

GEOMETRIC APPROACHES TO INTEGRABLE SYSTEMS AND THEIR PERTURBATIONS

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October 18th – October 21st, 2021

Monday to Thursday (5 sessions)

10:00 - 12:00 and October 21st also from 14:00-16:00 (a total of 10 hours)

This mini-course proposes an introduction to information Geometry and a special focus on its use introduced by Nakamura to study the Hierarchy of integrable gradient systems of Jacobi-Toda. A general construction is given for Hamiltonian systems of Euler-Poincaré types on Lie Groups. Other example of integrable systems (with a Darboux first integral) are further discussed, with the centers and the double centers for quadratic planar vector fields.

Outline for a 10-hour schedule:

1. The (open) Toda Lattice and information geometry:

Toda Lattice in Flaschka's form, Moser's system, equivalence with an averaged learning equation;

2. Information geometry and Hamiltonian systems of Euler-Poincaré on Lie groups:

Euler-Poincaré systems, Connections, their curvature and torsion, flat connections, Levi-Civita connection associated to a Riemannian metric, Information Geometry on Lie groups;

3. The Poincaré center-focus problem and the local Hilbert's 16th problem for quadratic planar vector fields:

Darboux integrability of the regular centers of planar quadratic vector fields. Nash space of arcs in the setting of perturbation theory

4. The double centers problem for quadratic planar vector fields:

Geometry of the double center set and the associated Darboux integrals

5. The Kahan-Kimura-Hirota mapping associated to a quadratic planar vector field:

Link with Runge-Kutta discretization, Invariant measure, application to integrable cases;

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- [3] Nakamura, Yoshimasa, A tau-function for the finite Toda molecule, and information spaces, Symplectic geometry and quantization (Sanda and Yokohama, 1993), 205-211, Contemp. Math. 179, Amer. Math. Soc., Providence, RI, 1994.
- [4] J.-P. Francoise, Information Geometry and Integrable Hamiltonian Systems F. Barbaresco and F. Nielsen (Eds.): SPIGL 2020, PROMS 361, pp. 141-153, 2021.
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- [6] J.-P. Françoise, L. Gavrilov and D. Xiao, Hilbert's 16th problem on a period annulus and Nash space of arcs Math. Proc. Camb. Phil. Soc. (2020), 169, 377-409.
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***Registration is free, but inscription is required before October 11^h, 2021:** So as to inscribe go to <https://forms.gle/Mdng87x4rbGRXDhj8> and fill the registration form.