basque center for applied mathematics

COURSE

(matematika mugaz bestalde)

December 2,3, 4,5, 2013 9:00-11:00
December 3, 2013 also at 15:00-17:00

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THE MELLIN TRANSFORM AND ITS APPLICATIONS

The aim of the course is to give an introduction to the useful tool of Mellin integral transform in an application driven spirit. Actually, even if it does not require mathematical notions higher than classical Laplace and Fourier transforms, it is much less famous. To highlight the role of Mellin transform in mathematical physics and applied mathematics, lectures are devoted to the derivation of several valuable results by applying the Mellin machinery to concrete questions.

In particular, it is shown that the problem to evaluate definite integrals can be successfully faced with a method mainly based on the reduction of the integrand function under consideration to functions whose Mellin transform is the ratio of product of Gamma functions. This method provides a powerful technique to solve a large family of problems.

Moreover, Mellin transform is strongly related to Mellin–Barnes integrals through the inverse transformation, so that in this framework Special functions emerge naturally in their Mellin–Barnes integral representation. Its relation with fractional diffusion equations is also shown.
For the discussed applications of Mellin transform, please see the program.

Keywords: Integral transforms, definite integrals evaluation, cosine and sine Fourier transform, solution of Fredholm integral equations, Mellin-Barnes integral representation, Special functions, subordination type formulae.

References

[1] Fikioris G., Integral evaluation using the Mellin transform and generalized hypergeometric functions: Tutorial and applications to antenna problems. IEEE Trans. Antennas Propag. 54, 3895–3907 (2006).

[2] Luchko Yu. and Kiryakova V., The Mellin integral transform in Fractional Calculus. Fract. Calcl. Appl. Anal. 16, 405–430 (2013).

[3] Marichev O.I., Handbook of Integral Transforms of Higher Transcendental Functions: Theory and Algorithmic Tables. Ellis Horwood Ltd, 1983.

[4] Paris R.B. and Kaminski D., Asymptotics and Mellin–Barnes Integrals. Cambridge University Press, Cambridge, 2001.

[5] Yakubovich S. and Luchko Yu., The Hypergeometric Approach to Integral Transforms and Convolutions. Series: Mathematics and its applications, Vol. 287, Kluwer Acad. Publ., Dordrecht - Boston - London, 1994.

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Program

First Lecture. Monday 2, 9-11

Definition of Mellin transform. Mellin transform inversion formula. Relation with Laplace and Fourier transforms. Basic properties of Mellin transform. The Mellin convolution. The Gamma function and its properties. Compu- tation of elementary Mellin transforms.

Second Lecture. Tuesday 3, 9-11

A method for the evaluation of definite integrals. Evaluation of some definite integrals. Computation of cosine and sine Fourier transformations. Com- putation of inverse Fourier transformations. Solution of Fredholm integral equations with kernel of the form k(xt) and kernel of the form k(x/t).

Third Lecture. Tuesday 3, 15-17

The pioneering work by Salvatore Pincherle. Mellin transform and Mellin– Barnes integrals. Mellin–Barnes integral representation. Special functions. Asymptotic expansion of functions.

Fourth Lecture. Wednesday 4, 9-11

Completely monotone functions via Mellin transform. Composition formu- lae for solutions of fractional diffusion equations. Subordination formulae for probability density functions.

Fifth Lecture. Thursday 5, 9-11

Lévy stable density. The Voigt function and its generalization. Mellin convolution for signal filtering. The Gaussianization of L evy noise.

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