Pricing and Hedging Derivative Securities

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Abstract:

Under the natural assumption of the absence of arbitrage opportunities, the theory of pricing and hedging derivative securities constitutes a fundamental tool in the world of quantitative financial engineering, and a such, is being widely used by financial companies throughout the world in continuously rebalancing their portfolios and managing their risk, trying to maximize their returns while maintaining a desired risk exposure.

This is an interdisciplinary theory, where applied mathematics, probability, statistics, computer science, econometrics and qualitative finance naturally come together to analyze and create financial models of use to the practitioners.

The absence of arbitrage assumption leads to the existence of the so-called risk neutral probabilities, which under the additional assumption of the existence of replicating portfolios make the markets complete, and therefore lead us to find the unique fair price of derivative securities and their hedging in both discrete and continuous time models.

There are three different ways to look at these problems in the continuous time framework that lead to the same answers: the PDE, the Probabilistic, and the Financial one. The tools that connect those three viewpoints are Stochastic Calculus/Ito's formula, the Feynman-Kac formula and the Martingale Representation Theorem together with Girsanov's theorem. This gives three different ways to attack these problems – sometimes they can be combined, or depending on the problem at hand, one will be more suited than the others.

Program:

1. No arbitrage hypothesis (complete vs. incomplete markets).
2. Risk neutral probabilities (pricing derivative securities).
3. Replicating portfolios (hedging derivative securities).
4. Stochastic differential equations, Black-Scholes PDE and relatives.
5. Stochastic optimal control, the dynamic programming principle and the Hamilton Jacobi Bellman (HJB) equation.
6. Jump processes.

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