NUMERICAL SIMULATION OF 3D DC BOREHOLE RESISTIVITY MEASUREMENTS USING AN HP-ADAPTIVE GOAL-ORIENTED FINITE ELEMENT FORMULATION

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ABSTRACT
We describe the development of a 3D Finite Element (FE) self-adaptive hp goal-oriented algorithm applied to the simulation of direct current (DC) borehole resistivity measurements for the assessment of rock formation properties. The self-adaptive algorithm delivers (without any user interaction) a sequence of optimal hp-grids that converges exponentially in terms of a user-prescribed quantity of interest with respect to the CPU time. Thus, it allows for highly accurate numerical simulations of a variety of 3D DC resistivity logging measurements, including those obtained in deviated cased wells. The corresponding version of the software for performing electrodynamic simulations (3D AC) is currently under development.

INTRODUCTION
During the last decades, a number of 3D simulators of resistivity logging measurements have been developed within the oil industry. These simulators have been successfully applied to study and analyze different physical effects occurring on three-dimensional problems (see, for example, [9], [8], [3], and [1]).

Despite these recent advances, there are still a large number of three-dimensional effects for which reliable simulations are not available. An example of this is the effect of a dip angle in a cased well. Furthermore, in most of the existing results, only a partial validation of those results is reported, typically obtained by comparing solutions of simplified model problems against the corresponding solutions obtained with a lower dimensional (2D or 1D) numerical method. This lack of 3D results (as opposed to 2D results) is due to major difficulties that are encountered when solving challenging 3D problems. Namely, for mesh-based methods (Finite Elements, Finite Differences, Boundary Elements, etc.), the size of the system of linear equations becomes excessively large to be solved within a few minutes/hours.

Different strategies can be taken to overcome this problem, including 1) the use of fast (iterative) solvers, 2) the use of parallel computations, and, 3) the use of computer-generated optimal grids that reduce the error without adding an unnecessarily large number of unknowns.

In this work, we combine all three techniques mentioned above, namely, an state-of-the-art self-adaptive goal-oriented hp-Finite Element strategy — for automatic generation of optimal grids —, a two-grid (multigrid) solver of linear equations — the fastest possible type of linear equation solver —, and a parallel implementation (see next extended abstract for details). In addition, we have also implemented a Perfectly Matched Layer (PML) for efficient truncation of the computational domain (see [4] and [7] for details), and an additional software that allows for transferring two-dimensional hp-refined optimal grids into the 3D code.

This methodology allows for the study of a large variety of 3D electrostatic effects in a borehole environment, such as the study of deviated wells, patched antennas, cased wells, laterolog and induction instruments, etc.

METHODOLOGY
We outline the main ideas of the numerical methodology used for simulating our 3D DC logging problems of interest. The method is based on combining the use of hp-Finite Elements, a self-adaptive algorithm designed for approximating a user-prescribed quantity of interest (also called goal-oriented adaptivity), and a two-grid iterative solver of linear equations.

A 2D and 3D hp-FEM. Our numerical method utilizes continuous elements of variable order of approximation. Thus, it supports hp-FE discretizations of electrostatic problems. Here h stands for the element size, and p denotes the polynomial element order (degree) of approximation, both varying locally throughout the grid.

The main motivation for using hp-FE is given by the following result:
For an optimal sequence of grids, both in terms of element size $h$ and polynomial order of approximation $p$, the corresponding sequence of solutions converges exponentially to the exact solution with respect to the number of unknowns (as well as the CPU time), independently of the number, intensity, and/or distribution of singularities in the solution.

An algorithm generating an 'optimal sequence of grids' delivering exponential convergence and highly accurate simulations is presented below.

**A Self-Adaptive $hp$ Goal-Oriented Algorithm.** The algorithm described in [6, 5] produces a sequence of optimally $hp$-refined meshes that delivers exponential convergence rates in terms of a user prescribed quantity of interest against the size of the discrete problem or CPU time. A given (coarse) conforming $hp$ mesh is first globally refined in both $h$ and $p$ to yield a fine mesh, i.e. each element is broken into eight new elements, and the discretization order of approximation $p$ is raised uniformly by one. Subsequently, the problem of interest is solved on the fine mesh. The next optimal coarse mesh is then determined as the one that maximizes the decrease of the projection based interpolation error [2] averaged by the added number of unknowns. Since the mesh optimization process is based on the minimization of the interpolation error rather than the residual, the algorithm is problem independent, and it can be applied to different physics (acoustics, elasticity, etc.), nonlinear and eigenvalue problems as well.

**A Two-grid (iterative) solver of Linear Equations.** We have developed a two-grid solver of linear equations based on block-Jacobi smoother iterations on the fine grid combined with a global solution on a coarse grid (also called coarse grid correction). A fine grid edge-based overlapping block-Jacobi smoother has been employed to avoid degeneration of convergence properties due to the presence of elongated elements. This two-grid cycle is accelerated by using a goal-oriented steepest-descent method.

Typically, the iterative solver converges in less than fifteen iterations, even in presence of elongated elements with an aspect ratio up to 10000:1.

**A Perfectly Matched Layer (PML).** A Perfectly Matched Layer (PML) has been utilized to efficiently truncate the computational domain. For details, see [4], where we demonstrate the robustness of the PML in presence of anisotropic materials at different frequencies and with high contrast in conductivity (for example, in cased wells).

**NUMERICAL RESULTS**

We consider a Through Casing Resistivity Instrument in a borehole environment, as described in Fig. 1 (left). The steel casing has a resistivity of $10^{-5} \Omega \cdot m$, and a thickness of $0.0127 cm = 0.5 in.$.

Three different logs are displayed in Fig. 1 (center). The first log (blue curve) has been obtained from the 2D self-adaptive goal-oriented $hp$-FE software. The second log (red circles) have been obtained by transferring the optimal 2D self-adaptive goal-oriented $hp$-grid to the 3D code, and solving the problem within the 3D code environment. Finally, the third log (red circles) has been obtained by performing a global p-enrichment to the transferred optimal 2D grid, and solving the problem within the 3D code. The corresponding relative errors are displayed in Fig. 1 (right), and they remain below 0.5%.

In Fig. 2 — left panel — we display the 2D optimal $hp$-grid transferred into the 3D code (different colors indicate average order of approximation $p$ within each element). The corresponding solution is displayed in Fig. 2 — right panel —.

Fig. 3 displays the logs — left panel — and errors — right panel — corresponding to the 30 degrees deviated well. Results indicate that a 2D optimal grid may deliver large errors (up to 300%) when utilized for deviated wells. From the physical point of view, we observe for the deviated well an slightly lower sensitivity with respect to the most conductive layer ($1 \Omega \cdot m$).

**CONCLUSIONS**

We have described a methodology that provides accurate simulations of various 3D DC resistivity logging problems. The electrodynamics (AC) version is currently under development.

Results indicate that it is possible to accurately simulate through casing resistivity measurements, and that optimal 2D grids may be utilized for obtaining 3D results, although the accuracy of such results may be compromised. It is more
Figure 1: TCRT problem. Left panel: 2D cross section of the geometry of the model. The model consists of one transmitter and three receiver electrodes, a conductive borehole, a metallic casing, and four layers in the formation material with varying resistivities. Center panel: Final log obtained with three different numerical techniques: 2D code, a 2D grid transferred to the 3D code, and a 2D grid transferred to the 3D code and globally p-enriched. Right panel: relative errors corresponding to the 2D grid transferred to the 3D code, and the 2D grid transferred to the 3D code and globally p-enriched. The 2D solution is utilized as the reference solution.

Figure 2: TCRT problem. Left panel: 2D optimal $hp$-grid transferred to the 3D code (different colors indicate average order of approximation $p$ within each element) for a 45 degrees deviated well. Right panel: corresponding solution (voltage).

accurate to directly generate optimal 3D grids.
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Figure 3: TCRT problem. Left panel: Final logs for the vertical well, and for the 30 degrees deviated well obtained with different numerical techniques: 1) an optimal 2D grid transferred to the 3D code, 2) an optimal 2D grid transferred to the 3D code and globally p-enriched, and 3) an optimal self-adaptive goal-oriented 3D hp-grid. Right panel: relative errors corresponding to the 2D grid transferred to the 3D code, and the 2D grid transferred to the 3D code and globally p-enriched. The 3D solution corresponding to the 3D self-adaptive (optimal) grid is utilized as the reference solution.


References