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Title: VARIATIONAL AND MULTISCALE MODELING: AN ANALYSIS OF THE FINITE ELEMENT METHOD FOR TRANSPORT PROBLEMS

Date and time: Mon Nov 19 (15:00 to 17:00), Tue Nov 20 (15:00-17:00), Thu Nov 22 (15:00 to 17:00)

Venue: BCAM Headquarters, Mazarredo 14, 48009 Bilbao

Abstract:
Diverse problems in engineering and physics are governed by partial differential equations in space and time. The spatial domain, in most relevant applied problems, involves complicated geometrical objects, such as, airplanes, cars, boats, semi-conductor devices, human anatomy, etc. Finite elements are the first general purpose approach able to deal with complicated geometries and solve a broad range of applications. While finite elements have been successful in areas, such as, solid thermo-mechanical problems, shortcomings in relevant areas, such as, mechanics, have been noted over the years.

To exemplify the shortcomings of conventional finite elements in mechanics, a model problem, advection-dominated diffusion in one spatial dimension, will be described. The observed poor numerical performance can be traced back to the lack of stability of the Galerkin method for non-symmetric and singularly perturbed operators. The notions of coercivity and continuity, used by the Lax-Milgram theorem, are revisited in the context of non-symmetric operators and their relation to discrete stability is described (Babuska's theorem is stated). Ad hoc procedures, such as artificial diffusion, yield stable discretizations, but render them inaccurate. Stabilization ideas are introduced and their relation to the variational multiscale (VMS) framework discussed.

In this context, a multiscale reformulation of the Galerkin method allows us to define consistent stable methods, such as, SUPG, G/LS and USPG. These stabilized methods can be described as “simplified” computable versions of VMS. To conclude, incompressible Navier-Stokes equations in strong and weak forms will be described. Revisiting the Variational multiscale formulation, we will describe a framework to develop novel turbulence models for large eddy simulations.

Outline:


Level:
The course aims at advanced graduate students in engineering, computer science, applied mathematics, and physical sciences interested in developing insights and skills that they may apply in their research. No formal prerequisites. Notes and papers will be provided as needed.