Schrödinger operators and their spectra

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Abstract:

Spectral theory is an extremely rich field that has found its application in many areas of classical as well as modern physics. One of the reason that makes it so attractive on the formal level is that it provides a unifying framework for problems in various branches of mathematics, for example, partial differential equations, calculus of variations, geometry, stochastic analysis, etc.

The goal of the course is to give an overview of the methods used in the spectral theory of linear differential operators arising in non-relativistic quantum mechanics. We intend at the same time to recall classical results and present recent developments in the field, and we wish to always do it by providing a physical interpretation of the mathematical theorems.

The targeted audience is mainly pure and applied mathematicians, more particularly interested in partial differential equations and spectral theory. We hope to make the presentation accessible to students, too. A familiarity with functional analysis and basic spectral theory is assumed.

Programme:

1. *Qualitative features of the spectrum.* The discrete and essential spectra. The role of the dimension of the Euclidean space. Criticality versus subcriticality. The Hardy inequality.
2. *Weakly coupled bound states.* Analytic versus asymptotic perturbation theory. The Birman-Schwinger analysis. Asymptotic formulae for the weakly coupled eigenvalues.
3. *The semiclassical limit.* Strongly coupled bound states. Weyl-type asymptotics. The Lieb-Thirring inequalities.
4. *The nature of the essential spectrum.* The absolutely and singular continuous spectra, embedded eigenvalues. The limiting absorption principle. Mourre's theory.
5. *Geometrical aspects of spectral theory.* Glazman's classification of Euclidean domains and their basic spectral properties. Geometrically induced bound states and Hardy-type inequalities.

Bibliography:

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