basque center for applied mathematics

Large time asymptotics for evolution problems: convergence to equilibrium, functional inequalities and hypocoercivity

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Description:

The goal of this course is to give an overview on *entropy methods* measuring the return to equilibrium of solutions of linear and nonlinear partial differential equations, or the convergence towards *intermediate asymptotics*.

1. In simple examples like the heat equation, the Poincaré inequality with Gaussian weight or the logarthmic Sobolev inequality can be used to describe the large time asymptotics of the solutions in various functional spaces. The *entropy/entropy production method*, or *Bakry-Emery method*, [1] and spectral approaches based on Beckner's work [2] will be presented.

2. The asymptotics of porous medium and fast diffusion equations will then be tackled, by the entropy/entropy production method [4], using functional inequalities of Gagliardo-Nirenberg type [5], gradient flow techniques based on mass transport [6, 12] and linearization of the entropy [3], which allow to give optimal asymptotic rates of convergence without restriction on the the exponent of the nonlinearity.

3. Some applications of entropy method to two-dimensional Navier-Stokes equations [10] or diffusive models like the Keller-Segel system will be briefly sketched [9]. Only preliminary results are available, as this topic is an active area of research.

4. All equations considered above are of diffusive type. Several of them can be achieved as *diffusive limits* of kinetic equations with various collision terms [7], which suggest how to establish *hypocoercivity* estimates [13, 14]. A special attention will be devoted to the case of time relaxation kernel [8, 11] for which hypocoercivity definitely differs from hypo-elliptic techniques.

References:

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