Control problems in quantum mechanics

Lecturer: Jean-Pierre Puel

Dates: 16–20 November 2009

Abstract:
Quantum mechanics present many specific difficulties, and among them are the questions related to the changes in the problems when one tries to observe the phenomena. Another question comes from the environment of the quantum system. Usually, one deals with an isolated quantum system whereas this is never the case and one has to take into account the environment of this quantum system.

It is of major interest to investigate the behavior of a quantum system under the effect of an external force, usually due to an external electric field. Trying to describe the possible behavior of the system leads to a control problem, the control variable acting through a potential in the equation, this potential coming from the external electric field. This question is important in quantum chemistry when one tries to break molecules or create new molecules. It is also important for physical reasons in order to drive a quantum system to a prescribed configuration. It could lead for example (in the future or is it only science fiction?) to quantum logical gates which could be a basis for future quantum computers. At the scale of a molecule or of an atom, in the non relativistic case, the basic equations to be considered are the Schrödinger equations. The environment can usually be described by microscopic or macroscopic Maxwell equations. Up to now, very few works have been done on the coupled Schrödinger-Maxwell systems and this could be a very important new subject for control theory. On the other hand, control of (isolated) Schrödinger equations has been extensively studied in the last years, but still many important questions remain unsolved.

In this series of lectures we will describe some of the problems arising in quantum control, including many open problems. We will also give the known results for the control of Schrödinger equations. The main properties of Schrödinger equations will be recalled, so that this series of lectures will be accessible to most of the people who are familiar with Hilbert spaces and possibly to Sobolev spaces.

Program:


2. Statement of some control problems for Schrödinger equation and coupled systems. Optimal control problems, controllability problems. Problems involving the so-called “density matrix”.

3. Basis of the mathematical study of Schrödinger equation. Conserved quantities, energy inequalities, existence results, regularity results.

5. Some open problems including discussions on the possible methods.

References:


