Diophantine Approximation through the Mass Transference Principle

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March 21st – 25th, 2022
Monday to Friday (5 sessions)
11:00-12:30, on Wednesday it will ends at 12:00 (a total of 7 hours)

The goal of this course is to prove Jarník’s theorem. In 1931 Jarnik studied the size of the set

\[ W(\tau) := \{ x \in [0, 1] \mid \left| x - \frac{p}{q} \right| \leq \frac{1}{q^\tau} \text{ for infinitely many fractions } \frac{p}{q} \}, \]

where \( r \geq 2 \), and concluded that its Hausdorff dimension is \( 2/r \) and that its Hausdorff measure at \( 2/r \) is infinite. In 2006 Beresnevich and Velani introduced the Mass Transference Principle, a powerful method to compute the dimension of lim sup-sets, so, in particular, the dimension of \( W(\tau) \). We will also see some applications.

CONTENT
- Diophantine approximation, history and motivation.
- Basic concepts: Hausdorff dimension and properties; Frostman measures; and some general principles.
- Statement and proof of the Mass Transference Principle.
- How to deduce Jarník’s theorem from the Mass Transference Principle and other applications.
- If time permits, recent extensions of the Mass Transference Principle.

PREREQUISITES:
None

REFERENCES:

*Registration is free, but inscription is required before March 14th, 2022: So as to inscribe go to https://forms.gle/FTb8GKtUjCyzwzmbA and fill the registration form.*