

# Courses 2015-16

January 26-29, 2016, 09:00 -13:00 h (4 sessions, a total of 16 hours)

BCAM-Basque Center for Applied Mathematics, Bilbao, Basque Country, Spain [www.bcamath.org](http://www.bcamath.org)

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## CERTIFIED REDUCED BASIS METHODS FOR COMPUTATIONAL MECHANICS

The course aims to provide the basic aspects of numerical approximation and efficient certified solution of parametrized PDEs for computational mechanics problems (heat and mass transfer, linear elasticity, viscous and potential flows).

In this course we present certified reduced basis (RB) approximation and associated a posteriori error estimation for rapid and reliable solution of parametrized partial differential equations (PDEs). The focus is on rapidly convergent Galerkin approximations on a subspace spanned by "snapshots"; rigorous and sharp a posteriori error estimators for the outputs/quantities of interest; efficient selection of quasi-optimal samples in general parameter domains; and Offline-Online computational procedures for rapid calculation in the many-query and real-time contexts. We develop the RB methodology for a wide range of (coercive and non-coercive) elliptic and parabolic PDEs with several examples drawn from heat transfer, elasticity and fracture, acoustics, and fluid dynamics. We introduce the concept of affine and non-affine parametric dependence, some elements of approximation and algebraic stability. Finally, we consider application of RB techniques to parameter estimation, optimization, optimal control, and a comparison with other reduced order techniques, like Proper Orthogonal Decomposition.

Some tutorials are prepared for the course based on FEniCS and Python (and/or Matlab). Lecture notes, slides and reading material will be provided during the classes in a digital repository.

### PROGRAMME

- Introduction to RB methods, offline-online computing, elliptic coercive affine problems
- Sampling, greedy algorithm, POD
- A posteriori error bounds
- Primal-Dual Approximation
- Time dependent problems: POD-greedy sampling
- Non-coercive problems
- Approximation of coercivity and inf-sup parametrized constants
- Geometrical parametrization
- Reference worked problems (scalar problem heat conduction problem, vectorial problem linear elasticity problem, scalar transport problem, time dependent heat conduction, non-affine problem/operator).
- Examples of Applications in CFD

### REFERENCES

- [1] J.S. Hesthaven, G. Rozza, B. Stamm, *Reduced Basis Method for Parametrized Partial Differential Equations*, Springer Briefs in Mathematics, 2015.
- [2] G. Rozza, D.B.P. Huynh, and A.T. Patera, *Reduced Basis Approximation and A Posteriori Error Estimation for Affinely Parametrized Elliptic Coercive Partial Differential Equations — Application to Transport and Continuum Mechanics*. *Archives of Computational Methods in Engineering* 15(3):229–275, 2008.
- [3] A. Quarteroni, G. Rozza, and A. Manzoni, *Certified Reduced Basis Approximation for Parametrized Partial Differential Equations and Applications*. *Journal of Mathematics in Industry* 2011, 1:3 (3 June 2011, open access). <http://www.mathematicsinindustry.com/content/1/1/3>
- [3] *Learning OpenCV: Computer Vision with the OpenCV Library* By Gary Bradski, Adrian Kaehler Publisher: O'Reilly Media, 2008

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