HARMONIC ANALYSIS

IOANNIS PARISSIS, LUZ RONCAL, MATEUS SOUSA

January 10th – February 23rd, 2022, and February 11th 2022 (15 sessions)
16:00 - 18:00 (a total of 30 hours)

1. Description

This course aims to cover topics in Harmonic Analysis that we consider fundamental within the current development of the field. It is oriented towards graduate students (and possibly undergraduate students in the final year of the Mathematics' degree). The students are expected to have a knowledge on the basic properties and objects related to real variable methods of Fourier analysis.

The lectures will be held face-to-face and live-streamed, at BCAM facilities, during the period **10th January 2022 - 23rd February 2022**, Mondays and Wednesdays from 16:00 to 18:00. It will also take place on **11th February 2022 from 16:00 to 18:00**. The total number of hours is 30. The language of the course will be English.

2. Topics for the course

The course is organised in two parts:
- The first part of the course will be a quick review of the basic background.
- The second part, is meant to introduce the students to a broader set of topics in harmonic analysis. The goal is to provide them with the basic tools and references one needs to understand modern research topics in harmonic analysis.

In what follows, we provide a sequence of topics meant to be explored in the two parts the course. The contents of the second part of the course could be modi ed depending on time and feedback from the students. The order of the contents could be also modi ed.

2.1. First part.

(1) Convolutions and approximate identities:
- Young's inequality;
- Maximal operators of convolution type;
- Pointwise and L^p convergence of approximate identities.

(2) Real and Complex Interpolation:
- Riesz-Thorin interpolation;
- Marcinkiewicz interpolation.

(3) Fourier Transform:
- $L^1 + L^2$ Theory;

(4) The Schwartz Class and Tempered Distributions.

(5) Hardy-Littlewood maximal function and Lebesgue differentiation:
- Vitali covering Lemma;
- Hardy-Littlewood-Wiener theorem;
- Lebesgue differentiation and differentiation bases.

(6) Calderón-Zygmund decomposition.

(7) Hilbert and Riesz Transform:
- Conjugate harmonic extension;
- Strong and weak L^p boundedness;
- Convergence of Fourier integrals.

(8) Singular Integrals:
- Calderón-Zygmund theorem;
- Littlewood-Paley Theory;
- Multipliers.

2.2 Second Part.

(1) Oscillatory Integrals:
- Phases without critical points;
- Van der Corput Lemma;
- Stationary Phase.

(2) Fourier Restriction:
- Stein-Tomas theorem;
- Knapp Examples;
- The restriction conjecture;
- Strichartz estimates;
- Bilinear estimates.

(3) Sparse Domination:
- Weighted inequalities;
- Rubio de Francia's extrapolation.
- Definition and properties of a sparse operator;
- Weighted inequalities via sparse domination.

(4) Kakeya conjecture and the ball multiplier.

REFERENCES:

[1] J. Duoandikoetxea, *Fourier Analysis. Graduate Studies in Mathematics*, 29, AMS, Providence, RI 2001.
[2] L. Grafakos, *Classical Fourier Analysis. Graduate Texts in Mathematics*, 249, Springer, New York, NY 2008.
[3] L. Grafakos, *Modern Fourier Analysis. Graduate Texts in Mathematics*, 250, Springer, New York, NY 2008.
[4] C. Muscalu, W. Schlag, *Classical and Multilinear Harmonic Analysis*. Cambridge University Press, New York NY 2013.
[5] E. Stein, *Singular Integrals and Differentiability Properties of Functions*. Princeton University Press, Princeton, NJ 1970.
[6] E. Stein, *Harmonic Analysis*. Princeton University Press, Princeton, NJ 1993.
[7] E. Stein and G. Weiss, *Introduction to Fourier Analysis on Euclidean Spaces*. Princeton University Press, Princeton, NJ 1971.
[8] T. Wolf, *Lectures on Harmonic Analysis*. American Mathematical Society, Providence, RI 2003.

***Registration is free, but inscription is required before 3rd January, 2022:**
So as toinscribe go to https://forms.gle/1Xr6ndNfB4f1fT2o6 and fill the registration form.