Lecture 2. Stochastic multistage NEP: Strategic scenario trees, multi-period Tactical scenario graphs and Operational two-stage trees. Scenario reduction. Risk Neutral

Laureano F. Escudero
Universidad Rey Juan Carlos (URJC), Móstoles (Madrid), laureano.escudero@urjc.es
Lecture 1. Introduction to Network Expansion Planning. Deterministic version.


Lecture 3. Risk averse policies in stochastic optimization.

Lecture 4. Stochastic-Equilibrium in Network Expansion Planning under Uncertainty, SE-NEP.

Lecture 5. Matheuristic Nested Stochastic Decomposition.
Multistage main concepts
Non-Anticipativity principle
Strategic multistage scenario trees.
Tactical multi-period scenario graphs rooted in strategic nodes. Application
Operational scenario two-stage trees rooted in strategic nodes. Application
Risk neutral modeling
Typologies of decomposition algos for problem solving
Our experience on large-scale stochastic optimization problems under uncertainty
Our papers on mixed 0-1 optimization under uncertainty, SMIP
Some computational experience
Deterministic mixed 0-1 optimization model

\[ z_{EV} = \max \sum_{t \in T} (a_t x_t + b_t y_t) \]

s.t. \[ \sum_{t' \in A_t} (A_{t'}^t x_{t'} + B_{t'}^t y_{t'}) = h_t \quad \forall t \in T \] (1)

\[ x_t \in \{0, 1\}^{n_x(t)}, \quad y_t \in \mathbb{R}^{+n_y(t)} \quad \forall t \in T. \]
Math optim under uncertainty

Multistage scenario tree

- A **stage** of a given horizon is a set of consecutive periods where the realization of the uncertain parameters takes place.
- A **scenario** is a realization of the uncertain parameters along the stages of a given horizon.
- A **node** for a given stage in the scenario tree has one-to-one correspondence with the group of scenarios that have the same realization of the uncertain parameters up to the stage.
- **Nonanticipativity principle**: The scenarios of a group have a unique solution for the stage which correspondent node belongs to.
Figura: Multistage nonsymmetric scenario tree

\(\Omega = \Omega^1 = \{10, 11, \ldots, 17\}; \quad \Omega^2 = \{10, 11, 12\}\)

\(\mathcal{N} = \{1, \ldots, 17\}; \quad \mathcal{N}_3 = \{5, 6, 7, 8, 9\}, \quad t^g = 3\)

\(\mathcal{A}^g = \{1, 4, 9\}, \quad \sigma^g = 4, \quad S^4 = \{8, 9, 15, 16, 17\}\)
Strategic multistage scenario tree. Notation

\( \mathcal{E} \), set of the lexicographically ordered stages along the horizon. Note: \( E = |\mathcal{E}| \) and \( \mathcal{E} = \{1, \ldots, E\} \).

\( \Omega \), set of scenarios.

\( \mathcal{N} \), set of the lexicographically ordered nodes in the tree. Note: \( N = |\mathcal{N}| \) and \( \mathcal{N} = \{1, \ldots, N\} \)

\( \Omega^n \subseteq \Omega \), set of scenarios in a group with one-to-one correspondence with node \( n \), for \( n \in \mathcal{N} \).

\( \mathcal{A}^n \), ancestor nodes to node \( n \) (including itself), for \( n \in \mathcal{N} \).

\( \mathcal{S}^n \), successor nodes of node \( n \) in the tree, for \( n \in \mathcal{N} \).

\( \mathcal{S}^n = \emptyset \), for \( n \in \mathcal{N}_E \).

\( w^\omega \), weight or probability assigned to scenario \( \omega \in \Omega \). Note: \( \sum_{\omega \in \Omega} w^\omega = 1 \)

\( w^n \), weight or probability assigned to node \( n \in \mathcal{N} \). Note: \( w^n = \sum_{\omega \in \Omega^n} w^\omega \).
Max objective function expected value over scenarios

\[ z_{RN} = \max \sum_{n \in N} w^n (a^n x^n + b^n y^n) \]

s.t. \( \sum_{n' \in A_n} (A_{n'} x^{n'} + B_{n'} y^{n'}) = h^n \quad \forall n \in N \quad (2) \)

\[ x^n \in \{0, 1\}^{nx(n)}, \quad y^n \in \mathbb{R}^{+ny(n)} \quad \forall n \in N. \]
Network Expansion Planning (NEP) along a time horizon: Deciding on location and timing of investment on network infrastructure elements.

Fields: SCM, RTND, TGEP, Production Planning Management (PPM), Fleet transportation investment, Forestry, DISMIT, FFM, etc.

In general: Cases where there is not too-much sense to consider strategic nodes as successor ones from operational nodes in multistage scenario trees.

That is, strategic decisions are not based on punctual operational decisions at the periods, besides the gigantic stochastic model that would result.
Strategic and Tactical / Operational math optim under uncertainty. Illustrative sectors

- RTND:
  *Strategic uncertain parameters*: investment costs, passenger demand levels.
  *Operational uncertain parameters*: passenger demand, network infrastructure elements’ disruption, tactical cost, local resources capacity.
  See Cadarso, LFE & Marín EJOR’18.

- Forestry:
  *Strategic uncertain parameters*: production yield of stand for harvesting (road building/upgrade investment costs).
  *Tactical uncertain parameters*: Product price & demand (harvesting, storing & market transportation capacities, tactical costs).
  See Alonso-Ayuso, LFE, Guignard & Weintraub, Sub’18.
Rationale behind partition of Strategic and Tactical / Operational uncertain parameters

- Individual Tactical / Operational uncertain parameters don’t influence on future strategic decisions.
- They just have influence on the related tactical / operational decisions for the given period in its related strategic node and,
- as a scenario set, they do influence on the strategic decisions to be made in the strategic node which the set belongs to, and on the ones in the related ancestor nodes.
- Two types of representing the Tactical / Operational uncertainty:
  - Operational two-stage tree, each one rooted with a strategic node.
  - Tactical graphs, each one rooted with a strategic node.

See LFE & Monge CMS’18.
A set of Operational scenarios can be represented in a two-stage tree, see:

**Kaut, Midthun, Werner, Tomasgard, Hellemo & Fodstad CMS’14;**

**Werner, Pichler, Midthun, Hellemo & Tomasgard, chap 8 edited book 2013;**

**Cadarso, LFE, & Marín EJOR’18:**

Field: Operational activities: Independent period services.

Example: Operations in RTND, etc.
Figura: Strategic multistage scenario tree

\[ s = 1 \quad s = 2 \quad s = 3 \]

\[ \Omega = \Omega^1 = \{4, 5, 6, 7\}; \Omega^2 = \{4, 5\} \quad N = \{1, \ldots, 7\}; \quad N^2 = \{2, 3\} \]

\[ A^3 = \{1, 3\}, \quad S^2 = \{4, 5\} \]
Figura: Strategic multistage scenario tree with Operational two-stage scenario trees rooted with strategic nodes

$\Pi^3 = \{a, b, c, d\}$
A set of Tactical scenarios is represented in a special multi-period graph to be rooted with replicas of the related strategic node.

Field: Inter-period influencing tactical activities: Inter-period ’material’ activities.


Raw material, subassemblies, end-products, commodity supplying stored in SCM and PPM, see LFE & Monge CMS’18

Forest harvesting and road construction/upgrading, see Alonso-Ayuso, LFE, Guignard & Weintraub, Submitted’18.

etc.
Stage $e = 1$, Stage $e = 2$, Stage $e = 3$.

$E = \{1, 2, 3\}$, $T = \{1, \ldots, 9\}$, $T_2 = \{4, 5, 6\}$, $\Omega = \{4, \ldots, 9\}$, $\Omega^3 = \{8, 9\}$.

$G = \{0, \ldots, 9\}$, $Q = \{01, \ldots, 99\}$, $Q_2 = \{21, \ldots, 29\}$, $Q_2^5 = \{24, 25, 26\}$, $L^2 = \{27, 28, 29\}$.

$e(0) = 1, e(3) = 2, \sigma(2) = 0$, $A^2 = \{0, 2\}$, $A^3_2 = \{21, 24, 27\}$.

Figura: Strategic multistage scenario tree with tactical multi-period graphs rooted with strategic nodes.
Strategic multistage scenario tree. Sets

- $\mathcal{E}$, lexicographically ordered stages along the horizon.
  \[ E = |\mathcal{E}|, \mathcal{E} = \{1, \ldots, E\}. \]

- $\Omega$, scenarios.

- $\mathcal{N}$, lexicographically ordered strategic nodes in the tree.
  \[ N = |\mathcal{N}|, \mathcal{N} = \{1, \ldots, N\} \]

- $\Omega^n \subseteq \Omega$, scenarios in a group with one-to-one correspondence with node $n$, for $n \in \mathcal{N}$.

- $\mathcal{N}_e \subset \mathcal{N}$, nodes that belong to stage $e$, for $e \in \mathcal{E}$.
  \[ |\mathcal{N}_1| = 1. \]

- $\mathcal{T}_e$, periods (e.g., years, semesters) in stage $e$, for $e \in \mathcal{E}$.

- $\mathcal{T}$, periods in the time horizon.
  \[ \mathcal{T} = \bigcup_{e \in \mathcal{E}} \mathcal{T}_e, \; T = |\mathcal{T}|. \]
\( A^n \), ancestor nodes to node \( n \) (including itself), for \( n \in \mathcal{N} \).

\( S^n \), successor nodes of node \( n \) in the tree, for \( n \in \mathcal{N} \).

\( S^n = \emptyset \), for \( n \in \mathcal{N}_E \).

\( S^n_1 \in S^n \), immediate successor nodes of node \( n \) in the tree, for \( n \in \mathcal{N} \).
\( e^n \), stage which node \( n \) belongs to, for \( n \in \mathcal{N} \).

\( \sigma^n \), immediate ancestor node to node \( n \).

\( \sigma^n \in \mathcal{A}^n \), for \( n \in \mathcal{N} \), \( \sigma(0) = \text{null} \).

\( t^n \), period in set \( \mathcal{T}_e \) of stage \( e^n \) which node \( n \) belongs to, for \( n \in \mathcal{N} \).

It is the first period in set \( \mathcal{T}_e \).

\( \bar{t}_e \), last period in set \( \mathcal{T}_e \).

\( w^\omega \), weight or probability assigned to scenario \( \omega \in \Omega \).

\( w^n \), weight assigned to node \( n \).

\( w^n = \sum_{\omega \in \Omega^n} w^\omega \).
Network infrastructure to be contracted. Sets

\[ I, \text{ infrastructure elements.} \]

\[ I^l, \text{ subset of infrastructure elements in set } I, \text{ where the value } 1 \text{ of the } 0-1 \text{ step variable } (x^n)_i; \text{ means that it starts to be built by period } t^n \text{ in strategic node } n, \text{ for } i \in I^l, n \in N. \]

Note: The capacity is a data, \( \bar{x}_i \).

Illustrative examples:

- power generation units as thermal plants and hydropower turbines in energy generation
- cables in electricity transmission lines
- stations and hedges in rapid transit networks
- road building in Forestry
- production units, machines, transportation means, sorting and market centers in supply chains; and others.
Network infrastructure to be contracted. Sets (c)

\( \mathcal{I}^C \), subset of infrastructure elements in set \( \mathcal{I} \), where:

- \((y^n)_i\), *continuous step variable* denoting the initial capacity of element \( i \) that is started to be built by period \( t^n \) in strategic node \( n \)

- \((y^{a,n})_i\), *continuous step variable* denoting the capacity increase by period \( t^n \) in strategic node \( n \)

- \( \overline{y}_i \), upper bound capacity

- **Illustrative examples:**
  - Intermittent power generation types as wind, solar and photovoltaic farms, among others.
  - Warehouses with variable capacity, etc.

Note: \( \mathcal{I} = \mathcal{I}^l \cup \mathcal{I}^C \)
\( \mathcal{I}_i \subset \mathcal{I} \), set of infrastructure elements whose construction cannot start until element \( i \) is available (i.e. its construction is over), for \( i \in \mathcal{I} \).
Tactical / Operational elements as activities in the network. Set $\mathcal{J}$

Illustrative examples:

- energy generation in the plants
- energy flow though a cable of a transmission line
- passengers transportation flow from one station to another in a rapid transit network.
- production in a factory, plant or unit
- water stored in reservoirs in a hydro-power system
- raw material, subassemblies and end-products, commodities stored in warehouses
- product production, transportation, storing and market sending, and others.

$\mathcal{I}_j \subset \mathcal{I}$, subset infrastructure elements that should be available when Tactical / Operational element $j$ is active, for $j \in \mathcal{J}$. 
For strategic node $n \in \mathcal{N}$:

Elements of Tactical scenario graph

$\mathcal{Q}^n$, (tactical) node set in the tactical scenario graph rooted with node $n$.

$\mathcal{Q}^n_t$, (tactical) node set that belongs to period $t$, for $t \in \mathcal{T}_e$ such that $\mathcal{Q}^n = \bigcup_{t \in \mathcal{T}_e} \mathcal{Q}^n_t$, for $e \equiv e^n$.

Note: Strategic node $n$ is not in $\mathcal{Q}^n$, but its replicas $\{q\}$ do it, for $q \in \mathcal{Q}^n_{tn}$; notice that $t_q = t^n$.

$t_q$, period which tactical node $q$ belongs to, for $q \in \mathcal{Q}^n$. 
Stage $e = 1$ Stage $e = 2$ Stage $e = 3$

$E = \{1, 2, 3\}, T = \{1, \ldots, 9\}, T_2 = \{4, 5, 6\}, \Omega = \{4, \ldots, 9\}, \Omega_3 = \{8, 9\}$

$G = \{0, \ldots, 9\}, Q = \{01, \ldots, 99\}, Q_2 = \{21, \ldots, 29\}, Q_2^3 = \{24, 25, 26\}, L^2 = \{27, 28, 29\}$

$e(0) = 1, e(3) = 2, \sigma(2) = 0, A^2 = \{0, 2\}, \tilde{A}_2^T = \{21, 24, 27\}$

Figura: Strategic multistage scenario tree with tactical multi-period graphs rooted with strategic nodes
For strategic node $n \in \mathcal{N}$:

Elements of Tactical scenario graph (c)

$\mathcal{L}^n$, leaf node set in the tactical graph for strategic node $n$. Note: $\mathcal{L}^n \equiv Q_t^n$ where $t \equiv t_e$, $e \equiv e^n$.

$\sigma_q$, immediate (tactical) ancestor to tactical node $q$, for $q \in Q^n$ in the graph. Note: $\sigma_q$=null for $t_q = t^n$.

$\mathcal{A}_\ell$, ancestor tactical nodes to tactical leaf node $\ell$ (including itself) in the tactical graph for strategic node $n$, for $\ell \in \mathcal{L}^n$, $n \in \mathcal{N}$.

That is, $\mathcal{A}_\ell$ gives the tactical scenario, for short so-named scenario $\ell$.

$w_\ell$, weight of tactical scenario associated to the H-path from tactical node $q$ to leaf tactical node $\ell$, for $q \in Q^n : t_q = t^n$ and $\ell \in \mathcal{L}^n$, such that $\sum_{\ell \in \mathcal{L}^n} w_\ell = 1$. 
For network infrastructure element $i$: latency $\tau_i$ for $i \in \mathcal{I}$

- Number of periods that are required between the period when the construction starts for infrastructure element $i$ (e.g., an edge as a connection of two nodes in the network, a station in a node, a plant, a production unit, a warehouse, etc.) and the period at which it becomes available.

Note: $\tau_i = 0$ means that element $i$ is available since the same period where its construction starts.
For strategic node $n \in \mathcal{N}$:

$\nu(t')_i$, latency-related ancestor strategic node for infrastructure element $i \in \mathcal{I}$

- Strategic node whose period $t^{\nu(t')}_i$ is the latest one by where element $i$ can start its construction, so that it is available for use in the network at period $t'$, where $t' \in \mathcal{T}_e$, $e \in \mathcal{E}$, for the tactical nodes $\{q\}$, such that

$q \in Q^n : t_q = t'$, where $n \in \mathcal{N}_e \cap S^{\nu(t')}_i$.

$\nu(t')_i = \arg\max_{n' \in \mathcal{A}^n} \{t^{n'} \in \mathcal{T} : t^{n'} \leq t - \tau_i\}$. 
Illustrative case for strategic multistage scenario tree with Operational two-stage subtrees rooted with strategic nodes

RTND: Rapid Transport Network Design problem

- Symmetric strategic scenario tree:
  \(|\mathcal{N}_1| = 1, \mathcal{N}_2| = 3, \mathcal{N}_3| = 9, \mathcal{N}_4| = 27, \mathcal{N}| = 40.\)

- \(|Q^n| = 8\) operational scenarios per each strategic node \(n\).

So, in total, there are 320 operational situations for representing the uncertainty to be dealt with, being partitioned in 40 groups.

- It means that there are 320 operational RTND submodels in the full strategic-operational one, see below.
\( s = 1 \quad s = 2 \quad s = 3 \)

\[
\begin{align*}
\Omega = \Omega^1 &= \{4, 5, 6, 7\}; \quad \Omega^2 = \{4, 5\} \quad N = \{1, \ldots, 7\}; \quad N^2 = \{2, 3\} \\
A^3 &= \{1, 3\}, \quad S^2 = \{4, 5\}
\end{align*}
\]

**Figura:** Strategic multistage scenario tree
Figura: Strategic multistage scenario tree with Operational two-stage scenario trees rooted with strategic nodes
FORESTRY: integrating strategic and tactical decision levels in forestry management under uncertainty.

Alternatives:
- Tactical graph is a multi-period two-stage tree.
- Tactical graph is a multi-period multistage tree.
Stage $e = 1$ Stage $e = 2$ Stage $e = 3$

$E = \{1, 2, 3\}$, $T = \{1, \ldots, 9\}$, $T_2 = \{4, 5, 6\}$, $\Omega = \{4, \ldots, 9\}$, $\Omega^3 = \{8, 9\}$

$G = \{0, \ldots, 9\}$, $Q = \{01, \ldots, 99\}$, $Q_2 = \{21, \ldots, 29\}$, $Q^2_2 = \{24, 25, 26\}$, $L^2 = \{27, 28, 29\}$

$e(0) = 1, e(3) = 2$, $\sigma(2) = 0$, $A^2 = \{0, 2\}$, $\tilde{A}^2_2 = \{21, 24, 27\}$

**Figura:** Strategic multistage scenario tree with tactical multi-period graphs rooted with strategic nodes
Illustrative case for strategic multistage scenario tree with Tactical multi-period two-stage subtrees rooted with strategic nodes

- Two period connected nodes in each stage.
- Symmetric strategic scenario tree:
  \[ |\mathcal{N}_1| = 1, |\mathcal{N}_2| = 12, |\mathcal{N}_3| = 72, |\mathcal{N}| = 85. \]
  *Uncertain strategic parameters*: production yield of stands for harvesting.
  *Strategic decisions*: stand selection for harvesting and road building/upgrade.

- \(|Q^n| = 8\) tactical scenarios for each strategic node \(n\):
  *Uncertain tactical parameters*: Product price & demand
  *Tactical decisions*: Product harvesting, network flow, product **storing** and market transportation operations.

- So, in total, there are 2 period \(85 \times 8 = 680\) tactical situations for representing the uncertainty to be dealt with, being partitioned in 85 groups.
Strategic decisions: $(x^n)_i$ -variable in mixed 0-1 model, for $n \in \mathcal{N} : t^n \in \mathcal{T} : t^n \leq T - \tau_i$, $i \in \mathcal{I}$

- **0-1 step variable** for infrastructure element $i$ in node $n$.
- Its value is 1 if element $i$ starts its construction by period $t^n$ and otherwise, 0.
- Let $x^n$ be the $|\mathcal{I}|$-dimensional vector.
- $x^n$ and $\bar{x}$: vectors of the variables $(x^n)_i$ and capacity bounds $\bar{x}_i$, res., for $i \in \mathcal{I}$
- Note: $(x^n)_i - (x^\sigma)_i = 1$ means that element $i$ starts its construction at node $n$. 

Laureano F. Escudero Universidad Rey Juan Carlos (URJC), Móstoles (Madrid), laureano.escudero@urjc.es

SMIP-NEP
Strategic decisions: \((y^n)_i\)-variable in mixed 0-1 model, for \(n \in \mathcal{N}: t^n \in \mathcal{T}: t^n \leq T - \tau_i, i \in \mathcal{I}^C\)

- **Continuous step variable** for infrastructure element \(i\) in node \(n\).
- Its positive value means that element \(i\) started the construction of its initial capacity \((y^n)_i\) by period \(t^n\) and otherwise, 0.
- \(y_i\) and \(\bar{y}_i\), conditional lower and upper bounds of capacity \((y^n)_i\), resp.

**Note 1:** \((y^n)_i - (y^\sigma)_i > 0\) means that element \(i\) starts its construction at node \(n\) (and, so, \((y^\sigma)_i = 0\)) and otherwise, 0.

**Note 2:** \((y^\sigma)_i > 0\) means that \((y^n)_i = (y^\sigma)_i\) and otherwise, 0.

- \(y^n\) and \(\bar{y}^n\): vectors of the variables \((y^n)_i\) and capacity bounds \(\bar{y}_i\), resp., for \(i \in \mathcal{I}^C\)
0-1 step variable for infrastructure element $i$ in node $n$. Its value 1 means that element $i$ started the construction of its initial capacity $(y^n)_i$ by period $t^n$ and otherwise, 0.

Remember data: $y_i$, conditional lower bound of the initial capacity of element $i$.

\[
(\delta^n)_i \in \{0, 1\}, \quad (\delta^n)^\sigma_i \leq (\delta^n)_i, \\
0 \leq (y^n)^\sigma_i \leq (y^n)_i \leq \overline{y}_i \cdot (\delta^n)_i, \\
y_i \cdot ((\delta^n)_i - (\delta^n)^\sigma_i) \leq (y^n)_i - (y^n)^\sigma_i \leq \overline{y}_i \cdot ((\delta^n)_i - (\delta^n)^\sigma_i).
\]
Strategic decisions: \((y^{a,n})_i\)-variable in mixed 0-1 model, for \(n \in \mathcal{N} : t^n \in \mathcal{T} : t^n \leq T - \tau_i, \ i \in \mathcal{I}^C\)

- **Continuous step variable** for infrastructure element \(i\) in node \(n\).
- \((y^{a,n})_i - (y^{a,\sigma^n})_i > 0\) means that element \(i\) started the construction of the capacity increase \((y^{a,n})_i\) by period \(t^n\) and, then, \((y^{a,\sigma^n})_i = 0\),

otherwise \((y^{a,n})_i - (y^{a,\sigma^n})_i = 0\) (i.e., either it already started or it did not start it yet).
- \(y^a_i\), lower bound of each capacity increase \((y^{a,n})_i\).
- \(y^{a,n}\), vector of the variables \((y^{a,n})_i\), for \(i \in \mathcal{I}^C\)
Strategic decisions: \( (\delta^{a,n})_i \)-instrumental variable in mixed 0-1 model, for 
\( n \in \mathcal{N} : t^n \in \mathcal{T} : t^n \leq T - \tau_i, \ i \in \mathcal{I}^C \)

0-1 step variable for infrastructure element \( i \) in node \( n \).

Its value 1 means that element \( i \) started the construction of the capacity increase \( (y^{a,n})_i - (y^{a,\sigma^n})_i > 0 \) by period \( t^n \) and otherwise, 0.

\[
(\delta^{a,n})_i \in \{0, 1\}, \ (\delta^{a,\sigma^n})_i \leq (\delta^{a,n})_i \leq (\delta^{\sigma^n})_i \\
0 \leq (y^{a,\sigma^n})_i \leq (y^{a,n})_i \leq \bar{y}_i - \underline{y}_i \cdot (\delta^n)_i, \\
y^a_i \cdot ((\delta^{a,n})_i - (\delta^{a,\sigma^n})_i) \leq (y^{a,n})_i - (y^{a,\sigma^n})_i \leq (\bar{y}_i - \underline{y}_i) \cdot ((\delta^{a,n})_i - (\delta^{a,\sigma^n})_i) \\
\]

Note: Available capacity by node \( n \): \( (y^n)_i + (y^{a,n})_i \leq \bar{y}_i \).
Strategic multistage tactical multi-period graphs stochastic mixed 0-1 model. $z$-tactical variables

$$(z_q)_j, \text{ for } q \in Q^n, \quad n \in N, \quad j \in J.$$ 

- **continuous impulse variable** for activity $j$ at tactical node $q$.

Remark: Any impulse variable $(z_q)_j$ cannot have a value greater than zero if any related step variable $(x^i(t_q))_j = 0$ for $i \in I^l \cap I^j$ and $(\delta^i(t_q))_j = 0$ for $i \in I^c \cap I^j$.

- $z_j, (\bar{z}_q)_j$, conditional lower and upper bounds of activity at tactical node $q$. resp.

- $z_q$, vector of the variables $(z_q)_j$ for $j \in J$. 

Laureano F. Escudero Universidad Rey Juan Carlos (URJC), Móstoles (Madrid), laureano.escudero@urjc.es

SMIP-NEP
Other parameters of the synthesized RN model for strategic node $n \in \mathcal{N}$

- $a^n$, $c^n$ and $d^n$, vector of objective function coeffs for the strategic variables in vectors $x^n$, $y^n$ and $y^{a,n}$, resp.
- $A^n_n$, $C^n_n$ and $D^n_n$, constraint matrix of the strategic variables in vector $x^n$, $y^n$ and $y^{a,n}$ of the ancestor strategic node $n'$ in the constraints of strategic node $n$, for $n' \in \tilde{A}^n$.
- $h^n$, rhs for the constraints.
Other parameters of the synthesized RN model for tactical node $q$ in strategic node $n$, for $q \in Q^n$, $n \in N$

- $b_q$, vector of objective function coeffs for the tactical variables in vector $z_q$.
- $B_{\ell}$, constraint matrix of the variables in leaf (tactical) vector $y_{\ell}$ of the tactical graph rooted with the immediate ancestor strategic node in the constraints of tactical node $q$ that belongs to the graph rooted with strategic node $n$, for $\ell \in \mathcal{L}_n^\sigma$.

So, node $q$ for $q : t_q = t^n$ (i.e., first tactical node of the graph).

- $B_{\sigma q}^q$, constraint matrix of the variables in (tactical) vector $z_{\sigma q}$ in the constraints of tactical node $q$, such that $t_q > t^n$.
- $B_q^q$, constraint matrix of the variables in (tactical) vector $z_q$ in the own constraints of tactical node $q$.
- $h_q$, rhs of the constraints of tactical node $q$.

Remember that $Q^n = \bigcup_{t \in T_n} Q^n_t$. 
DEM 0-1 RN model. Objective function

\[
\min \sum_{i \in I} \sum_{n \in N : t^n \leq T - \tau_i} \frac{1}{(1 + \rho_{tn}) t^n} w^n \\
\left[ (a^n(x^n - x^{\sigma^n})) : i \in I^l + (c^n(y^n - y^{\sigma^n}) + d^n(y^{a,n} - y^{a,\sigma^n})) : i \in I^c \right] \\
+ \sum_{n \in N} w^n \sum_{\ell \in L^n} w_\ell \sum_{q \in A_\ell} \frac{1}{(1 + \rho_{tq}) t_q} b_q z_q, \quad (3)
\]

where \( \rho_{tn} \) is the interest rate in period \( t^n \).
DEM 0-1 RN model. Strategic constraints for $x$-variable defining

$$(x^n)_i \in \{0, 1\}, \ (x^\sigma^n)_i \leq (x^n)_i,$$

$$\forall n \in \mathcal{N}: t^n \leq T - \tau_i, \ i \in \mathcal{I}^l \quad (4)$$

$$(x^n)_{i'} - (x^\sigma^n)_{i'} \leq (x^{\iota(t^n)}_i)$$

$$\forall n \in \mathcal{N}: t^n \leq T - \tau_{i'}, \ i' \in \mathcal{I}_i, \ i \in \mathcal{I}^l \quad (5)$$
DEM 0-1 RN model. Strategic constraints for $y$, $\delta$-variables relationship defining

\[
(\delta^n)_i \in \{0, 1\}, \ (\delta^{\sigma^n})_i \leq (\delta^n)_i,
0 \leq (y^{\sigma^n})_i \leq (y^n)_i \leq \bar{y}_i \cdot (\delta^n)_i,
\bar{y}_i \cdot ((\delta^n)_i - (\delta^{\sigma^n})_i) \leq (y^n)_i - (y^{\sigma^n})_i \leq (\bar{y}_i - \underline{y}_i) \cdot ((\delta^n)_i - (\delta^{\sigma^n})_i)
\]

\[\forall n \in \mathcal{N} : t^n \leq T - \tau_i, \ i \in \mathcal{I}^C \tag{6}\]

\[
(\delta^{a,n})_i \in \{0, 1\}, \ (\delta^{a,\sigma^n})_i \leq (\delta^{a,n})_i \leq (\delta^{\sigma^n})_i,
0 \leq (y^{a,\sigma^n})_i \leq (y^{a,n})_i \leq (\bar{y}_i - \underline{y}_i) \cdot (\delta^n)_i,
y^a_i \cdot ((\delta^{a,n})_i - (\delta^{a,\sigma^n})_i) \leq (y^{a,n})_i - (y^{a,\sigma^n})_i \leq
(\bar{y}_i - \underline{y}_i) \cdot ((\delta^{a,n})_i - (\delta^{a,\sigma^n})_i),
\]

\[\forall n \in \mathcal{N} : t^n \leq T - \tau_i, \ i \in \mathcal{I}^C \tag{7}\]

\[
y_{i'} \cdot (\delta^{\iota(t^n)}_i)_i \leq (y^n)_{i'} - (y^{\sigma^n})_{i'} \leq (\bar{y}_{i'} - \underline{y}_{i'}) \cdot (\delta^{\iota(t^n)}_i)_i
\]

\[\forall n \in \mathcal{N} : t^n \leq T - \tau_{i'}, \ i' \in \mathcal{I}_i, \ i \in \mathcal{I}^C \tag{8}\]
DEM 0-1 RN model. Strategic constraints for $y, \delta$-variables relationship defining.
Initial capacity disclosure: Constraint system (6)

$$(\delta^n)_i \in \{0, 1\}, \quad (\delta^{\sigma^n})_i \leq (\delta^n)_i,$$

$$0 \leq (y^{\sigma^n})_i \leq (y^n)_i \leq \bar{y}_i \cdot (\delta^n)_i,$$

$$y_i \cdot ((\delta^n)_i - (\delta^{\sigma^n})_i) \leq (y^n)_i - (y^{\sigma^n})_i \leq (\bar{y}_i - y_i) \cdot ((\delta^n)_i - (\delta^{\sigma^n})_i)$$

$$\forall n \in \mathcal{N} : t^n \leq T - \tau_i, \quad i \in \mathcal{I}^C$$
DEM 0-1 RN model. Strategic constraints for \( y, \delta \)-variables relationship defining. Capacity increase disclosure: Constraint system (7)

\[
(\delta^{a,n})_i \in \{0, 1\}, \quad (\delta^{a,\sigma^n})_i \leq (\delta^{a,n})_i \leq (\delta^{\sigma^n})_i,
0 \leq (y^{a,\sigma^n})_i \leq (y^{a,n})_i \leq (\overline{y}_i - \underline{y}_i) \cdot (\delta^n)_i,
\]

\[
y^a_i \cdot ((\delta^{a,n})_i - (\delta^{a,\sigma^n})_i) \leq (y^{a,n})_i - (y^{a,\sigma^n})_i \leq (\overline{y}_i - \underline{y}_i) \cdot ((\delta^{a,n})_i - (\delta^{a,\sigma^n})_i),
\]

\[
\forall n \in \mathcal{N} : t^n \leq T - \tau_i, \quad i \in \mathcal{I}^C
\]
DEM 0-1 RN model. Strategic constraints for $y, \delta$-variables relationship defining. Precedence relationships disclosure: Constraint system (8)

$$y_{i'} \cdot (\delta(t^n)_i) \leq (y^n)_{i'} - (y^\sigma)_{i'} \leq (\bar{y}_{i'} - \underline{y}_{i'}) \cdot (\delta(t^n)_i) \leq (\bar{y}_{i'} - \underline{y}_{i'}) \cdot (\delta(t^n)_i)$$

$$\forall n \in N : t^n \leq T - \tau_{i'}, i' \in \mathcal{I}_i, i \in \mathcal{I}^C$$
DEM 0-1 RN model. Strategic constraints for $x, y$-variables relationship defining

$$\sum_{n' \in \tilde{A}^n} (A_n^{n'} x^{n'} + C_n^{n'} y^{n'} + D_n^{n'} y^{a,n'}) = h^n \quad \forall n \in \mathcal{N} \quad (9)$$
Constraints linking strategic and tactical variables

$\forall q \in Q^n, \; n \in \mathcal{N}, \; j \in \mathcal{J}$

\[ z_j \cdot (x^{\ell(t_q)_i})_i \leq (z_q)_j \leq \bar{z}_j \cdot (x^{\ell(t_q)_i})_i \quad \forall i \in \mathcal{I}^l \cap \mathcal{I}^j \]  \hspace{1cm} (10)

\[ z_j \cdot (\delta^{\ell(t_q)_i})_i \leq (z_q)_j \leq \bar{z}_j \cdot (\delta^{\ell(t_q)_i})_i \quad \forall i \in \mathcal{I}^c \cap \mathcal{I}^j \]  \hspace{1cm} (11)

\[ A'^n x^n + C'^n (y^n + y^{a,n}) + D'_q z_q = h'_q \]  \hspace{1cm} (12)

\[ y^n + y^{a,n} \leq \bar{y} \]  \hspace{1cm} (13)
Constraints for activity in tactical node $q$ given the available infrastructure elements in the system for strategic node $n$, $\forall q \in Q^n, \ n \in N$

\[
\left( \sum_{\ell \in L^n} w_\ell B_\ell z_\ell : t_q = t_n + (B_{\sigma q}^q z_{\sigma q}) : t_q > t_n + B_q^q z_q = h_q \right) 
\]
\( E = \{1, 2, 3\}, \ T = \{1, \ldots, 9\}, \ T_2 = \{4, 5, 6\}, \ \Omega = \{4, \ldots, 9\}, \ \Omega^3 = \{8, 9\} \)

\( G = \{0, \ldots, 9\}, \ Q = \{01, \ldots, 99\}, \ Q_2 = \{21, \ldots, 29\}, \ Q_2^1 = \{24, 25, 26\}, \ L^2 = \{27, 28, 29\} \)

\( e(0) = 1, e(3) = 2, \ \sigma(2) = 0, \ \mathcal{A}_2 = \{0, 2\}, \ \bar{A}_2^{L} = \{21, 24, 27\} \)

**Figura:** Strategic multistage scenario tree with tactical multi-period graphs rooted with strategic nodes
TYPOLOGIES OF DECOMPOSITION ALGORITHMS
Benders Decomposition (BD) methodology (Benders, NM’62). Slyke, Wets SIAM’69 is the well-known first published algorithm in the subject. See also Aranburu et al. TOP’12; Lumbreras & Ramos WE’13; Qi & Sen MP’16, among many others. The (nested) version for multistage problems, Birge MP’95.


Multistage Clustering Lagrangean Decomposition (MCLD) heuristic methodology. See Mahlke PhD thesis 2011; Queiroz & Morton ORL’13; LFE et al. COR’15, COR’15a, COR’16; LFE et al. COAP’18.
Regularization. See Mulvey & Ruszczynski OR’95; Ruszczynski MOR’95; Ruszczynski & Swietanowski SIOPT’97; Li & Ierapetritou AIChE’12; Sen & Zhou EJOR’14; Asamov & Powell arXiv’15;

Progressive Hedging algorithm (PHA) for multistage primal decomposition was introduced in Rockafellar & Wets MOR’91; Watson & Woodruff CMS’11; Gade et al. MPB’16; LFE et al. Sub’18 (RCPA, RCSDPA).
Nested Stochastic Decomposition (NSD)

- SDDP methodology: Pereira & Pinto WRR’85, MP91; Ruszczynski, MP’93.
- Markovian scen tree: Cristobal, LFE & Monge COR’09; LFE, Monge & MoralesRomero TS’13; Aldasoro et al. TOP’15; Zou, Ahmed & Sun MP’18; LFE, Monge & Rodríguez-Chía Sub’19.
- CVaR risk averse measure: Philpott & de Matos EJOR’12; Philpott, de Matos & Finardi OR’13; Shapiro et al. EJOR’13; Guigues COAP’14; Kozmik & Morton MP’15.
- SD risk averse measures: LFE, Monge & RomeroMorales COR’15 (TSD); LFE, Monge & RomeroMorales COR’18 (TSD/ECSD).
Multistage cluster primal decomposition. Alonso-Ayuso, LFE & Ortuño EJOR’03; LFE et al. COR’10; Mahlke PhD thesis 2011; LFE et al. COR’12, EJOR’16, COR’18; Pages-Bernaus, Perez-Valdes & Tomasgard EJOR’13; Sandicki, Kong & Schaefer MP’13; Sandicki & Ozaltin OptimOnline ’14; Zenarosa, ’14; Boland et al.’16: LFE et al. Sub’18 (ENDO-ECSD); Baptista, LFE & Monge To be sub’19 (FFM: SFRR / SLRR)
BRIEF REF TO OUR OWN DECOMPOSITION METHODS FOR MULTISTAGE STOCHASTIC MIXED 0-1 PROBLEMS
Exact multistage decomposition methods

- Lagrangean lower bound and feasible solns providers:
  - MCLD-RN (LFE, Garín, Pérez & Unzueta COR’15)
  - MCLD-TSD (LFE, Garín & Unzueta COR’17)
  - MCLD-SLOC (LFE, Garín, Pizarro & Unzueta COAP’18)

- Exact Benders (Aramburu, LFE, Garín & Pérez TOP’12)

- Exact Branch-and-Fix Coordination:
  - BFC risk neutral (Alonso-Ayuso, LFE & Ortuño EJOR’03)
  - BFC risk neutral (LFE, Garín, Merino & Pérez COR’12)
  - PC-BFC (Aldasoro, LFE, Merino & Pérez COR’13)
  - BFC-TSD (LFE, Garín, Merino & Pérez EJOR’15)
• ELP (Beltrán-Royo, LFE, Monge & Rodríguez-Revines COR’14)
• PC SDP (Aldasoro, LFE, Merino, Monge & Pérez TOP’14)
• SDP-SD (LFE, Monge & Romero.Morales COR’15)
• PC DBFC-RN (Aldasoro, LFE, Merino, Monge & Pérez EJOR’17)
• SDP-TSD/ECSD (LFE, Monge, Romero.Morales COR’17)
• CDDA ENDO-ECSD(LFE, Garín, Monge & Unzueta COR’18)
• RCPA / RCSDPA ENDO-ECSD(LFE, Garín, Monge & Unzueta Sub’18)
• NSD-SE-NEP-RN(LFE, Monge & Rodrígez-Chía Sub’19).
• SFRR/SLRR(Baptista & LFE To be sub’19).
• NSD-SE-H-NEP-RN(Corberán, LFE, Monge, Peiró & Saldanha-da-Gama To be sub’19).
NSD versions for stage-independent uncertainties:

- SDDP for SLP, for solving large-sized LP instances, providing lower & upper bounds:
Pereira & Pinto WRR’85, MP’91 for Risk Neutral, and many others.
Philpott & de Matos EJOR’12; Philpott, de Matos & Finardi OR’13 with time-consistent and coherent measures

- SDDiP for SMIP, for solving large-sized mixed 0-1 instances with 0-1 state vars (finite convergence with probability one).
  Zou, Ahmed & Sun MP’18

Finite convergence with probability one NSD version for stage-dependent uncertainties:

- NSD for MSMIP, with 0-1 state vars.
Some refs. on NSD methodology (c)

Matheuristic NSD versions for mixed 0-1 state vars with stage-dependent uncertainties:

- **Markovian process** (influential variables from immediate ancestor node), for MSMIP:
  - SDP-RN. Cristóbal, LFE & Monge COR’09.
  - NSD-SE-NEP-RN. LFE, Monge & Rodríguez-Chía Sub’19.

- **Non-Markovian process** (influential variables from ancestor nodes), for MSMIP:
  - SDP-TSD, with Time Stochastic Dominance functional. LFE, Monge & RomeroMorales COR’15
  - SDP-ECSD, with Expected Conditional Stochastic Dominance functional. LFE, Monge & RomeroMorales COR’18
Our experience on large-scale stochastic optimization problems under uncertainty. Private sector

Production and Manufacturing sector
- production planning, sequencing and scheduling
- product selection
- plant capacity expansion planning
- vendor selection of key raw material
- strategic Supply Chain Management (SCM)
- tactical SCM
- closed-up SCM
- facilities selection and customers assignment in static and dynamic environments
Our experience on large-scale stochastic optimization problems under uncertainty. Private sector (c)

Energy sector

- power maintenance and operation
- generation and transmission capacity expansion planning and renewable energy sources location
- ‘smart energy’: prosumers (auto-production & consumption) planning management for buildings / Institutions
- tactical portfolio planning in Natural Gas SCM
- oil, hydrocarbon and chemical supplying
- oil refinery scheduling, and blending
- transformation and distribution logistics
- capacitated de-location of gas / oil stations in market restructuring
Our experience on large-scale stochastic optimization problems under uncertainty. Private sector (c)

Transport and Communications sector
- telecommunications network design planning
- air traffic flow management and air conflict reduction
- revenue management
  (ticket reservation: air flight, rent-a-car, hotel, etc.)
- Toll Assignment Problem
- p-median location / assignment Problem
- Hub-based network expansion planning and route assignment
Our experience on large-scale stochastic optimization problems under uncertainty. Private sector (c)

Finance sector
- assets / liabilities management
- financial derivatives structuring
- credit scoring
- customer management
- capacitated de-location of financial branches restructuring
- Multi-period multi-product advertising budget allocation
Natural resources

- cluster location in multi-period copper extraction planning in mining
- forestry harvesting planning and related road construction selection
- land irrigation scheduling
Our experience on large-scale stochastic optimization problems under uncertainty. Public sector

- air pollution reduction planning management
- pattern recognition (today, so-named Machine Learning) of satellite-driven big data for assessing crops volume, etc.
- water resources management planning
- national road network expansion system / budget allocation
- police stations location setting;
  police groups selection and assigning
- penitentiary network expansion system
- rapid transit network design and multi-period expansion planning
Our experience on large-scale stochastic optimization problems under uncertainty. Public sector (c)

- resources allocation planning and site location for natural disaster relief
- forest fire effect mitigation planning
Our papers on mixed 0-1 optimization under uncertainty, SMIP


A high combinatorial problem, RTND (Rapid Transit Network Design). Only 0-1 variables.

9 RTND nodes, 15 edges, 72 passenger groups.

4 stages.
Some computational experience. RTND (c)

- HW/SW platform: a WS Intel Xeon E5-2620, 2.4Ghz 6 cores, RAM 64GB.
- GAMS 24.5.6, CPLEX 12.6.2, optimality gap: 0.05%.
- 4 stages, 40 strategic nodes (1 + 3 + 9 + 27), 320 operational nodes (8 for each strategic one).
  
  Highest dimensions: m=2,641,429; n01=1,663,504.
  
  Decomposition algo: FLAggA (Fix-Lazy Aggregate /de-aggregate Algo).
  
  OG %=1.56, 3.58, 3.81 (but not by its own means).
  
  time (seconds): 1606, 16366, 43025 (11h 57 m).
  
  $\tau = 2, 1, 0$.

- Cadarso, LFE & Marín EJOR’18
Figura: Strategic multistage scenario tree with operational two-stage scenario trees rooted with strategic nodes
A difficult mixed 0-1 problem, FORESTRY (Forest harvesting & road building).

29 harvest forest stands, 3 products, 7 markets, 43 nodes, 11 potential roads, 23 and 33 existing roads in dirt and gravel, resp.

3 stages, 2 periods each stage.
Some computational experience. FORESTRY (c)

- HW/SW platform: a WS under the Linux operating system (version Ubuntu GNU/linus 14.04.1) with 64 bits, 2 processors Intel(R) Xeon(R) CPU E5-2630 @ 2.3 GHz, 64 Gb of RAM DDR3 1600MHz ECC and 24 virtual cores.

- GAMS 24.3.2, CPLEX 12.6.1, optimality gap:1 %.

- 3 stages, 2 periods each stage, 85 strategic nodes $(1 + 12 + 72)$, 8 tactical nodes each strategic one, 2 period 680 uncertain situations $(85 \times 8)$.

42,345 scenario nodes and 36,864 scenarios

Highest dimensions: $m=641,606$. $nc=1,391,949$, $n01=51,584$

OG % in testbed versus WS varies between 1.71 % and 6.38 %. 15h limit, time from 5h 30 to 15h.

- Alonso-Ayuso, LFE, Guignard & Weintraub, Sub’18.
Figura: Strategic multistage scenario tree with tactical multi-period graphs rooted with strategic nodes