Courses
2018-19

Entropy methods in PDE & Kinetic Theory

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12-16 November 2018 (5 sessions) I 09:00 - 11:00 (a total of 10 hours)

Lyapunov functionals are a very common tool in dynamical systems, used in order to study convergence to equilibrium of many systems. There are many partial differential equations in mathematical physics which have natural Lyapunov functionals that represent the entropy, energy, or free energy of a system. These PDE define infinite-dimensional dynamical systems and the application of these Lyapunov functionals in order to deduce any properties of the relaxation to equilibrium of these equations is often not straightforward.

In kinetic theory the so-called *entropy method* has been developed, which consists in studying equa- tions whose solution $u = u(t)$ satisfies

$$\frac{d}{dt} H\big(u(t)\big) \leq -D\big(u(t)\big),$$

where H and D are nonnegative functionals defined in whichever space the dynamical system $u(t)$ lives. Since D is nonnegative, the functional H is a Lyapunov functional for the system, and we can usually choose it so that it is equal to 0 at equilibrium. The entropy method essentially consists in looking for inequalities such as

$$\lambda H(u) \leq D(u)$$

for all suitable functions u. This is a functional inequality that directly implies exponential convergence to equilibrium for the associated PDE. These functional inequalities arise in many contexts: they are fundamental in the theory of Markov processes and involve spectral gap inequalities and (modified) logarithmic Sobolev inequalities, for example [1]. In kinetic theory, the application to the Boltzmann equation is known as *Cercignani's conjecture* [4]. In mathematical biology there are several recent applications using the *general relative entropy principle* [5].

The course aims to give a brief survey of these developments.

PROGRAMME:

1. Entropy and Lyapunov functionals. Introduction and examples.
2. The linear case
   a) Entropy for Markov processes
   b) The general relative entropy principle. Applications in mathematical biology

3. The nonlinear case
    a) Chemical reaction networks with detailed balance
    b) The Boltzmann equation
    c) Gradient ows

PREREQUISITES:
The course is intended for advanced graduate students and researchers. Some familiarity with partial differential equations is needed, especially with applied models in physics and mathematical biology.

REFERENCES:
[1] Dominique Bakry, Ivan Gentil, and Michel Ledoux. Analysis and Geometry of Markov Difusion Operators , volume 348 of Grundlehrender mathematischen Wissenschaften. Springer International Publishing, Cham, 2014. ISBN 978-3-319-00226-2.
[2] Canizo, J. A., & Lods, B. (2013). Exponential convergence to equilibrium for subcritical solutions of the Becker–Döring equations. Journal of Differential Equations, 255(5), 905-950.
[3] Carrillo, J. A., Jüngel, A., Markowich, P. A., Toscani, G., & Unterreiter, A. (2001). Entropy dissipation methods for degenerate parabolic problems and generalized Sobolev inequalities. Monatshefte für Mathematik, 133(1), 1-82.
[4] Laurent Desvillettes, Clément Mouhot, and Cédric Villani. Celebrating Cercignani's conjecture for the Boltzmann equation. Kinetic and Related Models, 4(1):277 294, January 2011. ISSN 1937-5093, arXiv:1009.4006.
[5] P. Michel, S. Mischler, and B. Perthame. General relative entropy inequality: an illustration on growth models. Journal de Mathématiques Pures et Appliqués, 84(9): 1235 1260, September 2005. ISSN 00217824.

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