



Propuesta de Curso de Postgrado en BCAM:
Advanced Aspects in Applied Mathematics

Structured population models - Modelling and analysis

Lecturer: Mats Gyllenberg (University of Helsinki)

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Abstract (written by Philipp Getto):

In a structured population model, the population state dynamics are described as the time evolution of the composition of a population with respect to the individual state, typically the individuals age or size [1, 3, 6]. Such models have rich applications in demography, ecology, epidemiology and cell biology [1, 6]. Traditionally, the models have been formulated as first order hyperbolic partial differential equations (PDE). The seemingly impossibility to establish, e.g., the principle of linearized stability for general nonlinear models inspired a rather recent reformulation of the models as integral equations, more specifically, Delay Equations (continuous delay) or Volterra functional equations [2]. For such equations, e.g., the principle of linearised stability was then established via the perturbation theory of dual semigroups (sun-star formalism). One aim of the course is to propagate the use of integral equations to population dynamical analysts with some dynamical systems background. Another aim is to show mathematicians working in applied infinite dimensional dynamical systems some challenges and open problems that can arise from population dynamical modelling.

Tentative list of topics:

1. Formulating structured population models as delay equations
2. Equivalence of delay equations and abstract integral equations
3. Solving abstract integral equations with perturbation theory
4. Stability and bifurcation theorems for delay equations
5. Application of stability and bifurcation theorems to structured population models

Bibliography:

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- [2] O. Diekmann, Ph. Getto and M. Gyllenberg. Stability and bifurcation analysis of Volterra functional equations in the light of suns and star. *SIAM J. Math. Anal.* **39** (2007) 1023–1069.
- [3] O. Diekmann, M. Gyllenberg, H. Huang, M. Kirkilionis, J. A. J. Metz and H. R. Thieme. On the formulation and analysis of general deterministic structured population models: II Nonlinear theory. *J. Math. Biol.* **43** (2001) 157–189.

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- [5] J. Hale. *Functional Differential Equations*. Springer Verlag, New York, 1977.
- [6] J. A. J. Metz and O. Diekmann (eds.). *The Dynamics of Physiologically Structured Populations*. Lecture Notes in Biomathematics, Vol. 68. Springer, Berlin, 1986.