for deviated wells.
NUMERICAL RESULTS: DC RESULTS

Through Casing Resistivity Measurements (Invasion)

30 degrees

60 degrees

Second difference of potential (V)

Vertical position of voltage electrodes (m)

NO INV

10 cm INV

50 cm INV

5 Ω·m

10000 Ω·m

1 Ω·m → 0.01 Ω·m

5 Ω·m

10000 Ω·m

1 Ω·m → 0.01 Ω·m

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NUMERICAL RESULTS: DC RESULTS

Through Casing Resistivity Measurements (Invasion)

30 degrees

Second difference of potential (V)

Vertical position of voltage electrodes (m)

5 Ω·m

10 Ω·m

100000 Ω·m

NO INV

10 cm INV

50 cm INV

60 degrees

Second difference of potential (V)

Vertical position of voltage electrodes (m)

5 Ω·m

10 Ω·m

100000 Ω·m

NO INV

10 cm INV

50 cm INV
NUMERICAL RESULTS: DC LATEROLOG

- Description of Tool
- Determination of Intensities ($W_j$) of Bucking Currents

Focusing Conditions

\[ V(M_1) = V(M_2) \]
\[ V(M_1') = V(M_2') \]

Relationships between $W_j$

\[ W_2 = (W_1 + c), \quad W_2 = (W_1' + c) \quad \text{for LLd} \]
\[ W_2 = -(W_1 + c), \quad W_2 = -(W_1' + c) \quad \text{for LLs} \]
NUMERICAL RESULTS: DC LATEROLOG

Coarse Grid

Synthetic focusing method

Solutions for

Potential on $M_i$ (Superposition)

Focusing conditions

Compute $W_j$

Solving one problem with several RHSs

Solution

hp-Refined Grid

Error Smaller than 1%?

No

Yes

Optimal Refinements

Optimal Grid, Optimal Intensities & Solution
NUMERICAL RESULTS: DC LATEROLOG

Model

Using the Tool Configuration of Halliburton Energy Services’ DLL
NUMERICAL RESULTS: DC LATEROLOG

$\theta = 0, 30$ and $60$ degrees

Relative errors of laterolog instruments in a homogeneous formation

![Graph showing relative error vs. number of Fourier modes for different angles.]
NUMERICAL RESULTS: DC LATEROLOG

Five layers: 100, 5, 1000, 0.5 and 100 ohm-m from top to bottom

Borehole: 0.1 m in radius
0.1 ohm-m in resistivity

Invasion

Anisotropy
NUMERICAL RESULTS: DC LATEROLOG

Comparison of Solutions by 1 and 9 Fourier Modes

Dip angle: 45 degrees
NUMERICAL RESULTS: DC LATEROLOG

Comparison of Solutions by 3 and 9 Fourier Modes

- Lld: 3 modes
- Lld: 9 modes

Dip angle: 45 degrees

Resistivity of Formation

Relative Depth (m)

Apparent Resistivity (Ω-m)

100 ohm-m
5 ohm-m
1,000 ohm-m
0.5 ohm-m
0.1 ohm-m
100 ohm-m
NUMERICAL RESULTS: DC LATEROLOG

Comparison of Solutions by 5 and 9 Fourier Modes

Resistivity of Formation

Dip angle: 45 degrees

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NUMERICAL RESULTS: DC LATEROLOG

Comparison of Solutions by 7 and 9 Fourier Modes

Dip angle: 45 degrees

Resistivity of Formation

Apparent Resistivity (Ω-m)

Relative Depth (m)
NUMERICAL RESULTS: DC LATEROLOG

Effects of dip angle: Conductive layer ↑

- LLD: 0°
- LLD: 10°
- LLD: 45°
- LLD: 60°
- LLs: 0°
- LLs: 10°
- LLs: 45°
- LLs: 60°

Resistivity of Formation

- 100 ohm-m
- 1,000 ohm-m
- 0.1 ohm-m
- 0.5 ohm-m
- 5 ohm-m

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NUMERICAL RESULTS: DC LATEROLOG

Effects of invasion to LLs: larger in a 45-degree deviated well than in a vertical well in conductive layer
NUMERICAL RESULTS: DC LATEROLOG

Effects of invasion to LLs: slightly smaller in a 60-degree deviated well than in a 45-degree deviated well in conductive layer.
NUMERICAL RESULTS: DC LATEROLOG

Effects of anisotropy: 45-degree deviated well ↑

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NUMERICAL RESULTS: DC LATEROLOG

Effects of anisotropy: 60-degree deviated well ↑
NUMERICAL RESULTS: AC RESULTS

Model Problem

150 kHz (Wireline)

Finite Size Loop Antenna

0.05 m

1.5 m

0.65 m

4 m

3 m

0.5 m

0.25 m

1.0 m

Fiber Glass Mandrel

$10^5 \, \Omega \cdot m$

Mandrel Radius: 0.04 m

Anisotropy (10:1)

SHALE

$
\begin{array}{c}
1 \, \Omega \cdot m \\
100 \, \Omega \cdot m \\
0.1 \, \Omega \cdot m \\
0.05 \, \Omega \cdot m \\
20 \, \Omega \cdot m \\
30 \, \Omega \cdot m \\
20 \, \Omega \cdot m \\
3 \, \Omega \cdot m \\
1 \, \Omega \cdot m
\end{array}$

OIL

SAND

SAND

SAND

SAND

SHALE

Anisotropy (5:1)

Anisotropy (10:1)

Anisotropy (10:1)
NUMERICAL RESULTS: AC RESULTS

Verification

Logging Instrument in a Homogeneous Formation

![Graph showing relative error vs. number of Fourier modes for different frequencies and angles]

- 20 KHz (30 degrees)
- 20 KHz (60 degrees)
- 150 KHz (30 degrees)
- 150 KHz (60 degrees)
NUMERICAL RESULTS: AC RESULTS

Verification

Logging Instrument in a Homogeneous Formation

Wireline, 150 Khz

Resistivity ($\Omega\cdot m$)

Real Part (V/m)

Imag Part (V/m)

For more info, visit: www.ices.utexas.edu/~pardo
NUMERICAL RESULTS: AC RESULTS

Verification

Logging Instrument in a Homogeneous Formation

Wireline, 150 Khz

Resistivity ($\Omega \cdot m$)

Real Part (V/m)

Imag Part (V/m)
NUMERICAL RESULTS: AC RESULTS

Verification

Logging Instrument in a Homogeneous Formation

Wireline, 150 Khz

Resistivity ($\Omega \cdot m$) vs. $10^{-2}$ to $10^{2}$

Real Part (V/m) vs. $10^{-2}$ to $10^{2}$

Imag Part (V/m) vs. $10^{-2}$ to $10^{2}$
NUMERICAL RESULTS: AC RESULTS

Verification

Logging Instrument in a Homogeneous Formation

![Graph showing resistivity and real/imaginary parts for AC results.](image)
NUMERICAL RESULTS: AC RESULTS

Verification

Logging Instrument in a Homogeneous Formation

Wireline, 150 Khz

Resistivity (Ω·m)

Real Part (V/m)

Imag Part (V/m)
NUMERICAL RESULTS: AC RESULTS

Dip Angle

Wireline, 150 Khz

Resistivity ($\Omega\cdot m$)

Real Part (V/m) $\times 10^{-3}$

Imag Part (V/m) $\times 10^{-3}$

0 degrees
30 degrees
60 degrees
NUMERICAL RESULTS: AC RESULTS

Dip Angle + Invasion

Wireline, 150 Khz

Resistivity (Ω·m)

Real Part (V/m) x 10^{-3}

Imag Part (V/m) x 10^{-3}

0 degrees
30 degrees
60 degrees

NO INVASION
INVASION
NUMERICAL RESULTS: AC RESULTS

Dip Angle + Anisotropy

Wireline, 150 KHz

Resistivity ($\Omega \cdot m$)

Real Part (V/m) $\times 10^{-3}$

Imag Part (V/m) $\times 10^{-3}$
NUMERICAL RESULTS: AC RESULTS

Dip Angle + Invasion + Anisotropy

Wireline, 150 Khz

Resistivity ($\Omega \cdot m$)

Real Part (V/m) $\times 10^{-3}$

Imag Part (V/m) $\times 10^{-3}$
NUMERICAL RESULTS: AC RESULTS

60-Degree Deviated Well

Wireline, 150 Khz

Resistivity (Ω·m)

Real Part (V/m) \times 10^{-3}

Imag Part (V/m) \times 10^{-3}

NO INV.

NO INV + ANI.

INV

INV + ANI

NO INV.

NO INV + ANI.

INV

INV + ANI

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NUMERICAL RESULTS: AC RESULTS

Vertical Well with 0.03 m Eccentricity

Wireline, 150 Khz

- Resistivity ($\Omega \cdot m$)
- Real Part (V/m)
- Imag Part (V/m)

NO INV.
NO INV + ECC.
INV
INV + ECC

NO INVASION
INVASION
NUMERICAL RESULTS: AC RESULTS

Model Problem

20 kHz (Wireline)

0.25 m

0.05 m

2.0 m

Fiber Glass Mandrel

$10^5 \, \Omega \cdot m$

Mandrel Radius: 0.04 m

Anisotropy (10:1)

SHALE

1 $\Omega \cdot m$

100 $\Omega \cdot m$

0.1 $\Omega \cdot m$

0.05 $\Omega \cdot m$

20 $\Omega \cdot m$

3 $\Omega \cdot m$

1 $\Omega \cdot m$

2.0 m

4 m

1.5 m

0.5 m

0.65 m

3 m

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NUMERICAL RESULTS: AC RESULTS

Verification

Logging Instrument in a Homogeneous Formation

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NUMERICAL RESULTS: AC RESULTS

Dip Angle

Wireline, 20 Khz.

Resistivity ($\Omega \cdot m$)

Real Part (V/m) $10^{-5}$

Imag Part (V/m) $10^{-5}$

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NUMERICAL RESULTS: AC RESULTS

Dip Angle + Invasion

Wireline, 20 Khz.

Resistivity (Ω·m)

Real Part (V/m)

Imag Part (V/m)

-10^{-2} -10^0 -10^2

10^{-6} -10^{-4} 0 2 4 6 8 10

0 degrees

30 degrees

60 degrees

0 degrees

30 degrees

60 degrees

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NUMERICAL RESULTS: AC RESULTS

Dip Angle + Anisotropy

Wireline, 20 Khz.

Resistivity ($\Omega\cdot m$)

Real Part (V/m) $\times 10^{-5}$

Imag Part (V/m) $\times 10^{-5}$

0 degrees
30 degrees
60 degrees
NUMERICAL RESULTS: AC RESULTS

Dip Angle + Invasion + Anisotropy

Wireline, 20 Khz.
NUMERICAL RESULTS: AC RESULTS

60-Degree Deviated Well

Wireline, 20 Khz.

Resistivity (Ω·m)

Real Part (V/m)

Imag Part (V/m)

NO INV.

NO INV + ANI.

INV

INV + ANI

NO INVASION

HORIZ. ρ

VERT. ρ

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NUMERICAL RESULTS: LWD

Model Problem and Verification

Finite Size Loop Antenna

Magnetic Buffer:
\[ \rho = 10000 \, \Omega \cdot m \]
\[ \mu_r = 10000 \]

Metallic Mandrel:
\[ \rho = 10^{-5} \, \Omega \cdot m \]
\[ \mu_r = 50 \]

Graph:
- x-axis: Number of Fourier Modes
- y-axis: Relative Error (in %)

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NUMERICAL RESULTS: LWD

Dip Angle

LWD, 2 Mhz

Resistivity ($\Omega\cdot$m)

Real Part (V/m)

Imag Part (V/m)
NUMERICAL RESULTS: LWD

Dip Angle + Invasion

LWD, 2 Mhz

Resistivity ($\Omega\text{-m}$)

Real Part (V/m)

Imag Part (V/m)

- Inversion
- No Inversion

For more info, visit: www.ices.utexas.edu/~pardo
NUMERICAL RESULTS: LWD

Dip Angle + Anisotropy

LWD, 2 Mhz

- Resistivity ($\Omega\cdot m$)
- Real Part (V/m)
- Imag Part (V/m)

- 0 degrees
- 30 degrees
- 60 degrees

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For more info, visit: www.ices.utexas.edu/~pardo
NUMERICAL RESULTS: LWD

Dip Angle + Invasion + Anisotropy

LWD, 2 Mhz

- Resistivity (Ω·m)
- Real Part (V/m)
- Imag Part (V/m)

0 degrees
30 degrees
60 degrees

NO INVASION
HORIZ.ρ
VERT.ρ

INVASION

For more info, visit: www.ices.utexas.edu/~pardo
60-Degree Deviated Well

LWD, 2 Mhz

NO INV.  
NO INV + ANI.  
INV  
INV + ANI

Resistivity ($\Omega \cdot m$)

Real Part (V/m)

Imag Part (V/m)
CONCLUSIONS AND FUTURE WORK

We have developed a new method based on a Fourier series expansion in a non-orthogonal system of coordinates.

- LIMITATION: Geometry of the problem.
- ADVANTAGE: It combines exponential convergence with sparse (penta-diagonal) matrices.

FURTHER APPLICABILITY OF THE METHOD:

- Eccentric measurements and tilted antennas.
- Time-domain simulations.
- Multi-Physics: Resistivity logging instruments, sonic logging instruments (acoustics + elasticity), fluid-flow, geomechanics, etc.
- Inverse problems.

FUTURE WORK: Iterative solver based on 2D-block Jacobi preconditioners.
ACKNOWLEDGMENTS

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