C O U R S E

(matematika mugaz bestalde)

June 4 and 5, 2012

12:00 - 13:00h

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NONLOCAL DIFFUSION, NONLOCAL MECHANICS, AND A NONLOCAL VECTOR CALCULUS

Based on notions for nonlocal fluxes between volumes and nonlocal balance laws and a nonlocal vector calculus we have developed, we introduce nonlocal models for diffusion and the nonlocal peridynamics continuum model for mechanics. A feature of the nonlocal problems that has important practical consequences are that constraints, e.g., of Dirichlet type, are applied over volumes and not along bounding surfaces.

PART I (June 4)

A review of the nonlocal calculus is given, including definitions of nonlocal divergence, gradient, and curl operators and derivations of a nonlocal Gauss theorem and Green's identities. Through appropriate limiting processes, relations between the nonlocal operators and their differential counterparts are established. Nonlocal problems analogous to boundary-value problems for partial differential equations are defined using the operators from the nonlocal calculus. The nonlocal calculus is used to define weak formulations of the nonlocal diffusion and mechanics problems which are then analyzed, showing, for example, that unlike elliptic partial differential equations, these problems do not necessarily result in the smoothing of data. We briefly consider connections to fractional Laplacian and fractional derivative problems, both of which are special cases of our models.

PART II (June 5)

We consider finite element methods for nonlocal problems, focusing on solutions containing jump discontinuities; in this setting, discontinuous Galerkin methods are conforming and can lead to optimally accurate approximations. We discuss the difficulties that arise in implementing such methods for nonlocal problems, including appropriate choices for quadrature rules. We also show how our models can be implemented so that they are multi-scale mono-models, i.e., they are single models that are valid and tractable over a wide range of scales. We also briefly consider control and optimization problems constrained by the nonlocal model equations.