

Contents of the course Topics on numerics for wave propagation

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BCAM and UPV/EHU courses 2011-2012:
Advanced aspects in applied mathematics
[Topics on numerics for wave propagation](#)
June 11-15, 2012, from 9:00 to 11:00

From Euclid to Fourier and much more...

Chapter 2: Background on Fourier analysis

- Wave equation and its versions.
- Harmonic oscillators: simple, damped, coupled, with driven forces.
- 1-d wave equation and d'Alembert formula
- Fourier transform and its applications: Schrödinger, wave and telegraph equation
- Spherical means and Hadamard method of descent to solve 3-d and 2-d wave equations
- Fourier series
- Resolution of the wave equation on bounded domains by the Fourier method
- Fourier series as numerical method.
- Damped wave equation
- Resolution of the wave equation by the semigroup theory

Chapter 3: Finite difference discretizations

- Lax equivalence theorem
- Finite difference approximations for the transport equations and stability through von Neumann method
- Convergence estimates for the centered three-points finite difference approximation of the Laplacian and of the wave equation.
- Brief introduction to the finite element method approximations of waves.

Chapter 4: Group velocity and concentrated waves

- Stationary phase lemma
- Fourier symbol, dispersion relation, phase velocity, group velocity, group acceleration
- Continuous and numerical wave packets
- Hamiltonian systems and rays of Geometric Optics

Chapter 5: Dispersive properties

- 1-d Schrödinger equation and its finite difference approximations
- Decay properties for the Schrödinger equation and its finite difference approximations
- Remedies for the lack of uniform dispersive properties for the discrete schemes: Fourier truncation and bi-grid algorithms.
- Gaussian wave packets for the Schrödinger equation and its dispersive schemes

Chapter 6: Control of vibrations

- Lack of uniform observability and controllability properties for the $1 - d$ wave equation
- Remedies for the lack of uniform observability properties: Fourier truncation and bi-grid algorithms
- Conjugate gradients algorithm to construct the optimal L^2 -control and implementation.

Chapter 7: Heterogeneous media

- Reflection transmission coefficients in layered media
- Rays of Geometric Optics for the wave equation with oscillating coefficients and for the numerical approximations of the wave equation on non-uniform meshes.