basque center for applied mathematics

Calculus of Variations and applications to Solid Mechanics

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Abstract: We start by presenting Cauchy's equation of motion, and the equations of nonlinear elasticity. Then, we interpret those equations as a minimization problem, and prove existence of minimizers. We will put examples of realistic constitutive equations, with emphasis on isotropic materials. Then, we present the notion of quasiconvexity as a necessary and sufficient condition for lower semicontinuity in the vectorial calculus of variations. As an aside, we make some comments on the scalar case. Finally, we show how variational models can explain phase transitions in crystal solids, and also the shape-memory effect.

Programme:

1. Introduction to Solid Mechanics. The equations of Elasticity.
2. Hyperelasticity. Nonlinear elasticity as a minimization problem. Polyconvexity. Existence of minimizers.
3. Constitutive equations. Isotropic materials.
4. Quasiconvexity and lower semicontinuity. Aside on the scalar case.
5. Phase transitions in crystal solids. Compatibility of gradients. The shape memory effect.

Bibliography:

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[3] Ph. G. Ciarlet, *Mathematical elasticity. Vol. I. Three-dimensional elasticity.* Studies in Mathematics and its Applications **20**. North-Holland. Amsterdam, 1988.

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