

GROUP COHOMOLOGY AND BIERI-STREBEL-NEUMANN-RENN INVARIANTS

CONCHITA MARTÍNEZ-PÉREZ

September 29th – November 17th, 2021
Monday and Wednesdays (17 sessions)
16:00 - 18:00 (a total of 34 hours)

Homological tools are important in many branches of mathematics. To fully understand them, some basic background in homological algebra is needed, however, homological algebra is usually considered too abstract and has disappeared from most programs in mathematics nowadays. In this course we want to give an overview of the basics of homological algebra, with a focus in group (co)homology. Homological algebra originated with group homology so this last theory provides a natural motivation for the more abstract setting of homological algebra. Moreover, groups are an excellent source of examples that can be used as an excuse to introduce some relevant families of subgroups which are interesting for their own sake such as right angled Artin groups, knot groups, Thompson group, solvable groups and also central constructions in modern group theory such as HNN extensions and free products with amalgamation.

The second part of the course is devoted to cohomological finiteness conditions. We will introduce some basic definitions and techniques and illustrate them with the above families of groups, trying to stress the interaction between algebra and geometry. To finish, we will give an introduction to the beautiful theory of the BNSR-invariants (Bieri-Neumann-Strebel-Renz).

The course is divided in two parts. Each part would consist of 12 hours of ordinary lectures plus 3 invited talks on additional topics. The ordinary lectures will be at master level and only a basic background in algebra and topology will be assumed. The invited talks will be on workshop level and the idea is that they can reach to research level, skipping details of proofs when necessary but giving the idea. In these talks, we intend to put the student in contact with active areas of research.

Additionally, during both the lectures and the invited talks we will discuss some relevant open problems in the subject. For example, in the first part we will mention problems such as the Eilenberg-Ganea conjecture, the conjecture that homological dimension one are free or the cohomological dimension of solvable groups in arbitrary characteristic and in the second part the weak form of Brown's conjecture the sigma conjecture for metabelian groups or several questions regarding the sigma invariants for various families of groups.

Follows a more detailed program of the two parts of the course.

Inaugural lecture: Peter Kropholler

Part 1: Introduction to homological algebra from the point of view of group cohomology

Cayley complex of a group. CW-complexes, simplicial complexes and actions of groups, classifying space and group cohomology. Geometric dimension of a group. Group rings and modules. Projective modules, projective resolutions. Ext and Tor functors and basic properties. Cohomological dimension. HNN extensions, free products with amalgamation and Mayer-Vietoris sequences. Right angled Artin groups as iterated HNN extensions and as iterated amalgamated products. Cube complex associated to a RAAG. Torsion free one relator groups as iterated HNN extensions. Knot groups and Alexander polynomials from the point of view of group cohomology.

Speakers: Ian Leary, Brita Nucinkis, Oihana Garaialde.

Additional topics: CAT(0) geometry, hyperbolic groups.

Part 2: Cohomological finiteness conditions and BNRS-invariants.

Finiteness conditions: FP_n , F_n , FP_∞ , F_∞ . Stability criteria for HNN extensions and free products with amalgamation. Brown's criterium via actions on spaces. Bredon cohomology and classifying spaces for proper actions. Finiteness conditions for RAAGs, knot groups and one-relator groups. Thompson group. Character sphere of a group. The invariant Σ^1 and its first properties. Σ^1 for metabelian groups. Brown's procedure for 1-relator groups. General BNRS invariants and examples: Thompson group, RAAGs

Speakers: Matt Zaremsky, Luis Mendoza, Dawid Kielak.

Additional topics: L^2 -invariants. Atiyah and Kaplansky conjectures.

Prerequisites: the student is assumed to have only basic knowledge of group theory and some basic topology.

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***Registration is free, but inscription is required before September 24th, 2021:** So as to inscribe go to <https://forms.gle/PU463DN7nAT6Pk576> and fill the registration form.