basque center for applied mathematics

Finite Element Methods: Adaptivity, Multiphysics and Parallelization

Lecturers: David Pardo (BCAM), Maciej Paszyński[1]

Date: 1–5 March 2010

Abstract:
In this course we will: 1) explain the main advantages and limitations of the h-, p-, k-, and r-methods, 2) describe various grid-refinement techniques, including h-, p- and hp-adaptivity, and goal-oriented (weighted based residual) adaptive algorithms, 3) discuss the theory and implementation of how to build a finite element software capable of solving multiphysics problems (for example, acoustics coupled with electromagnetism), 4) review different parallelization options for a finite element software, 5) describe efficient solvers for large sparse systems of linear equations, and 6) introduce more advanced topics such as inversion methods using finite element computations, un-refinement techniques, advanced solvers, non-linear analysis, etc.

Program:

1. Introduction: h-, p-, k-, and r-Finite Element Methods.
2. Adaptivity: h-adaptivity, p-adaptivity, and goal-oriented adaptivity.
3. Multiphysics: The "de Rham" diagram.
4. MATLAB Demo: 1D Finite Element Software.
5. Parallel implementation of a Finite Element Method.
6. Inversion methods and un-refinement techniques.
7. Introduction to solvers of linear equations. Direct vs. Iterative Solvers. Main properties.
8. Direct Multifrontal Solvers. Proper ordering of the unknowns.
9. Iterative solvers and parallel implementation. Krylov subspace optimization methods, smoothers, and multigrid. Solvers integrated in the application, and goal-oriented solvers.
10. MATLAB Demo: solvers of spare systems of linear equations.

Bibliography:

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[1]Department of Computer Science. Faculty of Electrical Engineering, Automatics, Computer Science and Electronics. AGH University of Science and Technology. Al.Mickiewicz 30, 30-059 Kraków, Poland. Email: paszynsk@agh.edu.pl

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