SELF-ADAPTIVE HP FINITE-ELEMENT SIMULATIONS OF MULTICOMPONENT INDUCTION MEASUREMENTS ACQUIRED IN DIPPING, INVADED, AND ANISOTROPIC FORMATIONS

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ABSTRACT
Tri-axial induction instruments have been designed to provide accurate formation evaluation by measuring both vertical and horizontal electrical conductivity. For accurate interpretation of tri-axial measurements in deviated wells penetrating reservoir rock formation showing extreme contrast of electrical conductivity, we have developed a simulation method by combining the use of a Fourier series expansion in a non-orthogonal system of coordinates with a two-dimensional goal-oriented high-order self-adaptive $hp$ finite element (FE) refinement strategy. Our algorithm provides highly accurate and reliable tri-axial induction measurements while reducing the computational complexity of the three-dimensional geometry of deviated wells. Numerical results indicate that dip effects are more emphasized in the most conductive layers.

INTRODUCTION
Conductivity of the reservoir rock formation is used to quantify hydrocarbon saturation based on the fact that the electrical conductivity of clay-free hydrocarbon-bearing rocks depends on pore volume, pore-volume connectivity, electrical conductivity of connate water, and interconnected pore volume occupied by connate water. The oil and gas exploration industry has widely used induction tools to estimate formation conductivity.

Standard induction logging tools only measure the horizontal resistivity, even though electrical anisotropy is critical to quantify formation conductivity. On the other hand, tri-axial induction tools are designed to estimate both horizontal and vertical conductivity [1].

Simulations of tri-axial measurements in deviated wells in a formation exhibiting extreme contrasts of electrical conductivity is still challenging due to the three-dimensional (3D) geometry of deviated wells. However, it becomes increasingly important to simulate those tri-axial measurements due to the increasing use of deviated wells within the oil industry. In this paper, we simulate tri-axial induction measurements in deviated wells in the presence of large contrasts of formation electrical conductivity.

We first outline the simulation method of tri-axial induction measurements described in [2], in which we consider 3D source and receiver configurations. Next, we validate our algorithm using analytical solutions. Finally, we describe numerical simulations of tri-axial measurements for deviated wells penetrating a formation exhibiting large contrasts of electrical conductivity.

METHOD
Maxwell’s equations in a domain $\Omega$ with time dependence of the form $e^{\frac{-j\omega t}{\lambda}}$ are given as

\[
\nabla \times \mathbf{E} = -j\omega \mathbf{H} - \mathbf{M}^{\text{imp}}, \quad \text{(Faraday's law)}
\]

\[
\nabla \times \mathbf{H} = (\sigma + j\omega \epsilon) \mathbf{E} + \mathbf{J}^{\text{imp}}, \quad \text{(Ampere's law)}
\]

where $\mathbf{H}$ and $\mathbf{E}$ denote the magnetic and electric fields, respectively, $\mathbf{J}^{\text{imp}}$ and $\mathbf{M}^{\text{imp}}$ are impressed electric and magnetic current densities, respectively, and $\epsilon$, $\mu$, $\sigma$ indicate dielectric permittivity, magnetic permeability, and electrical conductivity tensors, respectively. Tri-axial induction tools measure various components of the magnetic field $\mathbf{H}$. We assume an operating frequency equal to 20 kHz.

When considering a deviated well penetrating a 1D layered earth (Fig. 1), the resulting geometry is 3D. In the cylindrical system of coordinates, the vertical axis is along the center of the borehole. Employing a non-orthogonal coordinate system $\zeta = (\zeta_1, \zeta_2, \zeta_3)$ (Fig. 1), described in [2], material properties become invariant with respect to the quasi-azimuthal direction $\zeta_2$. Any function in the new coordinate system is periodic and thus, can be expressed in terms of its Fourier series expansion with respect to $\zeta_2$.

Using a Fourier series expansion of the resulting formulation from (1) in the non-orthogonal system of coordinates, we obtain a 3D formulation consisting of a sequence of 2D problems, in which each 2D problem only couples with a
maximum of five other 2D problems. Thus, the corresponding stiffness matrix becomes penta-diagonal. This sparsity becomes a major advantage over solving a general 3D problem (that consists of a sequence of fully coupled 2D problems).

For 3D sources, \(M_x\) and \(M_y\), we consider a Dirac’s delta function in the \(\varphi\)-direction, i.e., \(\delta(\varphi - \varphi_s)\) where \(\varphi_s\) is \(0^\circ\) and \(90^\circ\) for \(M_x\) and \(M_y\), respectively. On the other hand, \(M_z\) can be approximated by a small solenoidal coil (\(J_{\varphi}\)), and thus, it becomes a 2D source.

To solve the resulting 3D variational formulation, we employ the goal-oriented self-adaptive hp FE method [2]. When considering vertical wells, our new non-orthogonal system of coordinates is identical to the system of cylindrical coordinates, and the 3D formulation reduces to a so-called 2.5D problem.

**NUMERICAL RESULTS**

We modeled a tri-axial induction tool. 3D antennas have a finite-size cross-section (on the \(\rho-z\) plane), whose dimensions are 0.01 m by 0.01 m. Transmitter and receiver are 1.016 m (40 in.) apart from each other on a mandrel, which is \(10^5\) ohm-m in resistivity, 0.09 m in diameter and 2.016 m in length (Fig. 2).

To validate our algorithm, we compute \(H_{xx}\) (i.e., we consider a source equal to \(M_x\) and we measure \(H_x\)) without mandrel in a 60-degree deviated well penetrating a 1 ohm-m homogeneous formation. The borehole is assumed to have the same resistivity of that of formation, resulting in a homogeneous medium for which we have analytical solutions. Further, the source is fixed while the receiver moves. We can observe convergence of real and imaginary parts of \(H_{xx}\) to the analytical solution computed using “em1d” (K.H. Lee 1984, pers.comm.) as we increase the number of Fourier modes from 3 to 9 modes (Fig. 3).

We simulated tri-axial induction measurements using the tool described in Fig. 2 for 0- (vertical), 30- and 60-degree deviated wells in a layered formation composed of five layers. Each layer has a resistivity equal to 100, 0.05, 10000, 1 and 20 ohm-m, respectively (from top to bottom), while the thicknesses of the second, third and fourth layers are 1.5, 3 and 3 m, respectively. The borehole is 0.2 m in diameter and 100 ohm-m in resistivity. For a vertical well, \(H_{xx}\) at 2 kHz exhibit less variations (in both real and imaginary parts) than at 2 MHz (Fig. 4). The dip effects are notable in the most conductive (third) layers and increase with increasing dip angle (Fig. 5). We observe in Fig. 6 that \(H_{zz}\) (which is usually measured by standard induction tools) in a vertical well is insensitive to electrical anisotropy, and only becomes sensitive for a deviated well. On the other hand, \(H_{xx}\) is sensitive to anisotropy in both vertical and deviated wells (Fig. 7).

**CONCLUSIONS**

We have successfully simulated 3D tri-axial induction measurements in deviated wells by combining the use of a Fourier series expansion in a non-orthogonal system of coordinates with a high-order self-adaptive hp finite element method. We validated our algorithm against analytic solutions. Numerical experiments indicate that dip effects on tri-axial induction measurements of \(H_{xx}\) are large in conductive layers, while anisotropy effects on \(H_{xx}\) are emphasized in both vertical and deviated wells.

**ACKNOWLEDGEMENTS**

The work reported in this paper was funded by The University of Texas at Austin’s Research Consortium on Formation Evaluation, jointly sponsored by Aramco, Anadarko Petroleum Corporation, Baker-Hughes, British Gas, BHP Billiton, BP, Chevron, ConocoPhillips, ENI, ExxonMobil, Halliburton Energy Services, Marathon Oil Corporation, Mexican Institute for Petroleum, Petrobras, Schlumberger, Shell International E&P, StatoilHydro, TOTAL, and Weatherford.

**REFERENCES**


Fig. 1. Cross-section showing a deviated well with a dip angle $\theta$ penetrating a layered formation. The circles indicate the “quasi-azimuthal” direction $\zeta_2$ in a non-orthogonal system of coordinates while both the $x_3$- (in Cartesian system of coordinates) and $\zeta_3$-directions (in the non-orthogonal system of coordinates) correspond to the direction of the borehole.

Fig. 2. Tri-axial induction tool. The mandrel is $10^5$ ohm-m in resistivity, 9 cm in diameter and 2.016 m in length. Source and receiver antennas have a cross-section equal to 1 cm by 1 cm, and are located 1.016 m apart from each other. In (b), we describe tri-axial sources and receivers.

Fig. 3. Convergence history (left) of tri-axial induction measurements of $H_{xx}$ at 20 kHz for a 60-degree deviated well in a homogeneous formation (right) having a resistivity of 1 ohm-m as a function of number of employed Fourier modes (3, 7 and 9 modes). Both borehole and tool have the same resistivity as that of formation, resulting in a corresponding homogeneous medium. Simulated solutions by our algorithm are compared with the analytical solutions provided by “em1d” (K.H. Lee 1984, pers.comm.).

Fig. 4. Comparison of real (left) and imaginary (right) parts of $H_{xx}$ at 20 kHz (left triangles with solid lines) and at 2 MHz (right triangles with solid lines) for a vertical well. The formation has five layers having resistivities equal to 100, 0.05, 10000, 1 and 20 ohm-m, respectively (from top to bottom). The thicknesses of the second, third and fourth layers are 1.5, 3 and 3 m, respectively. The borehole is 0.2 m in diameter and 100 ohm-m in resistivity.

Fig. 5. Comparison of real (left) and imaginary (right) parts of $H_{xx}$ at 20 kHz for vertical (dashed lines), 30- (left triangles with solid lines) and 60-degree (right triangles with solid lines) deviated wells. The formation has five layers having resistivities equal to 100, 0.05, 10000, 1 and 20 ohm-m, respectively (from top to bottom). The thicknesses of the second, third and fourth layers are 1.5, 3 and 3 m, respectively. The borehole is 0.2 m in diameter and 100 ohm-m in resistivity.
Fig. 6. Comparison of real (top) and imaginary (bottom) parts of $H_{zz}$ at 20 kHz for vertical (left), and 60-degree (right) deviated wells in isotropic (left triangles with solid lines) and anisotropic (right triangle with solid lines) formations, respectively. The formation has five layers having horizontal resistivities equal to 100, 0.05, 10000, 1 and 20 ohm-m, respectively (from top to bottom). The second and fourth layers have anisotropy with vertical resistivities equal to 0.5 and 10 ohm-m, respectively. The thicknesses of the second, third and fourth layers are 1.5, 3 and 3 m, respectively. The borehole is 0.2 m in diameter and 100 ohm-m in resistivity, respectively (from top to bottom).

Fig. 7. Comparison of real (top) and imaginary (bottom) parts of $H_{xx}$ at 20 kHz for vertical (left), and 60-degree (right) deviated wells in isotropic (left triangles with solid lines) and anisotropic (right triangle with solid lines) formations, respectively. The formation has five layers having horizontal resistivities equal to 100, 0.05, 10000, 1 and 20 ohm-m, respectively (from top to bottom). The second and fourth layers have anisotropy with vertical resistivities equal to 0.5 and 10 ohm-m, respectively. The thicknesses of the second, third and fourth layers are 1.5, 3 and 3 m, respectively. The borehole is 0.2 m in diameter and 100 ohm-m in resistivity, respectively (from top to bottom).