

Arnold's classification of simple singularities of real-valued functions

Up to codimension 4:

symbol	codim	germ	unfolding	Thom's name
A_2	1	x^3	$x + u_1x$	fold
A_3	2	$\pm x^4$	$\pm x^4 + u_2x^2 + u_1x$	cuspid
A_4	3	x^5	$x^5 + u_3x^3 + u_2x^2 + u_1x$	swallowtail
A_5	4	$\pm x^6$	$\pm x^6 + u_4x^4 + u_3x^3 + u_2x^2 + u_1x$	butterfly
D_4^\pm	3	$xy^2 \pm x^3$	$xy^2 \pm x^3 + u_3x^2 + u_2x + u_1y$	umbilics
D_5	4	$xy^2 \pm x^4$	$xy^2 \pm x^4 + u_4x^3 + u_3x^2 + u_2x + u_1y$	parabolic umbilic

Higher codimension:

In addition to the A_n (codim $n - 1$) and D_n (codim $n - 1$) analogous to the above, there are

s	codim	germ	unfolding
E_6	5	$x^3 \pm y^4$	$x^3 \pm y^4 + u_5xy^2 + u_4y^2 + u_3xy + u_2x + u_1y$
E_7	6	$x^3 + xy^3$	$x^3 + xy^3 + u_6y^4 + u_5y^3 + u_4y^2 + u_3xy + u_2x + u_1y$
E_8	7	$x^3 + y^5$	$x^3 + y^5 + u_7xy^3 + u_6xy^2 + u_5y^3 + u_4y^2 + u_3xy + u_2x + u_1y$

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