basque center for applied mathematics

EXCELENCIA SEVERO OCHOA

BCAM Courses

PhD/PostDoc level course on FRACTIONAL KINETICS: ANALYTIC AND PROBABILISTIC APPROACHES

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November, Monday 10th - Friday 14th

Monday 10th - Wednesday 12th (Mon 11:30-13:30 / 15:00-19:00; Tue 9:00-12:00 / 15:00-16:00 / 17:00-19:00 ; Wed 9:00-12:00 / 15:00–18:00)
Thursday 13th - Friday 14th (Thu, Fri 9:30-12:00 / 15:00-17:30)

Aims:

The aim of the course is the presentation in a didactic way of results concerning some issues that strongly relates stochastic processes and fractional calculus.The course will start with an introduction of the methods of the theory of Markov processes and asymptotic analysis for the study of the scaling limits of CTRW (continuous time random walks)and related fractional, in space and time, differential equations.

Further small Lévy-noise perturbations of dynamical systems will be analysed. In particularin the lectures it will be considered how the trajectories of a differentiable dynamical system given by an ODE behave under a perturbation by an additive noise term given by a Lévy process.

In the so obtained Lévy-driven SDE the coupling constant of the noise will be assumed to be small. A main question considered is what are mean exit times and typical exit paths the perturbed system is likely to follow if the initial point is inside a given domain containing a stable attractor of the original ODE. The lectures will proceed in a comparative way by first choosing the perturbation to be Gaussian (i.e., Brownian motion). Next it will be chosen to be a non-Gaussian stable process, and finally it will be discussed how the exit patterns and mechanisms differ in function of the heavy/light tail properties of the perturbing noise.

If time allows, it will be also discussed related metastability phenomena.

Finally, the fractional Poisson process and its applications are discussed. The fractional Poisson process is a counting renewal process that was independently introduced in the first half of the previous decade by various authors and has recently been further characterised. It generalises the Poisson process and is naturally related to time-fractional diffusion. It finds applications in several applied fields including biophysics, finance and meteorology.

Programme

Markov processes, semigroups and generators.
Convergence of Markov processes.
Convergence of scaled CTRW and their position dependent analogs.
Controlled CTRW and their scaling limits.
Stable Lévy motions, stable-like processes and fractionally stable laws.
The theory of Fractional Hamilton-Jacobi-Bellmann equations.
Large deviation technique.
Definition of the fractional Poisson process. Properties of the fractional Poisson process.
The compound fractional Poisson process. Relationship with time-fractional diffusion. Applications