3aMU1. Adaptive stabilized finite element framework for simulation of vocal fold turbulent fluid-structure interaction

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As a step toward building a more complete model of voice production mechanics, we assess the feasibility of a fluid-structure simulation of the vocal fold mechanics in the Unicorn incompressible Unified Continuum framework. The Unicorn framework consists of conservation equations for mass and momentum, a phase function selecting solid or fluid constitutive laws, a convection equation for the phase function and moving mesh methods for tracking the interface, and discretization through an adaptive stabilized finite element method. The framework has been validated for turbulent flow for both low and high Reynolds numbers and has the following features: implicit turbulence modeling (turbulent dissipation only occurs through numerical stabilization), goal-oriented mesh adaptivity, strong, implicit fluid-structure coupling and good scaling on massively parallel computers. We have applied the framework for turbulent fluid-structure interaction simulation of vocal folds, and present initial results. Acoustic quantities have been extracted from the framework in the setting of an investigation of a configuration approximating an exhaust system with turbulent flow around a flexible triangular steel plate in a circular duct. We present some results of the investigation as well as results of the framework applied to other problems.

Published by the Acoustical Society of America through the American Institute of Physics

©2013 Acoustical Society of America [DOI: 10.1121/1.4799464]
Received 22 Jan 2013; published 2 Jun 2013
INTRODUCTION

The aim of this work is to investigate the feasibility of applying the Unicorn/FEniCS-HPC (Unicorn) open source finite element simulation framework for predicting aeroacoustic quantities in turbulent fluid-structure interaction applications, with the specific goal of eventually being able to simulate the mechanics and aeroacoustics of the vocal folds, in effect simulating the human voice, and other similar applications with flexible structures in turbulent flow.

The Unicorn framework consists of conservation equations for mass and momentum, a phase function selecting solid or fluid constitutive laws, a convection equation for the phase function and moving mesh methods for tracking the interface. The framework has been validated for turbulent flow and has the following features:

- If the structure is stiff or its motion prescribed, the framework reduces to the same as for pure flow.
- Framework for goal-oriented adaptivity
- Massively parallel strong linear scaling (can utilize performance increase in modern architectures)
- Strong, implicit fluid-structure coupling
- Parameter-free Implicit Large Eddy Simulation (ILES) turbulence model
- Sharp (discontinuous) fluid-structure interface, avoiding the need for refined mesh close to the interface

In this paper we model two applications in Unicorn: (i) a rubber model of the human vocal folds in a channel which comes from an experimental setup described in [3] and (ii) a configuration approximating an exhaust system with turbulent flow around a flexible triangular steel plate in a circular duct. For the vocal folds application we present early results demonstrating self-oscillation and an oscillation frequency in the range of human phonation. In the exhaust system application we extract acoustic quantities based on Curle's analogy and compare to experiments.

Background of vocal folds modeling

The physics of voice production is an intricate combination of processes which involve elastic bodies invoking non-linear vibrations of tissues, oscillatory flow, and turbulent flow, all of which combine to the generation of an acoustic wave. The vocal folds act on the airflow as a buzzing valve, a process called phonation. The vocal folds oscillate only under certain combinations of positioning, tensioning and driving pressure.

Conventionally, solid and fluid mechanics are dealt with using separate computational approaches and tools, which makes the treatment of this compound problem very intricate. Here, we use modern multi-physics simulation techniques, with a unified computational domain in which the dynamics of solids and fluids/gases are modelled simultaneously.

Background of exhaust system modeling: flexible steel plate in duct flow

We investigate a configuration approximating an exhaust system with turbulent flow around a flexible triangular steel plate in a circular duct. The plate is used to efficiently mix an aerosol into a low Mach number gas stream. One application is diesel exhaust after-treatment systems using selective catalytic reduction (SCR), where an additive is injected under low pressure into an exhaust stream.

In the results presented in this paper however, the main focus is the acoustic field. However effective as a mixer, it would disqualify for practical use if generating excessive levels of noise.
STATE OF THE ART

In 2003, de Oliveira Rosa et al [5] presented a FEM model of vocal fold mechanics with a steady fluid model. They pointed out that to achieve more realistic results, an unsteady fluid model and higher mesh resolution are both necessary. In the literature [21] FEM vocal fold mechanics models typically have:

- An iterative scheme between a fluid and structure solver
- Fixed mesh resolution
- No clear framework for adaptive error control

We believe that Unicorn could make a contribution to the field, and take some preliminary steps in this work.

To capture aeroacoustic quantities Kaltenbacher et. al. in [17] first solve for the flow using FEM and then extract acoustic sources using Lighthill’s acoustic analogy, which then also takes into account convective effects on the wave propagation.

THE FEniCS-HPC FINITE ELEMENT COMPUTATIONAL FRAMEWORK

FEniCS-HPC is an open source framework for automated solution of PDE on massively parallel architectures, providing automated evaluation of variational forms given a high-level description in mathematical notation, duality-based adaptive error control, implicit turbulence modeling by use of stabilized FEM and strong linear scaling up to thousands of cores [10, 12, 16, 11]. FEniCS-HPC is a branch of the FEniCS [19, 6] framework focusing on high performance on massively parallel architectures.

Unicorn-HPC is solver technology (models, methods, algorithms and software) with the goal of automated high performance simulation of realistic continuum mechanics applications, such as drag or lift computation for fixed or flexible objects (FSI) in turbulent incompressible or compressible flow. The basis for Unicorn is Unified Continuum (UC) modeling [8] formulated in Euler (laboratory) coordinates, together with a General Galerkin (G2) adaptive stabilized finite element discretization [15] with a moving mesh for tracking the phase interfaces. Unicorn formulates and implements the adaptive G2 method applied to the UC model, and interfaces to other components in the FEniCS-HPC chain.

FIGURE 1: Snapshot of the velocity field and flexible mixer plate (top) and pressure field and plate (bottom).

Unified continuum fluid-structure interaction

The incompressible UC model takes the form:

\[
\rho(D_t u_i + \theta u_j D_{x_j} u_i) = D_{x_j} \sigma_{ij} + f_i \\
D_{x_j} u_j = 0 \\
D_t \theta + D_{x_j}(u_j \theta) = 0
\]  

(1)
where \( u \) is velocity with components \( u_i \), \( \sigma \) is the stress tensor, \( f \) is the external force and \( \theta \) is the phase function that marks the continuum as either fluid or solid (\( \theta := 1 \) for fluid and \( \theta := 0 \) for solid). The three equations describe conservation of momentum, conservation of mass and phase convection respectively.

In this work the fluid-structure constitutive laws take the form (from [8]):

\[
\begin{align*}
\sigma &= \sigma_D - pI \\
\sigma_D &= \theta \sigma_f + (1 - \theta) \sigma_s \\
\sigma_f &= 2\mu_f \epsilon(u) \\
D_1 \sigma_s &= 2\mu_s \epsilon(u) + \nabla u \sigma_s + \sigma_s \nabla u^\top
\end{align*}
\]

(2)

where \( \sigma_D \) stands for deviatoric stress, \( p \) for pressure, \( \sigma_f \) and \( \sigma_s \) for fluid and solid stresses, \( \mu_f \) for viscosity, \( \mu_s \) for the shear modulus of the structure, which is the ratio of the shear stress to shear strain. The strain rate tensor \( \epsilon \) is defined as \( \epsilon(u) = \frac{1}{2} (\nabla u + \nabla u^\top) \). In 2D, the result of applying the operator \( \nabla \) on \( u = (u_0, u_1) \) is the tensor which is equal to the dyadic product of \( (D_{s0}, D_{s1}) \) and \( (u_0, u_1) \).

**General Galerkin finite element discretization**

Our computational approach is based on stabilized finite element methods, together with adjoint based adaptive algorithms, and residual based implicit turbulence modeling and shock capturing, for related work see e.g. [2, 7].

The General Galerkin (G2) method for high Reynolds number flow, including turbulent flow and shocks, takes the form of a standard Galerkin finite element discretization together with (i) least squares stabilization of the residual and (ii) residual based shock capturing.

With a G2 method, we define turbulent flow as the non-smooth parts of the flow where the residual measured in \( L_2 \)-norm increases as the mesh is refined, whereas in a negative \( H^{-1} \)-norm the residual decreases with mesh refinement [13]. That is, we characterize turbulence by a pointwise large residual which is small in average, corresponding to the equations being satisfied only in a mean value sense, which is sufficient to approximate mean value quantities of a turbulent flow field using G2.

We split the time interval \( I \) into subintervals \( I_n = (t_{n-1}, t_n) \), with associated space-time slabs \( S_n = \Omega \times I_n \), over which we define space-time finite element spaces, based on a spatial finite element space \( W^n \) over a spatial mesh \( T_n \) [13].

In a \( cG(1)cG(1) \) method [10, 9] we seek an approximate solution \( \hat{U} = (U, P) \) which is continuous piecewise linear in space and time. With \( W^n \) a standard finite element space of continuous piecewise linear functions, and \( W_0^n \) the functions in \( W^n \) which are zero on the boundary \( \Gamma \), the \( cG(1)cG(1) \) method for constant density incompressible flow with homogeneous Dirichlet boundary conditions for the velocity takes the form: for \( n = 1, ..., N \), find \( (U^n, P^n) \equiv (U(t_n), P(t_n)) \) with \( U^n \in V_0^n = [W_0^n]^3 \) and \( P^n \in W^n \), such that

\[
\begin{align*}
\left( (U^n - U^{n-1}) k_n^{-1} + (\hat{U}^n \cdot \nabla) \hat{U}^n, v \right) + (2\nu \epsilon(\hat{U}^n), \epsilon(v)) \\
- (P^n, \nabla \cdot v) + (\nabla \cdot \hat{U}^n, q) \\
+ SD_0^n(\hat{U}^n, P^n; v, q) = (f, v), \forall v = (v, q) \in V_0^n \times W^n
\end{align*}
\]

(3)

where \( \hat{U}^n = 1/2(U^n + U^{n-1}) \) is piecewise constant in time over \( I_n \), with the stabilizing term

\[
SD_0^n(\hat{U}^n, P^n; v, q) \equiv \\
(\delta_1(\hat{U}^n \cdot \nabla \hat{U}^n + \nabla P^n - f), \hat{U}^n \cdot \nabla v + \nabla q) \\
+ (\delta_2 \nabla \cdot \hat{U}^n, \nabla \cdot v),
\]

where we have dropped the shock capturing term, and where

\[
(v, w) = \sum_{K \in \mathcal{T}_h} v \cdot w \, dx,
\]

\[
(\varepsilon(v), \varepsilon(w)) = \sum_{i,j=1}^{3} (\varepsilon_{ij}(v), \varepsilon_{ij}(w)),
\]

with the stabilization parameters

\[
\delta_1 = \kappa_1 (k_n^{-2} + |U^{n-1}|^2 h_n^{-2})^{-1/2},
\]

\[
\delta_2 = \kappa_2 h_n
\]

where \( \kappa_1 \) and \( \kappa_2 \) are positive constants of unit size. For turbulent flow we choose a time step size

\[
k_n \sim \min_{x \in \Omega} (h_n/|U^{n-1}|).
\]

We note that the least squares stabilization omits the time derivative in the residual, which is a consequence of the test functions being piecewise constant in time for a CG(1) discretization of time [9].

For the Unified Continuum FSI model, we introduce a piecewise constant solid stress term \( S_s \), and the mesh motion adds an ALE mesh velocity \( \beta_h \) to the convective velocity:

\[
(\rho((U^n - U^{n-1})k_n^{-1} + (\bar{U} - \beta_h)^n \cdot v)U^n), v) + (1 - \theta)(S_s, \nabla v) + \theta(2\mu f(U^n), \varepsilon(v)) - (P^n, \nabla \cdot v) + (\nabla \cdot \bar{U}^n, q) + SD_\delta(U^n, P^n; v, q) = (f, v), \forall \hat{v} = (v, q) \in V^n_0 \times W^n
\]

**FLEXIBLE MIXER PLATE IN AN EXHAUST SYSTEM**

An experimental configuration approximating an exhaust system with a flexible triangular steel mixer plate in a circular duct flow has been studied in [18]. The Reynolds number is \( 4.37 \times 10^5 \) at a Mach number of 0.21. The flow induces a static deflection and oscillation of the plate. How this oscillation influences the aero-acoustical properties poses an interesting research question.

We set up the duct and flow conditions in Unicorn and introduce a flexible plate. Representative snapshots of the velocity and pressure together with the elastic plate are given in Figure 1. The generated sound spectrum computed from pressure probes in the the duct is given in 2, where we also plot experimental results for comparison. The peak at ca. 400Hz is captured by the simulation for the stiff case, and for the flexible case the simulation captures the reduction in sound level and has a good match to the experiment for the 300-1000Hz frequency band.

**Numerical results**

Using the numerical framework, three different plates are studied; one rigid, one flexible and one rigid but statically deflected. This static deflection, which is due to the air flow, was set to that of the flexible plate (observed in the numerical simulation). Common for all plates is that the thickness is 3 mm, simply due to computational cost associated with resolution of the plate. In order to obtain similar bending stiffness, the density for the flexible plate is assigned a density of \( 180 \text{ kg/m}^3 \) and a Young’s modulus of 9.3 MPa.

**Applying Curle’s Analogy**

As sound waves are density disturbances which propagate with the speed of sound, the incompressible solution obtained by the numerical method described in this chapter cannot contain sound. Nevertheless, the sound generation can be approximated from the incompressible solution by studying the instantaneous
pressure loss of the mixer plate. To do this, the acoustic analogy by Curle\cite{4} is here applied. Assuming a rigid solid in a low Mach number flow (i.e. the vortex mixer plate), Curle\cite{4} showed that the dominating source term is a dipole. Therefore, it is meaningful to model the vortex mixer source as a 1D dipole in the plane wave range. The sound field generated by such a dipole in a semi-infinite duct is given by \cite{1}
\[
\tilde{p}_\pm(x) = \pm \tilde{F}_e^{-ik_\pm x} \frac{A}{2A(1 \pm M)},
\]
where \(\tilde{F}_e\) is the dipole force and \(A\) is the duct cross section area, and where the sign in front of the Mach number in the denominator is positive in the upstream direction and negative in the downstream direction. As the wave propagation speed is infinite in the numerical calculations, Eq. (4) is simplified into
\[
\Delta \tilde{p} = \tilde{p}_+ - \tilde{p}_- = \tilde{F}_e \frac{A}{A}.
\]
Thus, the generated acoustic field is given directly by the resultant time varying force \(\tilde{F}_e\). In the case of the numerically calculated flow, this time varying force resultant divided by the duct cross section area, must equal the time varying pressure difference over the plate. This pressure loss, averaged over the duct cross section area, is thus taken to be the generated acoustic pressure. A notable difference between this acoustic pressure and the one generated in the experiments, is that there is no obvious way of dividing the calculated pressure into upstream and downstream directed parts, whereas in the experiments, these parts (consisting of the source cross spectrum elements, see \cite{14} are solved for. Another is that the convective effects on intensity and directivity of the source cannot be captured in the calculations, since the sound speed is infinite. To compare with experiments, the mean of the experimental upstream and downstream directed parts is used. In Figure 2, the calculated sound pressure level of the three plates is compared to the experimental sound pressure level of the 3 mm and the 0.5 mm plates.

\textbf{FIGURE 2}: Generated sound spectra at Mach 0.207. Black line: Experiments, 3 mm plate, Green line: Experiments, 0.5 mm plate, Red diamonds: Calculations, rigid plate, Blue diamonds: Calculations, rigid deflected plate, Red line: Calculations, flexible plate.

\section*{Vocal Folds}

In this case we study a geometric model of homogenous silicon rubber vocal folds and a channel together with boundary conditions from an experimental setup given by Becker et al \cite{3}, who kindly provided the
geometric model. We generate a fluid-structure mesh for the Unified Continuum framework, and run a parallel simulation.

To model the contact of the folds, we implement a basic contact model using the primitives of the UC model. When a point on the surface of the folds come closer than a threshold distance to the center of the gap between the folds, the constitutive law is switched to an elastic solid in a designated contact zone between the folds. This blocks the fluid flow and together with the pressure from the flow helps to push the folds apart again.

We apply a constant lung pressure and induce a self-oscillation in the vocal folds. A jet is generated in the small opening between the folds which is periodically cut off when the folds come into contact, see figure 3. We plot the y-displacement for the point with the maximum displacement on the vocal folds against time in figure 4. The frequency is ca. 110Hz and within the range of human phonation [3].

\textbf{FIGURE 3:} Self-oscillating vocal folds: snapshots of geometry and velocity during open and closed phases, and geometry of channel and vocal folds at the top.

\section*{Conclusion}

For the mixer plate case, it is evident from experimental and numerical results that the 3 mm, rigid plate generates significantly more noise and pressure loss. From the numerical results, it can be deduced that the main improvement of the flexible plate is due to the static deflection. However, the experimental results also indicate that an extra reduction of several dB occurs when there is coincidence in frequency, between the second bending mode of the plate and the broad band Strouhal peak of the sound spectrum. This is not seen in the numerical results. However, by inspecting the vibration spectra of the numerical solution, it is evident that in the computations, only the first bending mode is distinguishable from the broad band vibration signal. A possible reason for the drop in noise generation at the second bending mode, is that the second bending mode will radiate significantly less sound than the first in the plane wave region of the duct. This is because the shape of the second bending mode consists of two peaks in opposite phase, and will thus couple poorly with the plane wave. Whether this effect is strong enough to be significant, depends on e.g., the strength and position of each peak, and is yet to be determined. Considering the sound spectrum below 100 Hz say, i.e. below the low-frequency limit of the experimental setup, there is a strong peak for the flexible plate (0.5 mm). As the accuracy is uncertain, no real conclusion can be drawn, but it is interesting to note that the first
FIGURE 4: Displacement plot (y-coordinate) for the point with the maximum displacement on the vocal folds. The frequency is within the range of human phonation [3]. Note the shape of the displacement curve is essentially sinusoidal with non-smooth cut-offs arising from the contact of the folds.

The bending mode of the plate is found at this peak (±10 Hz).

We cannot say much quantitatively about the vocal folds results at this stage of development, but the setup generates self-oscillation with an estimated frequency within the range of human phonation. The displacement curve matches the expected “smooth sinusoidal interrupted by collision” as seen in experimental measurements by for example Rothenberg and Mahshie [20].

In summary, we have applied the Unicorn framework to two turbulent fluid-structure applications. In the mixer plate case we can predict acoustic quantities using Curle’s analogy, though we still have to investigate further if we can capture a noise-reducing effect seen in experiments. In the vocal folds case we can reproduce a self-oscillating system generating a frequency within the range of human phonation, but there is still work to be done in mesh-convergence studies and a more refined contact model before we can say anything quantitatively with confidence.

ACKNOWLEDGMENTS

The authors would like to acknowledge the financial support from the European Research Council, Swedish Foundation for Strategic Research, the Swedish Research Council, and the Swedish Energy Agency. The simulations were performed on resources provided by the Swedish National Infrastructure for Computing (SNIC) at PDC - Center for High-Performance Computing.

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