Lecturer: Ron DeVore, Texas A&M University, Texas, USA

Webpages: http://www.math.tamu.edu/~rdevore/

Title: HIGH DIMENSIONAL APPROXIMATION: THEORY AND ALGORITHMS

Date and time: Mon, April 22 from 10:00 to 12:00
Tue, April 23 to Fri, April 26, 2013, 9:00 to 11:00

Abstract:

This course will study scientific problems that are challenged by the fact that they live naturally in a Euclidean space $\mathbb{R}^N$ with $N$ very large. We mention three settings which will be considered in this short course.

*Broad-banded signals:* It is well known that a bandlimited signal (function on $\mathbb{R}$) can be captured exactly from its equispaced time samples taken at the Nyquist rate. However when the bandwidth of the signal is large then the sampling rate cannot be implemented accurately in circuitry. This leads us to consider other models for signals which are still realistic but enable capturing the signal with much fewer measurements. We shall study one such setting called Compressed Sensing (CS) which models signals as having a sparse or compressible representation in a suitably chosen basis of waveforms. We shall develop the mathematical foundations of compressed sensing culminating with a derivation of the optimal rate/distortion that can be achieved with CS and a discussion of the encoder/decoders which achieve this optimality.

*Mathematical learning:* The mathematical theory of data fitting in a stochastic setting is known as learning. Many interesting learning problems are set in high dimension and must engage the problem of approximating functions of a large number of variables. Classical approaches of approximation in several variables suffer from the curse of dimensionality: the approximation rate severely suffers as the dimension increases. We shall consider possible ways to avoid the curse of dimensionality through variable reduction and sparsity.

*Stochastic PDEs:* The classical approach to solving stochastic PDEs is Monte Carlo methods. While these methods are robust they have severe limits in terms of their rate/distortion bounds. We shall study alternatives to Monte Carlo methods which utilize Wiener chaos expansions to convert the stochastic PDE to a deterministic parametric PDE. The number of parameters in this approach is infinite and therefore its success depends capturing a function of an infinite number of variables in a numerically realistic way. We shall derive properties of the solutions to such parametric problems in the elliptic case that show these solutions have highly accurate sparse approximations. We shall then derive potential numerical approaches to capturing these sparse approximants.
Theory and algorithms for high dimensional problems are in their infancy and so this course will expose many interesting open questions.

**Bibliography:**


7. [A. Cohen, R. DeVore, C. Schwab, Convergence rates of best n term Galerkin approximations for a class of elliptic SPDEs, J. FOCM, 10 (2010), 615-646.]


