Towards Accurate Simulations of AC Dual-Laterolog Measurements with Tool Eccentricity Using $hp$-Finite Elements

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Outline

▶ Previous Work: 2D Dual-Laterolog (DLL)
  • $hp$ Adaptive Finite Element Method
  • Embedded Post-Processing Method

▶ 3D Methodology and DLL Simulations
  • Deviated Wells
  • Eccentered Measurements
  • Iterative Solver
  • Parallel Implementation

▶ Conclusions and Future Work
We vary locally the element size $h$ and the polynomial order of approximation $p$ throughout the grid.

Optimal grids are automatically generated by the hp-algorithm.

The self-adaptive goal-oriented hp-FEM provides exponential convergence rates in terms of the CPU time vs. the error in a user prescribed quantity of interest.
Dual-Laterolog (DLL)

- Description of Tool

- Determination of Intensities ($W_j$) of Bucking Currents

Focusing Conditions

\[ V(M_1) = V(M_2) \]
\[ V(M'_1) = V(M'_2) \]

\[ A_0 = 1 \]
\[ A_1 = W_1 \]
\[ A_1' = W_1' \]
\[ A_2 = W_2 \]
\[ A_2' = W_2' \]
One problem with several RHSs

Total potential on $M_i$

$\Rightarrow$ Superposition principle

$V(M_1) = W_1 V_{1,1} + V_{1,0} + W_{1,2} V_{1,0} + W_2 V_{1,1} + W_3 V_{1,2}$

$V(M_2) = W_2 V_{2,1} + V_{2,0} + W_{1,2} V_{2,0} + W_3 V_{2,1} + W_3 V_{2,2}$

$V(M_3) = W_3 V_{3,1} + V_{3,0} + W_{1,2} V_{3,0} + W_3 V_{3,1} + W_3 V_{3,2}$

$V(M_4) = W_3 V_{4,1} + V_{4,0} + W_{1,2} V_{4,0} + W_3 V_{4,1} + W_3 V_{4,2}$

$V(M_5) = W_3 V_{5,1} + V_{5,0} + W_{1,2} V_{5,0} + W_3 V_{5,1} + W_3 V_{5,2}$

(1) Focusing conditions

$V(M_1) = V(M_2)$

$V(M_3) = V(M_4)$

(2) Relationships between $W_j$

$W_1 = (W_1^0 + c)$, $W_2 = (W_2^0 + c)$ for LL.d

$W_3 = -(W_1^0 + c)$, $W_3 = -(W_3^0 + c)$ for LLs

with $c = 0.5$
Embedded Post-Processing Method (EPPM)

- Coarse Grid
  - Synthetic focusing method
    - Solutions for $M_i$
      - Potential on $M_i$ (Superposition)
        - Focusing conditions
          - Compute $W_j$
  - Solving one problem with several RHSs

- hp-Refined Grid
  - Error Smaller than 1%?
    - No
    - Yes
      - Optimal Refinements
      - Optimal Grid, Optimal Intensities & Solution

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Simulating the DLL tool

Using the Tool Configuration of Halliburton Energy Services’ DLL

Model

454.8 cm

20 cm

20 cm

100,000 ohm-m

0.00001 ohm-m

3.5 cm

2.2 cm

2.2 cm

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Invaded Formation (Vertical Well)

Effects of Invasion: LLs ↑

Borehole: 0.1 m in radius
0.1 ohm-m in resistivity

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Anisotropic Formation (Vertical Well)

Effects of anisotropy: LLs ↑

LLd: effects of anisotropy are negligible in conductive layer
3D Methodology and DLL Simulations I

- Deviated Wells
  - Non-orthogonal system of coordinates
  - Fourier series expansion
  - Numerical results

- Eccentered Measurements

- Iterative Solver

- Parallel Implementation
3D Deviated Well

Cartesian system of coordinates: \((x_1, x_2, x_3)\)

New non-orthogonal system of coordinates: \((\zeta_1, \zeta_2, \zeta_3)\)

Subdomain I

\[
\begin{align*}
x_1 &= \zeta_1 \cos \zeta_2 \\
x_2 &= \zeta_1 \sin \zeta_2 \\
x_3 &= \zeta_3
\end{align*}
\]

Subdomain II

\[
\begin{align*}
x_1 &= \zeta_1 \cos \zeta_2 \\
x_2 &= \zeta_1 \sin \zeta_2 \\
x_3 &= \zeta_3 + \tan \theta \frac{\zeta_1 - \rho_1}{\rho_2 - \rho_1} \rho_2 \cos \zeta_2
\end{align*}
\]

Subdomain III

\[
\begin{align*}
x_1 &= \zeta_1 \cos \zeta_2 \\
x_2 &= \zeta_1 \sin \zeta_2 \\
x_3 &= \zeta_3 + \zeta_1 \tan \theta \cos \zeta_2
\end{align*}
\]
3D Deviated Well

Cartesian system of coordinates: \((x_1, x_2, x_3)\)

New non-orthogonal system of coordinates: \((\zeta_1, \zeta_2, \zeta_3)\)

Constant material coefficients in the quasi-azimuthal direction \(\zeta_2\)
in the new non-orthogonal system of coordinates!!!!
DC problem: \(-\nabla \sigma \nabla u = f\)

Define Jacobian: 

\[
J = \left\{ \frac{\partial x_i}{\zeta_j} \right\}_{i,j=1,2,3}
\]

3D variational formulation in the new system of coordinates:

\[
\begin{cases}
\text{Find } \tilde{u} \in \tilde{u}_D + \tilde{H}^1_D(\Omega) \text{ such that:} \\
< \frac{\partial \tilde{v}}{\partial \zeta}, \tilde{\sigma}_{NEW} \frac{\partial \tilde{u}}{\partial \zeta} >_{L^2(\Omega)} = < \tilde{v}, \tilde{f}_{NEW} >_{L^2(\Omega)} + < \tilde{v}, \tilde{g}_{NEW} >_{L^2(\Omega)} \quad \forall \tilde{v} \in \tilde{H}^1_D(\Omega),
\end{cases}
\]

where

\[
\tilde{\sigma}_{NEW} := J^{-1} \tilde{\sigma} J^{-1T} \quad \tilde{f}_{NEW} := \tilde{f} \quad \tilde{g}_{NEW} := \tilde{g} \quad J_S
\]

The same concept can be applied to AC problems.
Fourier Series Expansion of a Function $\omega$ in $\zeta_2$:

$$\omega = \sum_{l=-\infty}^{l=\infty} \omega_l e^{j l \zeta_2} = \sum_{l=-\infty}^{l=\infty} F_l(\omega)e^{j l \zeta_2}$$

Final Variational Formulation after Fourier Series Expansion in $\zeta_2$:

Find $F_l(u) \in F_l(u_D) + H^1_D(\Omega_{2D})$ such that:

$$\sum_{k=-\infty}^{k=\infty} \sum_{l=k+2}^{l=k+2} <F_k\left(\frac{\partial v}{\partial \zeta}\right), F_{k-l}\left(\sigma_{NEW}\right)F_l\left(\frac{\partial u}{\partial \zeta}\right)>_{L^2(\Omega_{2D})}$$

$$= \sum_{k=-\infty}^{k=\infty} \left[<F_k(v), F_k(f_{NEW})>_{L^2(\Omega_{2D})} + <F_k(v), F_k(g_{NEW})>_{L^2(\Omega_{2D})}\right] \left.\forall F_k(v) \in H^1_D(\Omega), \right.$$ because $F_{k-l}(\sigma_{NEW}) = 0$ for every $|k - l| > 2$.

Only Five Fourier Modes ($l$) are enough to represent $\sigma_{NEW}$ EXACTLY for each $k$.

Therefore, we need to truncate only Fourier Modes ($k$) for 3D solution.
Example (9 Fourier Modes)

\[
\sum_{k=-4}^{4} \sum_{l=k-2}^{k+2} \langle F_k^\prime \left( \frac{\partial v}{\partial \zeta} \right), F_{k-l} \left( \sigma_{NEW} \right) F_i^\prime \left( \frac{\partial u}{\partial \zeta} \right) \rangle_{L^2(\Omega_{2D})} = \sum_{k=-4}^{4} \left[ \langle F_k (v), F_k (f_{NEW}) \rangle_{L^2(\Omega_{2D})} + \langle F_k (v), F_k (g_{NEW}) \rangle_{L^2(\Omega_{2D})} \right]
\]

\[
\sum_{k=-4}^{4} \sum_{l=k-2}^{k+2} d^k_i F_i(u) = \sum_{k=-4}^{4} b_k(F_k(v)) \quad d^k_i: \text{represents a 2D stiffness matrix}
\]
Verification of 3D Simulation

$\theta = 0, 30$ and $60$ degrees

Relative errors of laterolog measurements in a homogeneous formation

Reference Solutions: Solutions for $0^\circ$ deviated well
Convergence History of LLd Logs

Dip angle: 45 degrees

Solutions with 1 and 9 Fourier Modes

Relative Depth (m)

Resistivity of Formation

Apparent Resistivity (Ω-m)

Dip angle: 45 degrees

100 ohm-m

5 ohm-m

1,000 ohm-m

0.5 ohm-m

100 ohm-m

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Convergence History of LLd Logs

Solutions with 3 and 9 Fourier Modes

Dip angle: 45 degrees

Resistivity of Formation

Relative Depth (m) vs. Apparent Resistivity (Ω-m)

- LLd: 3 modes
- LLd: 9 modes

- Resistivity of Formation

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Convergence History of LLd Logs

Solutions with 5 and 9 Fourier Modes

Dip angle: 45 degrees

Relative Depth (m)

Resistivity of Formation

Apparent Resistivity ($\Omega$-m)

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Convergence History of LLd Logs

Solutions with 7 and 9 Fourier Modes

- LLd: 7 modes
- LLd: 9 modes

Dip angle: 45 degrees

Resistivity of Formation

Relative Depth (m)

Apparent Resistivity (Ω-m)

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Deviated Well (60, 45 and 10 degrees)

Effects of dip angle:
Thin layer ↑

Deviated Well

Relative Depth (m)

Apparent Resistivity (Ω-m)

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Deviated wells

Eccentered Measurements
- Non-orthogonal system of coordinates
- Fourier series expansion
- Numerical results

Iterative Solver

Parallel Implementation
New non-orthogonal system of coordinates: \((\zeta_1, \zeta_2, \zeta_3)\)

**Subdomain I**

\[
\begin{align*}
x_1 &= \rho_0 + \zeta_1 \cos \zeta_2 \\
x_2 &= \zeta_1 \sin \zeta_2 \\
x_3 &= \zeta_3
\end{align*}
\]

**Subdomain II**

\[
\begin{align*}
x_1 &= \frac{\zeta_1 - \rho_2}{\rho_1 - \rho_2} \rho_0 + \zeta_1 \cos \zeta_2 \\
x_2 &= \zeta_1 \sin \zeta_2 \\
x_3 &= \zeta_3
\end{align*}
\]

**Subdomain III**

\[
\begin{align*}
x_1 &= \zeta_1 \cos \zeta_2 \\
x_2 &= \zeta_1 \sin \zeta_2 \\
x_3 &= \zeta_3
\end{align*}
\]
Eccentricity

LLd readings with tool eccentricity

Effects of eccentricity: resistive layer ↑

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3D Methodology and DLL Simulations III

• Deviated Wells

• Eccentered Measurements

• **Iterative Solver**
  - 2D block Jacobi pre-conditioner
  - Numerical results

• Parallel Implementation
Iterative Solver I

Iterative Solver for Fast 3D Simulation:
- 2D Block Jacobi Pre-Conditioner
- Krylov-subspace optimization method (BI-Conjugate Gradient)

System of equations with 9 Fourier modes:
(deviated well)

\[
\begin{bmatrix}
\dd_{-4} & \dd_{-3} & \dd_{-2} & 0 & 0 & 0 & 0 & 0 & 0 \\
\dd_{-3} & \dd_{-2} & \dd_{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\
\dd_{-2} & \dd_{-1} & d_0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & d_{-1} & d_0 & d_1 & d_2 & d_3 & 0 & 0 & 0 \\
0 & 0 & d_2 & d_1 & d_2 & d_3 & d_4 & 0 & 0 \\
0 & 0 & 0 & d_0 & d_1 & d_2 & d_3 & d_4 & 0 \\
0 & 0 & 0 & 0 & d_0 & d_1 & d_2 & d_3 & d_4 \\
0 & 0 & 0 & 0 & 0 & d_2 & d_3 & d_4 & d_4 \\
0 & 0 & 0 & 0 & 0 & 0 & d_3 & d_4 & d_4 \\
\end{bmatrix}

= 

\begin{bmatrix}
F_4(u) \\
F_3(u) \\
F_2(u) \\
F_1(u) \\
F_0(u) \\
F_1(u) \\
F_2(u) \\
F_3(u) \\
F_4(u) \\
\end{bmatrix}

= 

\begin{bmatrix}
l_{-4} \\
l_{-3} \\
l_{-2} \\
l_{-1} \\
l_0 \\
l_1 \\
l_2 \\
l_3 \\
l_4 \\
\end{bmatrix}

\[
\begin{bmatrix}
\dd_{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \dd_{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \dd_{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \dd_{1} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \dd_{0} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \dd_{1} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \dd_{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \dd_{3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dd_{4} \\
\end{bmatrix}
\]

\(d^k_i\): represents a 2D stiffness matrix
Iterative Solver II (results)

Direct vs. Iterative (Time)

\[ \theta = 60 \text{ degrees} \]

7 Fourier modes for solution

Direct vs. Iterative (Memory)
Deviated wells

Eccentered Measurements

Iterative Solver

Parallel Implementation
  - Shared domain decomposition
  - Numerical results
3D Parallelization Implementation

Distributed Domain Decomposition

Shared Domain Decomposition!!

Processor 1
Processor 2
Processor 3
Processor 4
Processor 5
Processor 6
Processor 7
Processor 8

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3D Parallelization Implementation

Scalability of the Parallel Multi-Frontal Solver (Direct Solver)

Parallel computations performed on Texas Advance Computing Center (TACC) 60% relative efficiency up to 200 processors.
Parallel direct solver is 125 times faster on 200 processors.
Conclusions

• We have successfully simulated 3D DLL measurements by combining the use of a Fourier series expansion in a non-orthogonal system of coordinates with a 2D higher-order self-adaptive \( hp \) finite element method, and by using an embedded post-processing method.

• Iterative Solver for Fast 3D Simulation.

• Parallelization of Direct Solver
Future Work

- Simulation of Non-Zero Dual-Laterolog Measurements
- Simulation of Highly Eccentered Measurements
- Parallelization of Iterative Solver.
- Multi-Frequency and Time-Domain Simulations
- User Friendly Interface
  For setting up DLL tools and formations
  For implementing new monitoring conditions
Sponsors of UT Austin’s consortium on Formation Evaluation: