basque center for applied mathematics

EUSKO JAURLARITZA
GOBIERNO VASCO
HEZKUNTZA SAILA
DEPARTAMENTO DE EDUCACIÓN

Courses
2020-21

Optimal Transport and Geometric Optics in complex media

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01-05 February 2021 (5 sessions) l 14:00 - 16:00 (a total of 10 hours)

A classical optical inverse problem is the following: given a fixed unit direction m and two distinct media, if monochromatic light rays emanate from the origin inside medium I, what is the surface (interface between medium I and medium II) that refracts all these rays into rays parallel to the direction m?

This course provides an introduction to such optical refraction problems in complex media. In particular, we will discuss refraction in materials that have a negative refractive index. The far field version of this refraction problem can be treated with Optimal Mass Transport techniques. As such, a fully nonlinear PDE of Monge-Ampere type arises. We will also discuss related optics problems, including the near field analog (which cannot be treated with Optimal Mass Transport methods) and a lens design problem.

PROGRAMME:

Day 1: Introduction to negative refractive index materials and Snell's Law of Refraction:

We will give an introduction to negative refractive index materials and metamaterials. We will also discuss the classical Snell Law of refraction, its vector formulation, and surfaces of uniform refraction in the context of negative refractive index materials;

Day 2: Optimal Mass Transport:

We will review some of the basics of Optimal Mass Transportation. In particular, we will show existence of a minimizer to a Kantorovich dual problem;

Day 3: Application of Optimal Mass Transport to Refractor problem:

We will apply Optimal Mass Transport techniques to solve a far field refraction problem by using a particular cost function. We will use results from Day 1 on surfaces of uniform refraction;

Day 4: The Monge-Ampere Equation:

We will derive explicitly the nonlinear PDE of Monge-Ampere type that arises in the previous far field problem. We will show that certain geometric conditions (so-called A3w and A3s conditions of Ma, Trudinger, and Wang) hold depending on the refractive index of the material;

Day 5: Other applications:

We will end this course by discussing related optics problems, including the near field refractor problem. This problem entails illuminating a single point rather than refracting light into a prescribed direction, and cannot be treated with Optimal Mass Transport methods.

REFERENCES:

[1] G. De Philippis and A. Figalli, The Monge-Ampere equation and its link to Optimal Transportation. Bulletin of the American Mathematical Society, 51(4), 527--580, 2014.

[2] C. E. Gutierrez, Refraction Problems in Geometric Optics. Lecture Notes in Mathematics, Volume 2087, Springer, 2014, pp. 95--150.

[3] C. E. Gutierrez and E. Stachura, Uniform refraction in negative refractive index materials. Journal of the Optical Society of America A, 32 (11), 2110--2122, 2015.

[4] X.-N. Ma, N. Trudinger, and X.-J. Wang, Regularity of Potential functions of the optimal transport problem. Archive for Rational Mechanics and Analysis, 177, 151--183, 2005.

[5] E. Stachura, Existence of weak solutions to refraction problems in Negative Refractive Index Materials. Nonlinear Analysis, 157, 76--103, 2017.

***Registration is free, but mandatory before Thursday, 28th January 2021.** To sign-up go to **https://forms.gle/eQvVbXyXzgLeFQfv8** and fill the registration form.