

Courses 2017-18

January 29-February 2, 2018 (9:30 - 11:30)

(5 sessions, a total of 10 hours)

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WEAK SOLUTIONS FOR THE INCOMPRESSIBLE EULER EQUATIONS

In this course I will explain in detail the construction of infinitely many, compactly supported, L_2 weak solutions to the incompressible Euler equations following the theory developed by C. De Lellis and L. Székelyhidi in [2]. Such results relies on the technique of convex integration developed by M. Gromov in [4] in order to study $C_{0,\alpha}$ isometric embeddings of compact, closed Riemannian manifolds in Euclidean spaces of codimension greater or equal than one. Such toolbox of techniques seems to be rather efficient and flexible, and can be adapted to work in different settings. Another striking example of the application of convex integration to nonlinear PDEs is the work [6] of S. Müller and V. Šverák. Since the publication of [2] such technique has been studied in many different systems of vector conservation law obtaining weak nonunique solutions for large classes of hyperbolic nonlinear PDEs.

If the time will allow it I will as well explain how to construct weak continuous solutions for the incompressible Euler equations supported in $Td \times [0, 1]$ as it is done in [3]. Such result relies again formally on the concept of convex integration, but the techniques used differ substantially from the ones developed in [2], and have been lately perfected and refined up to the point to provide a complete proof of Onsager's conjecture ([1], [5]).

Prerequisites: Basic functional analysis.

Some lecture notes (in Italian) will be provided.

References:

- [1] Tristan Buckmaster, Camillo De Lellis, Philip Isett, and László Székelyhidi, Jr., Onsager's conjecture for admissible weak solutions, <https://arxiv.org/abs/1701.08678>.
- [2] Camillo De Lellis and László Székelyhidi, Jr., The Euler equations as a differential inclusion, *Ann. Of Math.* (2) 170 (2009), no. 3, 1417–1436.
- [3] _____, Dissipative continuous Euler flows, *Invent. Math.* 193 (2013), no. 2, 377–407.
- [4] Mikhael Gromov, Partial differential relations, *Ergebnisse der Mathematik und ihrer Grenzgebiete (3) [Results in Mathematics and Related Areas (3)]*, vol. 9, Springer-Verlag, Berlin, 1986.
- [5] Philip Isett, A proof of Onsager's conjecture, <https://arxiv.org/abs/1608.08301>.
- [6] S. Müller and V. Šverák, Convex integration for Lipschitz mappings and counterexamples to regularity, *Ann. of Math.* (2) 157 (2003), no. 3, 715–742. MR 1983780.

Registration is free, but **inscription is required before 24th January**: So as to inscribe send an e-mail to roldan@bcamath.org. Student grants are available. Please, let us know if you need support for travel and accommodation expenses