Numerical schemes for dispersive equations

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Abstract:

Dispersive equations have their origin within physics in subjects such as general relativity, quantum mechanics, water waves, nonlinear elasticity or various field theories. From a mathematical viewpoint, the numerical approximations of these equations is a challenging topic between the theory of Partial Differential Equations, Harmonic Analysis and Numerical Analysis.

In this series of lectures we shall describe some tools that can be used in the numerical approximation of these equations, and illustrate some possible directions of research and relevant applications, as those mentioned above. We shall mainly focus on numerical approximations of the nonlinear Schrödinger equation.

The course is addressed to researchers with a basic background in the theory of Partial Differential equations and numerical analysis by means of finite differences.

Program:

1. Fourier analysis tools in PDE and Numerical analysis
2. Well-posedness results for Nonlinear Schrödinger equation
3. Finite differences methods
4. Conservative schemes
5. Transparent boundary conditions
6. Splitting methods
7. Order of error

Bibliography:

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